

Economics 101A

(Lecture 24)

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Outline

1. Walrasian Equilibrium II
2. Example of General Equilibrium
3. Existence and Welfare Theorems
4. Asymmetric Information: Introduction

1 Walrasian Equilibrium II

- **Walrasian Equilibrium.** $((x_1^{1*}, x_2^{1*}), (x_1^{2*}, x_2^{2*}), p_1^*, p_2^*)$ is a Walrasian Equilibrium if:

- Each consumer maximizes utility subject to budget constraint:

$$(x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i((x_1^i, x_2^i))$$
$$s.t. p_1^* x_1^i + p_2^* x_2^i \leq p_1^* \omega_1^i + p_2^* \omega_2^i$$

- All markets clear:

$$x_j^{1*} + x_j^{2*} \leq \omega_j^1 + \omega_j^2 \text{ for all } j.$$

- Compare with partial (Marshallian) equilibrium:
 - each consumer maximizes utility
 - market for good i clears.
 - (no requirement that all markets clear)

- How do we find the Walrasian Equilibria?

- **Graphical method.**

1. Compute first for each consumer set of utility-maximizing points as function of prices
2. Check that market-clearing condition holds

- *Step 1.* Compute optimal points as prices p_1 and p_2 vary

- Start with Consumer 1. Find points of tangency between budget sets and indifference curves

- Figure

- **Offer curve** for consumer 1:

$$(x_1^{1*}(p_1, p_2, (\omega_1, \omega_2)), x_2^{1*}(p_1, p_2, (\omega_1, \omega_2)))$$

- Offer curve is set of points that maximize utility as function of prices p_1 and p_2 .

- Then find offer curve for consumer 2:

$$(x_1^{2*}(p_1, p_2, (\omega_1, \omega_2)), x_2^{2*}(p_1, p_2, (\omega_1, \omega_2)))$$

- Figure

- *Step 2.* Find intersection(s) of two offer curves
- Walrasian Equilibrium is intersection of the two offer curves!
 - Both individuals maximize utility given prices
 - Total quantity demanded equals total endowment

- Relate Walrasian Equilibrium to barter equilibrium.
- Walrasian Equilibrium is a subset of barter equilibrium:
 - Does WE satisfy Individual Rationality condition?
 - Does WE satisfy the Pareto Efficiency condition?
- Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.

2 Example

- Consumer 1 has Leontieff preferences:

$$u(x_1, x_2) = \min(x_1^1, x_2^1)$$

- Bundle demanded by consumer 1:

$$\begin{aligned} x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \\ &= \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} \end{aligned}$$

- Graphically

- Comparative statics:

- increase in ω

- increase in p_2/p_1 :

$$\begin{aligned} \frac{dx_1^{1*}}{dp_2/p_1} &= \frac{\omega_2^1 (1 + (p_2/p_1)) - (\omega_1^1 + (p_2/p_1) \omega_2^1)}{(1 + (p_2/p_1))^2} = \\ &= \frac{\omega_2^1 - \omega_1^1}{(1 + (p_2/p_1))^2} \end{aligned}$$

- Effect depends on income effect through endowments:

- * A lot of good 2 \rightarrow increase in price of good 2 makes richer

- * Little good 2 \rightarrow increase in price of good 2 makes poorer

- Notice: Only ratio of prices matters (general feature)

- Consumer 2 has Cobb-Douglas preferences:

$$u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5}$$

- Demands of consumer 2:

$$x_1^{2*} = \frac{.5 (p_1 \omega_1^2 + p_2 \omega_2^2)}{p_1} = .5 \left(\omega_1^2 + \frac{p_2}{p_1} \omega_2^2 \right)$$

and

$$x_2^{2*} = \frac{.5 (p_1 \omega_1^2 + p_2 \omega_2^2)}{p_2} = .5 \left(\frac{p_1}{p_2} \omega_1^2 + \omega_2^2 \right)$$

- Comparative statics:

- increase in $\omega \rightarrow$ Increase in final consumption

- increase in $p_2/p_1 \rightarrow$ Unambiguous increase in x_1^{2*} and decrease in x_2^{2*}

- Impose Walrasian equilibrium in market 1:

$$x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$$

This implies

$$\frac{\omega_1^1 + (p_2/p_1)\omega_2^1}{1 + (p_2/p_1)} + .5 \left(\omega_1^2 + \frac{p_2}{p_1}\omega_2^2 \right) = \omega_1^1 + \omega_1^2$$

or

$$\frac{.5 - .5(p_2/p_1)}{1 + (p_2/p_1)}\omega_1^1 + \frac{.5(p_2/p_1) + .5(p_2/p_1)^2 - 1}{1 + (p_2/p_1)}\omega_2^1 = 0$$

or

$$\left(\omega_1^1 - 2\omega_2^1 \right) + \left(\omega_1^1 + \omega_2^1 \right) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0$$

- Solution for p_2/p_1 :

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\left(\omega_1^1 + \omega_2^1\right)^2 - 4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

- Some complicated solution!

- Problem set has solution that is easier to compute (and interpret)

3 Existence and Welfare Theorems

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex
- Is Walrasian Equilibrium always unique? Not necessarily
- Is Walrasian Equilibrium efficient? Yes.

- **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).
- Figure

- **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation $((x_1^1, x_1^1), (x_1^2, x_2^2))$ there exists some endowment (ω_1, ω_2) such that $((x_1^1, x_1^1), (x_1^2, x_2^2))$ is a Walrasian Equilibrium for endowment (ω_1, ω_2) .
- Figure

- Significance of these results:
 - First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
 - BUT: problems with externalities and public good
 - BUT: what about distribution?

- Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
- But redistribution is hard to implement, and distortive.

4 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 641-645
- Common economic relationship
- Contract between two parties:
 - Principal
 - Agent
- Two parties have asymmetric information
 - Principal offers a contract to the agent
 - Agent chooses an action
 - Action of agent (or his type) is not observed by principle

- Example 1: *Manager and worker*
 - Manager employs worker and offers wage
 - Worker exerts effort (not observed)
 - Manager pays worker as function of output

- Example 2: *Car Insurance*
 - Car insurance company offers insurance contract
 - Driver chooses quality of driving (not observed)
 - Insurance company pays for accidents

- Example 3: *Shareholders and CEO*
 - Shareholders choose compensation for CEO
 - CEO puts effort
 - CEO paid as function of stock price

- In all of these cases (and many more!), common structure
 - Principal would like to observe effort (of worker, of CEO, of driver)
 - Unfortunately, this is not observable
 - Only a related, noisy proxy is observable: output, accident, success
 - Contract offered by principal is function of this proxy
- This means that occasionally an agent that put a lot of effort but has bad luck is ‘punished’
- Also, agents that shirked may instead be compensated
- These principle-agent problems are called *hidden action* or *moral hazard*

- Second category (next lecture): *hidden type* or *adverse selection*
- Example 1: *Manager and worker*
 - Manager employs worker and offers wage
 - Worker can be hard-working or lazy
- Example 2: *Car Insurance*
 - Car insurance company offers insurance contract
 - Drivers ex ante can be careful or careless
- Example 3: *Shareholders and CEO*
 - Shareholders choose compensation for CEO
 - CEO is high-quality or thief

- Problem is similar (action is not observed), but with a twist
 - *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives
 - *Hidden type*: agent's behavior is not affected by incentives, but by her type
- Different task for principal:
 - *Hidden action*: Principal wants to incentivize agent to work hard
 - *Hidden type*: Principal wants to make sure to recruit 'good' agent, not 'bad' one
- Two look similar, but analysis is different
- Start from *Hidden Action*

5 Next lecture

- Asymmetric Information
- Moral Hazard