Econ 101A – Midterm 2
Th 2 November 2006.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Profit Maximization with Uncertainty (65 points) Allison owns a company up in Napa that produces wine. Her income is given by the profits of the company that she runs. Allison is risk-averse, with utility function $u(c)$ satisfying $u'(c) > 0$ for all c, and $u''(c) < 0$ for all c. Allison maximizes her utility from the profits of the company. Wine y is a labor-intensive operation. It is produced using labor with the production function $y = l^\alpha$, with $0 < \alpha < 1$. Workers in the vine are paid the wage w. Allison maximizes utility from her only income source, the profits from wine sales:

$$\max_l u\left[pl^\alpha - wl\right]. \tag{1}$$

1. Define the concept of returns to scale, that is, what we mean by constant/increasing/decreasing returns to scale. Under what conditions for α the production of wine has decreasing returns to scale? Show the steps of your reasoning. (Here assume only $\alpha > 0$, from the next point on assume $0 < \alpha < 1$) (4 points)
2. Consider the maximization of utility of Allison, and obtain first-order conditions.(3 points)
3. Are the second order conditions satisfied? (3 points)
4. Solve for the utility-maximizing l^*. How does the optimal employment of workers in the vineyard vary as the wage w of workers increases? And when the price p increases? Discuss the economic intuition. (3 points)
5. Using the envelope theorem, compute the effect on the utility of Allison at the optimum $u\left[p\left(l^*\right)^\alpha - wl^*\right]$ of an increase in price p. Does the finding make sense? Discuss. (4 points)
6. In a nearby terrain, Wei runs his operation with a standard profit-maximization. He has the same production function and costs as Allison, and hence maximizes

$$\max_l pl^\alpha - wl. \tag{2}$$

   Solve for the profit-maximizing l^* for Wei. (3 points)
7. Allison and Wei are neighbors to a chatty economist trained in Stanford. (a concession to Prasad) The economist, after drinking a little too much of their wine, is on-the-record on the local TV as saying "it is a pity that Allison is so risk-averse. Unlike Allison, Wei just maximizes profits, which is much better for the company". Discuss this assertion in light of what you found so far. Provide economic and mathematical intuition. (6 points)
8. The world of wine-growing is a treacherous one, with much uncertainty. In particular, the price of wine fluctuates from year-to-year, and the decisions on how much to produce and how many workers to hire have to be made before the level of price is known. The price p of a bottle is 10 with probability q and 5 with probability $1-q$. Let's consider first the problem of Wei, (2). Wei now maximizes *expected* profits. Set up the maximization of expected profits by Wei, explaining why you are setting up the problem the way you are. (6 points)
9. Derive Wei's first order conditions and solve for the profit-maximizing level of labor hired l^* by Wei. (5 points)
10. How does the quantity of labor hired l^* vary as the probability of a high price of wine q increases? Provide economic intuition (3 points)
11. Suppose that instead of facing uncertainty over the price of wine, Wei knew that the price of wine was going to be for sure $q10 + (1-q)5$, which is the expected value of the uncertain price. Using the solution you obtained in point 6, compute the quantity of labor hired by Wei in this case, and compare it to the quantity hired when the price is uncertain. How do the two quantities compare? (I mean, the one in point 9 and the one you just derived) Discuss the reason for this result. (5 points)

12. Let us now go back to Allison's problem (1). Allison now faces uncertainty over the price of wine as well. As in Wei's case, the price p of a bottle is 10 with probability q and 5 with probability $1-q$. Allison now maximizes *expected* utility. Set up the maximization of expected utility by Allison. Derive the first-order conditions. (6 points)

13. Solving this problem formally is tricky. Using your economic intuition, discuss whether you expect that the quantity of labor hired by Allison l^* in this case with uncertainty will be the same, smaller, or larger than the quantity hired by Wei (that is, what you derived in point 10. Compared to the discussion in point 7, can the utility function u matter here? (6 points)

14. (Hard) Can you prove formally whether the quantity of labor hired by Allison l^* in this case with uncertainty will be the same, smaller, or larger than the quantity hired by Wei? (Hint: use the property that $u'(y) < u'(x)$ for $y > x$ if u is concave) (8 points)

Problem 2. Investment in Risky Asset (30 points) Consider a standard problem of investment in risky assets, similar to the one that we covered in class. The agent can invest in bonds or stocks. Bonds have a return $r = 0$. (that is, they return back your capital at the end of the period) Stocks have a stochastic return, .10 (ten percent) with probability 1/2, and $-.05$ (minus 5 percent) with probability 1/2. The agent has income w and utility function $u(x)$. The agents wants to decide the amount s of wealth to invest in stocks; we assume $0 \leq s \leq w$. The remaining of his wealth, $w - s$, is invested in bonds. (Note: s is not the share of wealth, but the amount of wealth) The agent's utility function (defined over wealth at the end of the period) is linear, $u(x) = a + bx$, with $b > 0$.

1. At the beginning, before investing, the agent has wealth w. What is the wealth at the end, after investing, if the return of the stock is high? What is the wealth if the return of the stock is low?(4 points)

2. Write down the expected utility Eu of the agent, explaining how the expected utility is defined. (4 points)

3. Derive the first order condition. (4 points)

4. Derive the second order condition. Is it satisfied? (3 points)

5. Find the solution for s^*. Argue the steps of the solution as precisely as you can. (Hint: Consider your answer to point 4) (10 points)

6. How does the solution for s^* change if bonds have a return $r = .05$ (5 percent)? (5 points)