Econ 101A – Midterm 2
Th 5 April 2012.

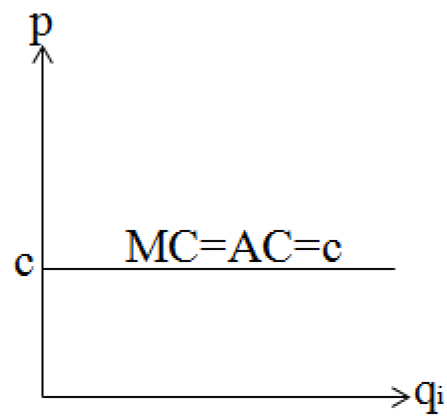
You have approximately 1 hour and 20 minutes to answer the questions in the midterm. We will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Demand, Supply, and Taxes. (55 points). Assume the case of perfect competition, with each firm i producing quantity q_i with total cost $C_i(q_i) = cq_i$. That is, costs are linear in the quantity produced.

1. Determine the marginal cost function C'_q and the average cost function $C(q)/q$, and plot the two functions in a graph with x-axis quantity q_i and y-axis cost/price. (5 points)
2. Plot graphically the supply function for each firm. Also, write it analytically, in the form $q^* = S(p)$, that is, what the quantity supplied q^* is as a function of p. (5 points)
3. Assume now that in perfect competition there are 5 firms, all with the same cost function $C_i(q_i) = cq_i$. How does the aggregate supply function differ from the individual supply function, if at all? Plot, and write analytically. (5 points)
4. Assume now that aggregate demand is given by the linear (inverse) demand function $p(Q) = A - bQ$, with $A > c$. Draw in the graph with the marginal cost function of point (3). If you want, assume $A = 10$, $c = 5$, $b = 1$. Solve (graphically or otherwise) for the equilibrium perfect competition price p^*_{PC}, as well as for the overall quantity produced Q^*, and obtain also analytical solutions. Notice that the equilibrium will be where demand equates supply. How do quantity and price depend on A and c? (5 points)
5. Highlight graphically the consumer surplus and compute the area in term of the parameters c, A, and b. (To do so, remember how to compute the area of a triangle) How does the consumer surplus depend on c and on A? Comment (5 points)
6. Highlight graphically the producer surplus and compute the area in term of the parameters c, A, and b. How does the producer surplus depend on c and on A? Comment (5 points)
7. Suppose now that a tax of amount t is imposed in this perfectly competitive market, so that the price that consumers pay is p', but producers only earn $p' - t$. Assume $t < A - c$. Show graphically the new equilibrium price p'^* and quantity Q'^*, and solve analytically for these variables. (5 points)
8. Discuss in light of the answer to the previous question what the incidence of the tax is in this case (that is, who bears the burden of the tax), and interpret in light of what we discussed in class about elasticities. (5 points)
9. Highlight graphically the new consumer and producer surplus. (5 points)
10. Highlight graphically, and then compute analytically, the deadweight loss due to taxation. (5 points)
11. In light of your previous answer, discuss this statement: 'Taxes generate a deadweight loss which is a linear function of the tax rate'. Is this correct? (5 points)

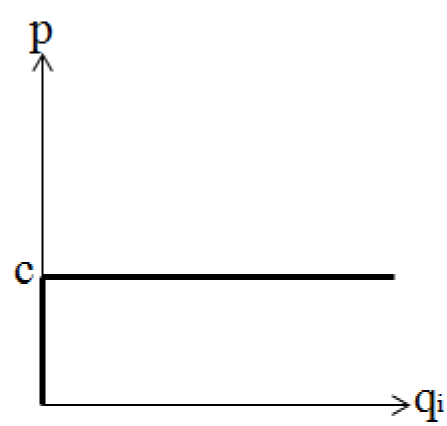
Solution to Problem 1.

1. The marginal cost and average cost both equal c: $C'_q = C(q)/q = c$.

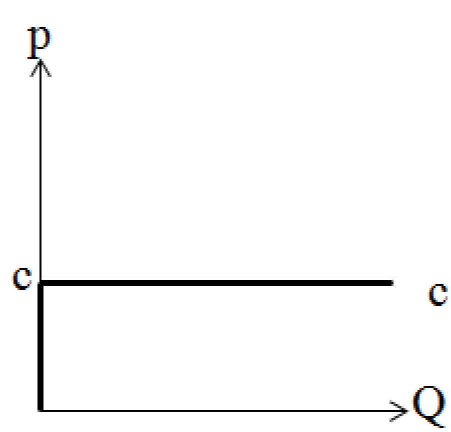


2. The supply function is

$$S(p) = \begin{cases} +\infty & \text{if } p > c \\ \text{any quantity } q \in [0, \infty) & \text{if } p = c \\ 0 & \text{if } p < c \end{cases}$$



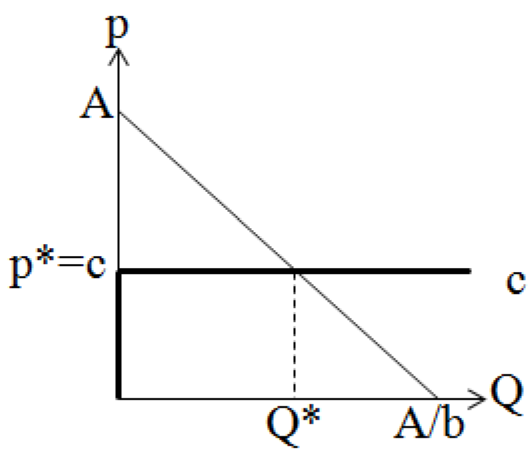
3. The aggregate supply function which comes from horizontal aggregation in this case is exactly identical.



4. Demand will equate supply for a price p^*_{PC} equal to c. Given this price, the quantity produced will be given by the demand function, and hence

$$\begin{aligned} p^*_{PC} &= c = A - bQ, \text{ and hence} \\ Q^*_{PC} &= \frac{A-c}{b} \end{aligned}$$

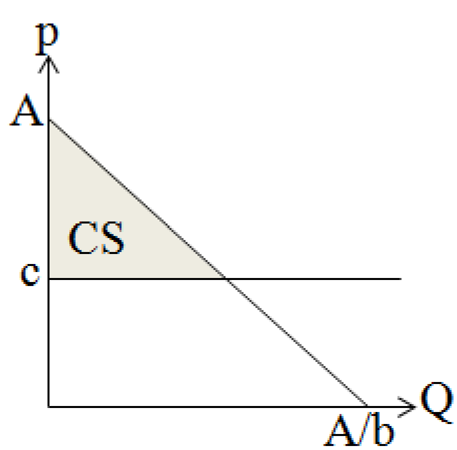
The equilibrium price is increasing in marginal cost c, but independent of the demand intercept A. The equilibrium quantity is increasing in the demand intercept A and decreasing in marginal cost c.



5. The consumer surplus is given by the area to the left of the demand curve and above the price $p^*_{PC} = c$. In this case, it is a triangle with base equal to Q^*_{PC} and height equal to $A - p^*_{PC} = A - c$. Hence, the consumer surplus equals

$$CS = \frac{\frac{A-c}{b} * (A-c)}{2} = \frac{(A-c)^2}{2b}$$

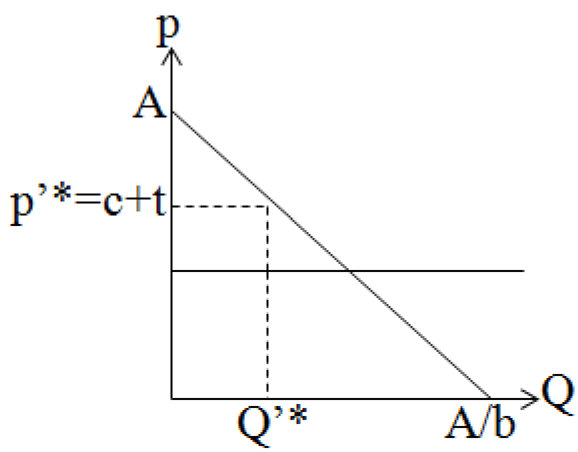
The consumer surplus is increasing in the demand intercept A and decreasing in the marginal cost c.



6. The producer surplus is given by one of two methods: (i) the integral of the area between the marginal cost and the price, net of the fixed cost (zero here); (ii) the rectangle given by the quantity produced and the difference between the price and the average cost. Either method shows that the producer surplus is zero and hence independent of A and c.

7. Graphically, it is easy to see how the new equilibrium price will equal $p^* = c + t$, and hence the price that producers receive, $p^* - t$, will equal c. This has to be the case as firms will not produce below the marginal cost. Given this, the new quantity produced is found on the demand curve as

$$\begin{aligned} p'^*_{PC} &= c + t = A - bQ, \text{ and hence} \\ Q'^*_{PC} &= \frac{A - c - t}{b}. \end{aligned}$$

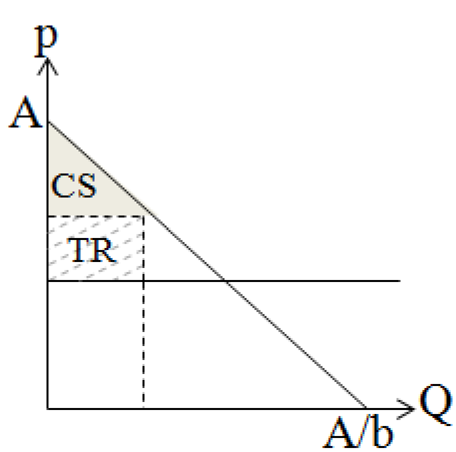


8. In this example, the consumers bear all the burden of the tax, as the consumer price increases by the full amount of this tax t, and the producer price remains the same. This is due to the fact that the supply curve is infinitely elastic, and hence consumers bear the burden (the side that bears the burden is the one that is relatively inelastic). Another way to see this is that producers were already making zero profits, they could not bear any of the burden.

9. The new producer surplus is still zero, while the consumer surplus is now

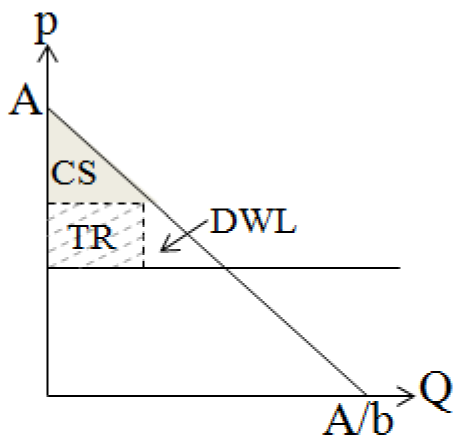
$$CS = \frac{\frac{A-c-t}{b} * (A - c - t)}{2} = \frac{(A - c - t)^2}{2b}$$

[You did not need to compute this analytically]



10. The deadweight loss due to taxation is given by the triangle in the figure. We can compute its area as

$$DL = \frac{t * \left(\frac{t}{b}\right)}{2} = \frac{t^2}{2b}.$$



11. The deadweight loss due to taxation is a quadratic function of the tax imposed, not linear. Hence, the statement is not correct. This implies that taxes are increasingly distortive as the tax rate increases.

Problem 2. Insurance. (30 points)

Tim drives a Toyota and expects to find himself in an accident within the next year with probability p, with $0 \leq p \leq 1$. If an accident occurs, the damage amounts to loss L; there is no damage if no accident occurs. Tim's utility over wealth is $u(w) = \ln(w)$, and Tim starts the year with wealth w_0. Tim chooses how much (if any) of and insurance policy to buy. The policy is as follows. Tim pays premium q per each dollar paid to him in the event of an accident. He chooses how many units for insurance α to purchase, with $0 \leq \alpha \leq L$. (The set-up hence is as in class, except that we specified the form of the utility function)

1. Write down the expected utility of Tim. (5 points)
2. Maximize the expected utility with respect to α, derive the first-order conditions. (5 points)
3. Without worrying for now about the boundary conditions ($0 \leq \alpha \leq L$), solve for α^* [If you get stuck here, move to next point and use your intuition] (10 points)
4. What is the solution for α^* in the case $p = q$? How much insurance does the agent purchase? Comment and provide intuition. [If you could not solve the point above, write what you expect to find, and why] (5 points)
5. What is qualitative feature of the solution for α^* in the case $p < q$? How much insurance does the agent purchase? Comment and provide intuition. [If you could not solve the point above, write what you expect to find, and why] (5 points)

Solution to Problem 2.

1. The utility in case of no accident is $\ln(w_0 - \alpha q)$, and in case of accident it is $\ln(w_0 - L + \alpha - \alpha q)$. Hence, the expected utility is
$$EU = (1-p)\{\ln(w_0 - \alpha q)\} + p\{\ln(w_0 - L + \alpha - \alpha q)\}$$
2. The driver maximizes
$$\begin{aligned} & \max_{\alpha}(1-p)\{\ln(w_0 - \alpha q)\} + p\{\ln(w_0 - L + \alpha - \alpha q)\} \\ s.t. 0 \leq & \ \alpha \leq L \end{aligned}$$
The f.o.c. is
$$-q\frac{1-p}{w_0 - \alpha q} + (1-q)\frac{p}{w_0 - L + \alpha(1-q)} = 0$$
3. We solve the f.o.c as follows
$$\begin{aligned} -q\frac{1-p}{w_0 - \alpha q} + (1-q)\frac{p}{w_0 - L + \alpha(1-q)} &= 0 \\ \frac{q}{(1-q)}\frac{1-p}{p}(w_0 - L + \alpha(1-q)) &= w_0 - \alpha q \\ \alpha q\left(\frac{1-p}{p} + 1\right) &= w_0 - \frac{q}{(1-q)}\frac{1-p}{p}(w_0 - L) \\ \alpha^* &= \frac{p}{q}w_0 - \frac{1-p}{1-q}(w_0 - L) \end{aligned}$$
4. In the case $p = q$, the solution for α^* simplifies to $\alpha^* = L$, that is, full insurance. This is the result which we had derived in class more generally for any concave utlity function (of which $\ln()$ is an example). Risk-averse agents will fully insure if insurance is fair, that is, if $p = q$. Notice that the constraints are satisfied in this case.
5. For $p < q$, notice that the expression for α^* is decreasing in q. Hence, α^* will be smaller than in the case of $p = q$, and hence smaller than L. That is, in this case, $\alpha^* < L$. This is the familiar under-insurance result we saw in class. Any risk-averse agent will not seek full insurance any more once the insurance is not actuarially fair in its pricing. Notice that here is the constraint which may be binding is $\alpha \geq 0$.