

**Econ 101A – Midterm 2**  
**Th 10 April 2014.**

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 11.00 sharp. Show your work, and good luck!

**Problem 1. Monopoly with Fixed Cost.** (55 points + extra credit). Consider the case of a monopolist producing quantity  $Q$  with total cost  $C(Q) = F + cQ$ , with  $c > 0$  and  $F \geq 0$ .

1. Comment briefly on the cost function. Does it involve a fixed cost? (5 points)
2. Determine the marginal cost function  $C'_q$  and the average cost function  $C(Q)/Q$ , and plot the two functions in a graph with x-axis quantity  $Q$  and y-axis cost/price. (5 points)
3. Assume now that aggregate demand is given by the linear (inverse) demand function  $p(Q) = A - bQ$ , with  $A > c$ . Draw it in the graph with the marginal cost function of point (2). If you want, assume  $A = 10$ ,  $c = 5$ ,  $b = 1$ ,  $F = 1$  (just for the graph) Also, derive the marginal revenue  $MR$  function, by differentiating with respect to  $Q$  the revenue  $R$ , and draw the  $MR = R'$  function in the graph. (5 points)
4. Find graphically the solution for monopoly quantity and price (5 points)
5. Still graphically, indicate the producer surplus or profits of the monopolist. Indicate which of the two methods you are using. (5 points)
6. Now solve analytically for the monopoly solution, by maximizing

$$\max_Q Q * p(Q) - C(Q) = Q(A - bQ) - F - cQ.$$

Obtain the first order conditions and solve for  $Q^*$  and  $p^*$ , the monopoly solution. (5 points)

7. Compute now the profits analytically, plugging in  $Q^*$  and  $p^*$  in the solution above. (5 points)
8. A consultant to the government claims, regarding this monopoly: ‘A big problem with monopoly in this industry is that profits are unfairly high’. Is that always true? Can you give conditions under which monopoly profits in this case would be *zero*? (10 points)
9. Continuing on the point above, can you give conditions under which monopoly would have *negative* profits? Going back to the figure, what would the average cost curve look like in that case? What would the monopolist do in that case? (5 points)
10. Given the above point (9), revise the solution at point (6) for the optimal monopoly solution. (5 points)
11. (Extra credit, do not attempt till you have done Problem 2) If we were in perfect competition, and each firm had the same cost curve as above, and faced the same demand curve as above, what would be the supply function and what would be the perfect competition equilibrium, if any? Why? (15 points)

**Solution to Problem 1.**

1. The cost function involves a constant marginal cost  $c$  and a fixed cost  $F$ .
2. The marginal cost  $C'_q = c$ , so the marginal cost is constant, and the average cost  $C(Q)/Q = F/Q + c$  is decreasing, asymptoting to  $c$  for very large  $Q$ .
3. The marginal revenue is

$$\frac{\partial}{\partial Q} (Q(A - bQ)) = A - 2bQ,$$

which is a line with the same intercept, but twice as steep.

4. The monopoly solution for quantity is at the intersection of MR and  $C'$  curves, then one finds the price on the demand curve.
5. The first method to indicate the profits is to take the integral of the area above the marginal cost and below the price  $p^*$  up until  $Q^*$ , and then subtract the fixed costs. We obtain the rectangle between  $c$  (the marginal cost) and  $p^*$ , until  $Q^*$ , and then need to subtract  $F$ . Alternatively, we take the rectangle of height equal to the difference between  $p^*$  and average costs, and base equal to  $Q^*$ .
6. The solution leads to f.o.c

$$A - 2bQ - c = 0,$$

which is just marginal revenue equal to marginal cost. The solution is

$$Q_m^* = \frac{A - c}{2b}$$

and the price is

$$p_m^* = A - bQ^* = A - b\frac{A - c}{2b} = \frac{A + c}{2}.$$

7. The profits are

$$\begin{aligned} Q(A - bQ) - F - cQ &= \frac{A - c}{2b} \left( \frac{A + c}{2} \right) - F - c\frac{A - c}{2b} \\ &= \frac{A - c}{2b} \left( \frac{A - c}{2} \right) - F = \frac{(A - c)^2}{4b} - F \end{aligned}$$

8. The statement is not always true. Monopoly profits are zero if fixed cost are such that they are equal to the difference between revenue and variable cost, i.e.,

$$F^{\pi=0} = Q_m^*(A - bQ_m^*) - cQ_m^*.$$

In this case, that occurs for

$$F^{\pi=0} = \frac{(A - c)^2}{4b}.$$

Graphically this is where the  $AC$  function is tangent to the demand function at  $Q_m^*$ .

9. Monopoly can have *negative* profits in the short-run if  $F > Q^*(A - bQ^*) - cQ^*$  or  $F > (A - c)^2 / 4b$  (equivalently: if  $AC > p$  at  $Q_m^*$ ). The average cost curve lies to the right of the demand function. In this case the monopolist does not enter the market (or exits if already in the market) and  $Q^* = 0$ .
10. The solution is

$$\begin{aligned} Q_m^* &= \frac{A - c}{2b} \text{ if } F \leq \frac{(A - c)^2}{4b} \text{ and} \\ Q_m^* &= 0 \text{ if } F > \frac{(A - c)^2}{4b} \end{aligned}$$

11. Under perfect competition, each firm's supply decision will be

$$q_{PC}^* = \begin{cases} q \rightarrow \infty & \text{if } p > c; \\ 0 & \text{if } p \leq c. \end{cases}$$

To see this, notice that for any price level below the marginal cost  $c$ , the firm will make negative profits. But given the fixed costs, the firm will also make negative profits if producing when price equals marginal cost  $c$ . Instead, when price is above marginal cost  $c$ , the firm will want to increase production indefinitely (that is, up to infinity) to take advantage of the fact that it can produce each unit above marginal cost, and make up the initial losses from the fixed cost, and then accumulate profits. Given this, *there is no competitive equilibrium* in this case. If we draw the usual graph with

demand and marginal cost, the perfectly competitive equilibrium would be for  $p = c$ . But at this price, no firm wants to produce. Another possibility is an equilibrium for price  $p > c$ , but in this case every firm is going to produce an infinity supply, again not a competitive equilibrium. This is a case in which only a monopoly (or an oligopoly) will be able to produce in this market – and only if the fixed costs are not too large. This was a tricky question, hence the extra credit.

**Problem 2. Search for the Unemployed.** (45 points)

We consider the problem of Ivan, an unemployed worker looking for a job in period  $t$  (sorry Ivan!). Ivan has utility over consumption  $u(c)$ , with  $u$  concave, that  $u'(c) > 0$  and  $u''(c) < 0$  for all  $c \geq 0$ . As long as Ivan is unemployed, he consumes the unemployment benefits  $b$ , that is  $c = b$ . If he can get a job, he consumes instead the wage  $w > b$ , and thus  $c = w$ . Assume that  $s$  (for search effort) is the probability that Ivan will get a job in period  $t + 1$ . Thus, with probability  $s$  Ivan earns  $w$  at  $t + 1$ , while with probability  $1 - s$ , Ivan earns  $b$ .

1. Assume first that the probability  $s$  is given, and write down the expected utility of consumption of Ivan in period  $t + 1$ . (5 points)
2. Now we assume that Ivan chooses the probability  $s$  of finding a job optimally. Namely, Ivan's utility in period  $t$  is given by the cost of effort of searching for a job  $-c(s)$ , with  $c'(0) = 0$ ,  $c'(s) > 0$  and  $c''(s) > 0$  for all  $s \in [0, 1]$ . That is, search is costly, and increasingly so at the margin. The utility in period  $t + 1$  is as in point (1). Write the intertemporal utility as of period  $t$ , that is, the sum of the period- $t$  utility (the cost of search) and the discounted period- $t+1$  utility (the consumption). Assume that the utility at period  $t+1$  is discounted by the discount rate  $\delta$ . (5 points)
3. Maximize the expected utility with respect to  $s$  and derive the first-order conditions. Interpret the first order conditions in terms of marginal cost of effort  $s$ , and marginal benefit of effort  $s$ . Do not worry about the corner solutions (that is, the constraint that  $0 \leq s \leq 1$ ) (5 points)
4. Write down the second order conditions. Are they satisfied in light of the properties of  $c(s)$ ? (5 points)
5. An advisor to the president would like to know how the intensity of search by the unemployed would be affected by an increase in unemployment benefits. Using the first order condition and the implicit function theorem, obtain the expression for  $\partial s^* / \partial b$ . What sign does it have, and what is the intuition? (5 points)
6. Similarly using the first order condition and the implicit function theorem, obtain the expression for  $\partial s^* / \partial w$ . What sign does it have, and what is the intuition? (5 points)
7. The advisor also is studying a subpopulation that is very impatient, that is, has high discounting  $\delta$ , and would like to know how that affects optimal search. Similarly using the first order condition and the implicit function theorem, obtain the expression for  $\partial s^* / \partial \delta$ . What sign does it have, and what is the intuition? (5 points)
8. The same advisor now would like to know whether the unemployed would be made happier by higher unemployment benefits. Use the envelope theorem to show how the utility at the optimum would be affected by an increase in benefits. What sign does it have, and what is the intuition? (10 points)

### Solution for Problem 2.

1. The expected utility is  $su(w) + (1 - s)u(b)$ .
2. The expected discounted utility is

$$-c(s) + \frac{1}{1 + \delta} [su(w) + (1 - s)u(b)]$$

3. We maximize

$$\max_{s \in [0,1]} -c(s) + \frac{1}{1 + \delta} [su(w) + (1 - s)u(b)]$$

The first order condition is

$$-c'(s) + \frac{1}{1 + \delta} [u(w) - u(b)] = 0. \quad (1)$$

The term  $-c'(s)$  is the marginal cost of effort which must equate the marginal return to effort, which is  $u(w) - u(b)$ , the return in terms of consumption to finding a job.

4. The second order conditions are

$$-c''(s) < 0$$

which is satisfied by the assumption above.

5. Using the implicit function theorem on (1), we derive

$$\frac{\partial s^*}{\partial b} = -\frac{-\frac{1}{1+\delta}u'(b)}{-c''(s)} = \frac{1}{1 + \delta} \frac{u'(b)}{-c''(s)} < 0.$$

The higher the unemployment benefits, the less the individual has an incentive to work hard.

6. Using the implicit function theorem on (1), we derive

$$\frac{\partial s^*}{\partial w} = -\frac{1}{1 + \delta} \frac{u'(w)}{-c''(s)} > 0.$$

The higher the future wage, the more the individual has an incentive to work hard.

7. Using the implicit function theorem on (1), we derive

$$\frac{\partial s^*}{\partial \delta} = -\frac{-\left(\frac{1}{1+\delta}\right)^2 [u(w) - u(b)]}{-c''(s)} < 0.$$

The more impatient the individual is (the higher  $\delta$ ), the less the individual searches, since the costs of search are at present while the returns are in the future.

8. To evaluate the impact on the utility at the optimum, we use the envelope theorem and compute the derivative of the objective function evaluated at the optimum with respect to  $b$ :

$$\frac{\partial \left[ -c(s^*) + \frac{1}{1+\delta} [s^*u(w) + (1 - s^*)u(b)] \right]}{\partial b} = \frac{1}{1 + \delta} (1 - s^*) u'(b) > 0.$$

An increase in unemployment benefits would lead to an increase in utility since Ivan would be better off under the state in which he is still unemployed.

PROBLEM 1

