Econ 101A – Final exam
May 14, 2013.

Do not turn the page until instructed to.
Do not forget to write Problems 1 in the first Blue Book and Problems 2, 3 and 4 in the second Blue Book.

Econ 101A – Final Exam
May 14, 2013.

Do not forget to write Problems 1 in the first Blue Book and Problems 2, 3 and 4 in the second Blue Book. Good luck on solving the problems!

Problem 1. Second-price auction (85 points) Two individuals participate in a sealed bid auction. Individual 1 values the good v_1 and individual 2 values it v_2. Assume $v_1 > v_2$. Assume that both individuals know their own value, as well as the value for the opponent. The two individuals write their valuation inside a sealed envelope and submit it to the auctioneer simultaneously (that is, they cannot observe the other bidder's bid). The auctioneer assigns the good to the individual with the highest bid at the *second* highest price. (In the case of tied bids, the auctioneer assigns the good with probability 0.5 to the first player) Remember, this is to a first approximation the auction format used by eBay. Formally, denote by b_i the bid of individual i. The payoff function of individual i is

$$u_i(b_i) = \begin{cases} v_i - b_{-i} & \text{if} \quad b_i > b_{-i} \\ (v_i - b_{-i})/2 & \text{if} \quad b_i = b_{-i} \\ 0 & \text{if} \quad b_i < b_{-i} \end{cases} \tag{1}$$

where b_{-i} is the bid of the other player.

1. Explain why the payoff function is given by the expression (1). (5 points)
2. Give the general definition of an equilibrium in dominant strategies and of Nash Equilibrium. (10 points)
3. Show that, no matter what player $-i$ bids, it is always (weakly) optimal for player i to bid v_i, that is, $(b_1^*, b_2^*) = (v_1, v_2)$ is an equilibrium in dominant strategies (20 points)
4. Using the definitions in (1), conclude that $(b_1^*, b_2^*) = (v_1, v_2)$ is a Nash Equilibrium. (10 points)
5. Assume now that players play sequentially. Player 1 bids b_1 first, and player 2 bids next after observing the bid b_1. How do we find the sub-game perfect equilibria in this case? (10 points)
6. Is it a sub-game perfect equilibrium that player 1 bids $b_1^* = v_1$ and player 2 bids $b_2^* = v_2$, no matter what the history? Go through all the steps of the argument. (15 points)
7. Consider now a first-price simultaneous sealed bid auction with two players, that is, the top bidder pays his bid. The payoff function of player i is

   $$u_i(b_i) = \begin{cases} v_i - b_i & \text{if} \quad b_i > b_{-i} \\ (v_i - b_i)/2 & \text{if} \quad b_i = b_{-i} \\ 0 & \text{if} \quad b_i < b_{-i}. \end{cases}$$

   Show that $(b_1^*, b_2^*) = (v_1, v_2)$ is NOT a Nash Equilibrium for the case $v_1 \neq v_2$. [Do not attempt to characterize the Nash equilibria, just show that something is not a Nash equilibrium] (15 points)

Problem 2. Short questions. (70 points)

1. **Cost curves.** For each of the following cost functions, plot as well as derive analytically the marginal cost $c'(y)$, the average cost $c(y)/y$ and the supply function $y^*(p)$:

   (a) $c(y) = 10y^2$ (10 points)

   (b) $c(y) = 10 + 10y^2$ (10 points)

   (c) $c(y) = 5y$ (10 points)

   (d) For each of the cost functions above, say what the returns to scale are for the underlying technology and whether there is a fixed cost. (10 points)

2. **Game Theory.** Compute the pure-strategy and mixed strategy equilibria of the following coordination game. Call u the probability that player 1 plays Up, $1-u$ the probability that player 1 plays Down, l the probability that Player 2 plays Left, and $1-l$ the probability that Player 2 plays Right. (30 points)

1\2	Left	Right
Up	3, 3	1, 1
Down	1, 1	2, 2

Problem 3. Monopoly and Duopoly. (90 points). Initially there is one firm in a market for cars. The firm has a linear cost function: $C(q) = q$. The market inverse demand function is given by $P(Q) = 10 - Q$.

1. What price will the monopolist firm charge? What quantity of cars will the firm sell? (10 points)
2. Plot the solution graphically: derive and plot the marginal cost, the marginal revenue, and the demand curve, and find the solution. (10 points)
3. How much profit will the firm make? (5 points)
4. Now, a second firm enters the market. The second firm has an identical cost function. What will the Cournot equilibrium output for each firm be? Compare the quantity produced and the price to the monopoly case (10 points)
5. Similarly to what you did in point (2) derive the Cournot solution graphically by plotting the best response curves in a graph with q_1 in the x axis and q_2 in the y axis – how do you find the equilibrium? (10 points)
6. What is the Stackelberg equilibrium output for each firm if firm 2 enters second? (10 points)
7. How much profit will each firm make in the Cournot game? How much in Stackelberg? (10 points)
8. Using a graphical plot of the demand function, derive a formula for the consumer surplus as the area below the demand curve and above the equilibrium price. Derive it for the cases of monopoly, Cournot and Stackelberg. Which yields the higher consumer surplus? (15 points)
9. Debate this assertion in light of your answer at the point above: 'Market power generates consumer losses because it leads to a higher market price. In particular, the welfare losses are linearly increasing in the market price.' (10 points)

Problem 4. Worker Effort and Altruism. (90 points) Consider the case of case of a firm with employees. The employee chooses the effort e. The firm pays the employees both a salary W and a piece-rate w times the units of effort e. Unlike in the moral hazard case we dealt with in class, in this case the effort is observable by the firm. The firm earns revenue pe for every unit e produced by the worker. The timing is such that the firm first sets the pay package (W, w) and then the worker chooses the optimal effort e. The worker has linear utility over money (that is, is risk-neutral) and thus gains utility $W + we$ from earnings at the workplace. In addition, the worker pays a cost of effort $ce^2/2$ (with $c > 0$) from exerting effort at the workplace. Hence the worker maximizes

$$\max_e W + we - ce^2/2$$

1. Solve for the optimal e^* of the worker as a function of W, w, and c. (10 points)

2. How does the optimal effort of the worker depend on the piece rate w? How about on the flat pay W? How about on the cost parameter c? Discuss the intuition. (10 points)

3. Consider now the problem of the firm. Remember that the firm revenue is pe. Thus the firm seeks to maximize

$$\begin{aligned} & \max_{W,w} pe - W - we \\ s.t. e^* &= e^*(W, w) \end{aligned}$$

   The firm chooses optimally the piece rate w and the flat-pay W subject to $w \geq 0$ and $W \geq 0$. Plug into the maximization problem the e^* you derived above. First, use a qualitative argument to find the optimal W^*. Then take first order conditions with respect to w. What is the solution? [If you have trouble here, just skip to the next point] (10 points)

4. Let's go back now to the employee problem and consider the case in which the worker is altruistic towards the employer by putting a weight α on the profits of the firm, with $0 < \alpha < 1$. In this case, the worker solves the problem

$$\max_e W + we - ce^2/2 + \alpha [pe - W - we].$$

   Derive the optimal effort e^*_α in this case. (10 points)

5. Now we compare the optimal effort with altruism e^*_α derived in point (4) for $0 < \alpha < 1$ to the optimal effort without altruism e^* derived in point (1). In doing the comparison, hold constant c, p, and also w, and W. Also, assume $p > w$. Which is larger, e^*_α or e^*? (Remember $0 < \alpha < 1$) Discuss the intuition. (10 points)

6. How do the two efforts e^* and e^*_α respond to an increase in the value of the product p? Compare the derivatives of e^* and e^*_α with respect to p . Discuss the intuition. (10 points)

7. How do the two efforts e^* and e^*_α respond to an increase in the value of the piece-rate w? Compare the derivative of e^* and e^*_α with respect to w and discuss which effort is more sensitive to the piece rate. Discuss the intuition. (10 points)

8. Consider now the case of a social planner who aims to maximize overall welfare in the economy. This particular social planner wants to maximize the sum of the utility of the employee and of the firm. Therefore, this planner maximizes

$$\left(W + we - ce^2/2\right) + (pe - W - we).$$

   Find the solution for e^*_{SP} which maximizes this expression. (10 points)

9. For which value of altruism α does the solution e^*_α equate the one set by the social planner? Explain the intuition. (10 points)