**Econ 101A – Final exam**
**Tu 15 December, 2009.**

Do not turn the page until instructed to.
Do not forget to write Problems 1 and 2 in the first Blue Book and Problems 3 and 4 in the second Blue Book.

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**Problem 1. Short questions.** (60 points)

1. **Cost curves.** For each of the following cost functions, plot as well as derive analytically the marginal cost $c'(y)$, the average cost $c(y)/y$ and the supply function $y^*(p)$:
   (a) $c(y) = 5y^2$ (8 points)
   (b) $c(y) = 10 + 5y^2$ (8 points)
   (c) $c(y) = 5y^{1/2}$ (8 points)
   (d) For each of the cost functions above, say what the returns to scale are for the underlying technology and whether there is a fixed cost. (6 points)

2. **Expected Utility.** An agent is considering a lottery which returns with probability $p$ a sum $M$ and with probability $1-p$ returns 0. The agents has utility function $u(x)$, where $x$ is the winning of the lottery (which is zero if he/she loses the lottery).
   (a) Write the expected value of the lottery and the expected utility of the agent (5 points)
   (b) Consider now a simplified version of the so-called St. Petersburg paradox. Consider the case in which $p = 1/n$ and $M = n^2$. For $n$ sufficiently large, this is a lottery that with very small probability pays out a very large amount of money. You can think of it as the probability of beating the odds against the casino. Compute the expected *value* (not the expected utility) of this lottery for finite $n$ and for $n$ that converges to infinity. (5 points)
   (c) How much would a risk-neutral decision marker be willing to pay to play this lottery as $n$ converges to infinity? Is this intuitively reasonable? Argue carefully the steps of this argument. (7 points)
   (d) Now assume that the agent has utility $u(x) = \sqrt{x}$. Solve for the expected utility of the lottery as $n$ converges to infinity. (6 points)
   (e) How much is this risk-averse agent willing to pay for the lottery? Does this solve the puzzle? (7 points)

**Problem 2.** (48 points + 20 extra) In this question we will assume that there are just two countries in the world (China and the US) that are relevant for mitigating the effects of global climate change. If the two countries cooperate in reducing their green house gas emissions then the potential impact of global climate change can be greatly reduced. However, if only one country reduces their greenhouse gas emissions there will be little impact on global green house gas emissions and therefor little impact on future economic damages due to climate change. It is costly in both current expenditures and forgone future economic output for each country to reduce their green house gas emissions. Thus, neither country wants to reduce their emissions unless the other country also reduces their emissions.

1. We model the decision for each country to reduce their emissions (Abate) or not to reduce their emissions (Not Abate) as a 1-time **simultaneous** decision. We can represent the choices and payoffs using the familiar normal form game setup. The US is the column player and China is the row player in the grid below. (Remember: The payoff to the US is the 1st term in each pair of payoffs in each cell and the payoff to China is the 2nd term).

| US\China | Abate | Not Abate |
|---|---|---|
| Abate | $-2, -2$ | $-6, -1$ |
| Not Abate | $-1, -6$ | $-5, -5$ |

   (a) Define as precisely as you can the meaning of a dominant strategy and what it means to have an equilibrium in dominant strategy (you can use either math notation or explain succinctly in words). Is there an equilibrium in dominant strategy for this game? Either state what the equilibrium in dominant strategy is and prove why it is an equilibrium or show that there is no dominant strategy equilibrium. (7 points)

   (b) Define as precisely as you can the definition of a Nash Equilibrium (you can use either math notation or explain succinctly in words). Is there a Nash Equilibrium (in pure strategies) in this game? Either state the Nash Equilibrium equilibrium and prove why it is an equilibrium or show that there is no Nash Equilibrium. (7 points)

   (c) Solve for *all* Nash equilibria *in mixed strategies* of the game above. Denote by $u$ (up) the probability that US abates and by $l$ (left) the probability that China abates. You need to derive this going through the steps. Plot the best responses in the usual graph relating $l$ to $u$ (and vice versa). Are you surprised about the equilibria you find given your answer in point a? (10 points)

2. Now assume that the payoffs remain as above, but that the game is now sequential, in that the US moves first and then China moves second after observing the move of the US.

   (a) Draw a tree of the game. Define as precisely as you can the definition of a Sub-Game Perfect Nash Equilibrium (you can use either math notation or explain succinctly in words). How does a Sub-Game Perfect Nash Equilibrium differ from a Nash Equilibrium? (7 points)

   (b) Solve for all the subgame perfect equilibria. (Be careful to denote the strategies correctly) (7 points)

3. Now let's go back to the assumption that the game is played simultaneously. However, let's assume that there are now $N > 1$ years in which the game above is repeated (with $N$ finite). You can think of the grid above as representing the payoffs for one year. At the end of each year, US and China observe the abatement decision of the other country and then again decide simultaneously. Each country maximizes the sum of payoffs over the different years. What are the Subgame Perfect equilibria in this case? Provide all the steps for the argument. (10 points)

4. (Extra credit) Consider now a game that has the same payoff structure as in part 1 except that it is repeated an infinite number of times. After each year both countries decide again whether to Abate or Not Abate. Each country can observe whether the other country Abated or Did Not Abate in the previous year before making their next decision. Each country wants to maximize the Present Value (PV) of their infinite stream of payoffs and has the same discount rate $\delta$. (The formula for the sum of an infinite geometric series (as $t$ goes to infinity) is: $X + \delta X + \delta^2 X + ... + \delta^t X = \frac{X}{1-\delta}$) Suppose each country plays a "Grim Trigger" strategy. In this problem the "Grim Trigger" strategy is one where

each country starts in year 1 Abating and will continue to Abate as long as the other country Abates. If the other country Doesn't Abate, then in the next year the 1st country will switch to only playing Not Abate for the rest of the (infinite) game. If each country announces and plays a "Grim Trigger" strategy can there be a Sub-Game Perfect Nash Equilibrium where each country always abates in this game? Compare this to the answer in point 3. (20 points)

(Hint: Try comparing the PV for a country that follows the announced "Grim Trigger" strategy with the PV when that country deviates from the "Grim Trigger" in the 1st period)

**Problem 3. Externalities in Production** (59 points) In this exercise, we consider the problem of externalities due to pollution generated by one firm. An externality occurs when a firm does not take into account in its maximization problem the effect of one of its decisions on other firms. The idea of this problem is that each of the firms produces a pollutant that increases the costs of production of the other firm. Assume that two firms compete á la Cournot. Firm $i$ produces $q_i$ units of the good at total cost $cq_i + dq_i q_{-i}$, with $c > 0$ and $d \geq 0$. For example, the total production cost of firm 1 is $cq_1 + dq_2 q_1$. The first part of the cost function captures the standard assumption of constant marginal cost $c$, while the second part captures the fact that increased production by the competitor increases the production costs. For $d = 0$, we obtain a standard Cournot case with marginal cost of production $c$. The inverse demand function is $P(Q) = a - bQ = a - b(q_1 + q_2)$.

1. Consider first the Cournot solution. Write down the profit function for firm $i$ as a function of the production of the competitor, $q_{-i}$. Write down the first order condition of firm $i$ and solve for $q_i^*$ as a function of $q_{-i}^*$, that is, find the best response function for firm $i$, for $i = 1, 2$. (8 points)

2. Holding constant for now the production of the competitor $q_{-i}^*$, discuss what happens to the quantity produced by firm $i$, $q_i^*$, as $a$, $c$, and $d$ vary. Provide intuition for each. (5 points)

3. Find the solution for the Cournot oligopoly by requiring that both best response functions hold. In other words, solve the system of two first order conditions for the two firms. Find the solutions for $q_1^* = q_2^*$ and for $p^*$ (6 points)

4. In the plane $(q_1, q_2)$ graph the best response function of both firms for $a = b = 1$, $c = 1/2$ and $d = 0$. Indicate the Cournot Nash equilibrium in the graph. (6 points)

5. Now assume that the two firms merge together and become one monopolist. The monopolist maximizes the total profits from the two plants, that is $(a - bq_1 - bq_2)(q_1 + q_2) - cq_1 - cq_2 - 2dq_1q_2$. Write down the first order conditions with respect to $q_1$ and $q_2$. Compare these first order conditions to the ones of the Cournot case in point 1. There are two differences between the f.o.c.s in the monopoly case and in the Cournot case. Identify them. Comment and explain: 'The monopolist internalizes externalities' (10 points)

6. Find the optimum for $q_1^M$ and $q_2^M$, that is the quantity that the merged monopolist produces in each plant. (the monopolist will produce an equal quantity $q_M$ in both plants) (6 points)

7. Compare the quantities produced in monopoly and duopoly. In particular, the ratio $q^*/q^M$ should come out to be
$$\frac{q^*}{q_M} = 2\frac{2b + d}{3b + d}.$$
Show that this ratio is increasing in $d$. What is the intuition for the fact that the 'overproduction' of duopoly relative to monopoly is more accentuated when $d$ is large? (8 points)

8. Now suppose that the result of the Copenhagen Climate talks is the imposition of a tax on polluters. To simplify, each firm $i$ pays a tax $tq_i$, where $t$ is the per-unit tax. Rewrite the maximization problem for each firm in a Cournot setting. Find the level of tax such that the government by imposing a tax $t$ can induce the same production $q_{D,t}^*$ under Cournot (with taxes) as in monopoly $q_M$ (without taxes). (10 points)

9. Qualitatively discuss whether the imposition of the tax above can raise welfare. Clarify the assumptions you are making. (6 points)

**Problem 4. General Equilibrium** (55 points) Consider the case of pure exchange with two consumers. Both consumers have Cobb-Douglas preferences, but with different parameters. Consumer 1 has utility function $u(x_1^1, x_2^1) = (x_1^1)^\alpha (x_2^1)^{1-\alpha}$. Consumer 2 has utility function $u(x_1^2, x_2^2) = (x_1^2)^\beta (x_2^2)^{1-\beta}$. The endowment of good j owned by consumer $i$ is $\omega_j^i$. The price of good 1 is $p_1$, while the price of good 2 is normalized to 1 without loss of generality.

1. Assuming $\alpha = 1/2$, $\beta = 1/2$, draw the Pareto set and the contract curve for this economy in an Edgeworth box for the following two economies with different endowments (you do not need to give the exact solutions, only a graphical representation) (notice, this is different from the problem set, look at the endowments carefully)
   (a) Assume $\omega_1^1 = 3, \omega_1^2 = 1, \omega_2^1 = 1, \omega_2^2 = 3$.(6 points)
   (b) Assume $\omega_1^1 = 3, \omega_1^2 = 1, \omega_2^1 = 3, \omega_2^2 = 1$. (6 points)
   (c) What is the set of points that could be the outcome under barter in these two economies? (6 points)
2. For each consumer, compute the utility maximization problem. Solve for $x_j^{i*}$ for $j = 1, 2$ and $i = 1, 2$ as a function of the price $p_1$ and of the endowments. (6 points)
3. Now, solve analytically for the general equilibrium. Require that the total sum of the demands for good 1 equals the total sum of the endowments, that is, that $x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$. Solve for the general equilibrium price $p_1^*$. (8 points)
4. What is the comparative statics of $p_1^*$ with respect to the endowment of good 1, that is, with respect to $\omega_1^i$ for $i = 1, 2$? What about with respect to the endowment of the other good? Does this make sense? What is the comparative statics of $p_1^*$ with respect to the taste for good 1, that is, with respect to $\alpha$ and $\beta$? Does this make sense? (8 points)
5. What is the general equilibrium price $p_1^*$ and allocations $x_j^{i*}$ for the set of parameters in point 1.a and 1.b? Relate to your answer in points 1.a and 1.b and find these points on the graphs. (10 points)
6. What does the previous question suggest is the relationship between the contract curve and the Walrasian equilibrium? (5 points)