Econ 101A – Final Exam
We May 9, 2012.

You have 3 hours minutes to answer the questions in the final exam. We will collect the exams at 2.30 sharp. Show your work, and good luck!

Problem 1. Utility Maximization. (45 points) Consider the following utility maximization problem:

$$\begin{aligned} \max_{x,y} u(x,y) &= \alpha \log(x) + (1-\alpha)\log(y) \\ \text{s.t. } p_x x + p_y y &= M, \end{aligned}$$

with $0 < \alpha < 1$. As usual, interpret p_x as the price of good x, p_y as the price of good y, and M as income.

1. Write down the Lagrangean function and derive the first order conditions for this problem with respect to x, y, and λ. (5 point)
2. Solve explicitly for x^* and y^* as a function of p_x, p_y, M,and α. (10 points)
3. How do these solutions for x^* and y^* compare to the solution for a standard Cobb-Douglas utility function $u(x,y) = x^\alpha y^{1-\alpha}$? Explain as clearly as you can the reasons for the relationship between the solutions for these two utility functions. (10 points)
4. Notice that the utility function is defined only for $x > 0, y > 0$. Does your solution for x^* and y^* satisfies these constraints? What assumptions you need to make about p_x, p_y and M so that $x^* > 0$ and $y^* > 0$? (5 points)
5. Now, consider the fact that the government needs to raise extra revenue to pay for an expensive war. It is considering two possibilities: an income tax t_s and an indirect tax t_i. The utility maximization problem becomes

$$\begin{aligned} \max_{x,y} u(x,y) &= \alpha \log(x) + (1-\alpha)\log(y) \\ \text{s.t. } p_x(1+t_i)x + p_y(1+t_i)y &= M(1-t_s). \end{aligned}$$

   Interpret the revised budget constraint (5 points)
6. Solve for the revised solutions for x^* and y^* and provide intuition for the effects of the added taxes on optimal consumption. (5 points)
7. Solve for the share of income s spent on good x, that is, in this case, $s = p_x(1+t_i)x^*/M(1-t_s)$. How is this share s affected by the two types of taxes? Interpret. (5 points)

Problem 2. Shorter Questions. (55 points)

Problem 2a (*Expected Utility.*) Consider an individual with utility function $u(c)$ where c is consumption, with u which satisfies $u'(c) > 0$ for all c. The individual has a job with uncertain income, she earns w_H with probability p and $w_L < w_H$ with probability $1-p$. The agents has no other income and hence consumes $c = w$.

1. Write the expected utility of the individual. (5 points)

2. The government is considering providing an insurance policy which would provide a flat wage W which is constant, rather than fluctuating across states of the world, as an alternative to getting the uncertain market wage defined above. The government would like to know what is the level of sure pay W which would make the person *indifferent* between taking this insurance policy (and hence earning $\bar{W}$ for sure) and not taking it (and hence earning expected utility as in the point above). Write down the equation which implicitly defines this level $\bar{W}$. (5 points)

3. The government wants to know whether the insurance level $\bar{W}$ which makes the person indifferent is smaller than the expected wage $pw_H + (1-p)w_L$. As a consultant to the government, provide conditions under which it is indeed smaller. If you can, provide necessary and sufficient conditions. Explain as clearly as you can. (15 points)

Problem 2b (*Moral Hazard*). Consider the moral hazard (hidden action) problem which we considered in lecture. An agent is offered a contract $w = a + by$, where w is the wage, and y is the output, with $y = e + \varepsilon$, where e is (unobservable) effort and ε is noise. Remember that the agent has exponential utility which leads to the expected utility of a contract being equal to

$$EU(w) = a + be - \frac{\gamma}{2}b^2\sigma^2 - c(e)$$

4. Interpret the three parts in this expression (5 points)

5. Remember that effort is costly with $c(e)$ cost of effort which satisfies $c'(e) > 0$ and $c''(e) > 0$ for all e. Without solving for the overall problem, explain intuitively, but also as clearly as you can, why the firm will not set $b^* = 0$ in the optimum. What effort would workers choose for $b = 0$? (10 points)

6. Without solving for the overall problem, explain intuitively why the firm will set $b^* < b_{FB}$ in the optimum, where b_{FB} is the level of incentives which would achieve the optimal (i.e., first-best) level of production. (Remember that the optimal level of production is the one achieved when e is observable) (5 points)

7. In light of this, explain qualitatively what it means that the firm is facing a risk-incentive trade-off in setting the optimal piece rate b^*. (10 points)

Problem 3. Static Games and Dynamic Games. (60 points).

1. State the definition of a Nash equilibrium for a game in which there are 2 players $i = 1$ and $i = 2$, each of which with utility function $u_i(s_i, s_{-i})$, as a function of strategies s_i and s_{-i}. (5 points)

2. Consider the following 2×2 game played simultaneously:

		P2	
		L	R
P1	T	1 , 10	1 , 1
	B	2 , α	0 , 1

   First, assume $\alpha = 2$ for this and the next points, until stated otherwise. Find the pure Nash equilibrium (if any). (That is, do not worry yet about equilibria in mixed strategies. (5 points)

3. Now, Consider the dynamic game in which player 1 moves before player 2. The payoffs remain the same, draw the decision tree for this dynamic game. Write down the set of possible strategies for each player. (10 points)

4. Solve for the sub-game perfect equilibrium *strategies* and also compute the equilibrium payoff. Is there an advantage to moving first in this game (versus moving simultaneously)? How about to moving second (versus moving simultaneously)? (5 points)

5. In light of the solution of the previous point, comment on this – is this right or wrong, and why: '*Having player 1 move first advantages player 2 because Player 2 threatens to go R if player 1 goes B, and to go L if player 1 goes R. Facing this threat, player 1 goes T, because he prefers a payoff of 1 (from T,L) to a payoff of 0 (from B,R). The threat works because in equilibrium player 1 goes T, and hence never finds out whether player 2 would actually play R if player 1 were to go B. Hence, player 2 can get 10 in equilibrium*' (10 points)

6. Now consider the original static game, but from now on assume $\alpha = 0$. Find all the Nash equilibria of this game, including the mixed strategies. Denote by p the probability player 1 plays T, and by q the probability that player 2 plays L. Compute and graph the best response functions, and show where they intersect, which are the Nash equilibria (or equilibrium if there is only one). (10 points)

7. Compute the expected payoffs for each player in the Nash equilibrium above. (5 points)

8. Now, let player 1 move first again in a dynamic game. Draw the tree and solve for the sub-game perfect equilibrium *strategies* and also compute the equilibrium payoff. (5 points)

9. Is there an advantage to moving first in this game (versus moving simultaneously)? How about to moving second (versus moving simultaneously)? (5 points)

Problem 4. (Production) (65 points)

International Toys L.L.C. produces action figures for the Japanese and American markets. For one particular doll, the Auto-Action-Rambo-Teletubby doll (that's one doll, it's both lethal and huggable at the same time, a Matt Leister unique design), International Toys L.L.C. sells exclusively to Toy-Japan in Japan and Toys-R-Us in America. Toy-Japan's demand function is $D_J(p_J) = \frac{40-2p_J}{3}$ (giving inverse demand $p_J(y_J) = 20-\frac{3}{2}y_J$), and Toys-R-Us's demand is $D_A(p_A) = 16-p_A$ (giving inverse demand $p_A(y_A) = 16-y_A$), where quantity is measured in thousands of dolls. International Toys faces a cost function $C(Y) = 4+Y^2$, where Y is the total production of Auto-Action-Rambo-Teletubby dolls in thousands of units.

1. Assume first that International Toys L.L.C. sells in a perfectly competitive market and faces price p_{PC}. Derive the marginal and average cost and plot them in a graph with Y on the x axis. (5 points)

2. Derive the supply function of Auto-Action-Rambo-Teletubby dolls $Y^S(p_{PC})$. Use the graph if it helps, but make sure to write down the supply function analytically. (10 points)

3. Suppose now that International Toys L.L.C. acts as a monopolist in both countries, and can choose separate prices p_J and p_A to charge the two chain stores (third-degree price discrimination). Write down the profit function from selling in both countries as function of the quantities sold Y_J and Y_A. [Remember that the costs are given by $C(Y) = 4+(Y_J+Y_A)$] (5 points)

4. Solve the maximization problem and determine the optimal quantities sold Y_J^* and Y_A^*. Using this, determine also what prices p_J and p_A the monopolist will set. (10 points)

5. Now assume that a Japanese-American law does not allow separate pricing between the two countries, so $p_J = p_A = p$. Derive the aggregate demand $D(p) = D_J(p)+D_A(p)$ which International Toys faces. [Note: Here keep in mind that demand in each country cannot be negative, that is, $D_J(p) \geq 0$ and $D_A(p) \geq 0$, it may help to draw the demand curves and add them] (10 points)

6. Using the aggregate demand function, solve for the profit maximizing price p that International Toys will charge. [Hint: Here you can assume $p < 16$] Here it may be easier to write the profit maximization of the monopolist as maximization with respect to the price p. Find the implied sales in each country (that is, y_J and y_A). (5 points)

7. Continue assuming that International Toys L.L.C. charges one monopoly price in the two countries. Now, however, a Chinese company makes an identical toy which is also sold in America. Since consumers are indifferent between the Chinese version and the International Toys version, the price in America will drop to the price of production of the Chinese toy, which is 11. International Toys L.L.C. now has two strategies: (i) to keep selling in both markets at $p = 11$; (ii) to sell only in Japan at the profit-maximizing price. Compute the profits for each of the two possibilities and hence the overall optimum. Does the firm continue to sell in America? (10 points)

8. Has the consumer surplus increased or decreased in America as a function of the entry of the Chinese toy? [You should not need to solve for the consumer surplus to answer the question] How about in Japan? Explain. (10 points)