Problem set on Behavioral Finance – Due on April 22

Question (Behavioral Finance – Noise Traders)

This Question elaborates on the DeLong, Shleifer, Summers, Waldman (JPE 1990) paper. The idea is to consider what happens to asset prices when a share of the traders have irrational expectations about future dividends. These traders in the literature are called noise traders. Consider the set-up of DeLong, Shleifer, Summers, Waldman (JPE 1990), which I summarize here. There is a share of noise traders,\(\lambda_n\) of arbitrages. The arbitrages are risk averse and have a short horizon, that is, they have to sell the shares at the end of period to consumer. Formally, consider an OLG model where in period 1 the agents have initial endowment and trade, and in period 2 they consume. There are two assets with identical dividend\(\rho\) : a safe asset with perfectly elastic supply, whose price we will set to 1 (numeraire), and an unsafe asset in inelastic supply (1 unit) and a price\(p\) that is determined by supply and demand. We denote the demand for unsafe asset: \(\lambda^a\) and \(\lambda^n\). The investors have CARA utility function \(U(w) = -e^{-2(\gamma w)}\) with \(w\) being the wealth in period 2, which is what the investor consumes. Compared to the arbitrages, the noise traders believe that in period \(t\) the asset with higher return \(\rho_t\).

a) Assume that the wealth\(w\) is distributed \(N(\mu, \sigma^2_w)\). Show that maximizing \(EU(w)\) is equivalent to maximizing \(w - \gamma \sigma^2_w\), that is, the problem reduces to one of mean-variance optimization.

b) Show that arbitrages maximize the problem
\[
\max(w_t - \lambda^a_t p_t)(1 + r) + \lambda^a_t (E_t[p_{t+1}] + r) - \gamma (\lambda^a_t)^2 Var_t(p_{t+1}).
\]
Derive the first order condition and solve for \(\lambda^a_t^*\).

c) Show that noise traders maximize the problem
\[
\max(w_t - \lambda^n_t p_t)(1 + r) + \lambda^n_t (E_t[p_{t+1}] + \rho_t + r) - \gamma (\lambda^n_t)^2 Var_t(p_{t+1}).
\]
Derive the first order condition and solve for \(\lambda^n_t^*\).

d) Discuss how the optimal demand of the risky asset will depend on the expected returns \((r + E_t[p_{t+1}] - (1 + r)p_t)\), on risk aversion \((\gamma)\), on the variance of returns \((Var_t(p_{t+1}))\), and on the overestimation \(\rho_t\).

e) Under what conditions noise traders hold more of the risky asset than arbitrages do?

f) To solve for the price\(p_t\), we impose the market-clearing condition \(\lambda^n \mu + \lambda^a (1 - \mu) = 1\). Use this condition to solve for \(p_t\) as a function of \(E_t[p_{t+1}], Var_t(p_{t+1})\), and the other parameters.

g) To solve for the equilibrium, assume that the average price is not time-varying (that is, \(E_t[p_t] = E_t[p_{t+1}] = E[p]\)), and take expectations on the right and left of the expression

\[
\]
for \( p_t \). Solve for \( E[p] \), and substitute into the expression for \( p_t \). Now, use this expression to compute \( Var[ p_t ] \). Finally, substitute the expression for \( Var[ p_t ] \) in the updated expression for \( p_t \). In the end, you should obtain

\[
p_t = 1 + \frac{\mu (\rho_t - \rho^*)}{1 + r} + \frac{\mu \rho^*}{r} - \frac{2\gamma \mu^2 \sigma^2_{\rho}}{r(1 + r)^2}. \tag{1}
\]

h) Analyze how the price \( p \) responds to an increase in \( \mu \), in \( \rho_t \), in \( \rho^* \), in \( \gamma \), and in \( \sigma^2_{\rho} \). For each of these terms provide intuition.

i) In light of expression (1), comment on the following statement: ‘Biases of investors do not matter in financial markets because they do not affect prices’. What are the key assumptions in the set-up driving this result?

j) (Extra credit) The returns for traders of group \( j \) (\( j = a, n \)) are \( R^j = (w_t - \lambda^a_t p_t)(1+r) + \lambda^a_t (p_{t+1} + r) - w \). Straightforwardly, this implies that \( \Delta R = R^n - R^a = (\lambda^n_t - \lambda^n_t)(p_{t+1} + r - p_t (1 + r)) \). Solve that \( E(\Delta R|\rho_t) \), that is, the expected return to noise traders relative to arbitrageurs conditional on \( \rho_t \), is

\[
E(\Delta R|\rho_t) = \rho_t - \frac{(1 + r)^2 \mu^2}{2\gamma \mu \sigma^2_{\rho}}. \tag{2}
\]

k) Using (2), discuss whether it is possible that noise traders outperform in expectations the arbitrageurs, and under what conditions.

l) What is the intuition for why noise traders may outperform in expectations the arbitrageurs?

m) Does your answer in (l) imply that noise traders can achieve a higher expected utility than arbitrageurs? (Note: I intend when utility is evaluated with the actual returns, not with naive expectations that noise traders have)