Economics 101A
(Lecture 16)

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March 17, 2015
Outline

1. Cost Curves II

2. One-step Profit Maximization

3. Second-Order Conditions

4. Introduction to Market Equilibrium

5. Aggregation

6. Market Equilibrium in the Short-Run
1 Cost Curves II

• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2 One-step Profit Maximization

• Nicholson, Ch. 11, pp. 383-393

• One-step procedure: maximize profits

• Perfect competition. Price $p$ is given
  – Firms are small relative to market
  – Firms do not affect market price $p_M$

  – Will firm produce at $p > p_M$?
  – Will firm produce at $p < p_M$?
  – $\implies p = p_M$
- Revenue: $py = pf(L, K)$

- Cost: $wL + rK$

- Profit $pf(L, K) - wL - rK$
Agent optimization:

\[
\max_{L,K} pf (L, K) - wL - rK
\]

First order conditions:

\[
pf'_L (L, K) - w = 0
\]

and

\[
pf'_K (L, K) - r = 0
\]

Second order conditions?

\[
 pf''_{L,L} (L, K) < 0 \quad \text{and}
\]

\[
|H| = \begin{vmatrix}
 pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\
 pf''_{L,K} (L, K) & pf''_{K,K} (L, K)
\end{vmatrix} = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0
\]

Need \( f''_{L,K} \) not too large for maximum
• Comparative statics with respect to $p$, $w$, and $r$.

• What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K} (L, K) \\ 0 & pf''_{K,K} (L, K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\ pf''_{L,K} (L, K) & pf''_{K,K} (L, K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

• Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$. 
3 Second Order Conditions in P-Max: Cobb-Douglas

• How do the second order conditions relate for:
  – Cost Minimization
  – Profit Maximization?

• Check for Cobb-Douglas production function

\[ y = AK^\alpha L^\beta \]

• **Cost Minimization.** S.o.c.:

\[ c_y''(y^*, w, r) > 0 \]

• As we showed, for CD prod. ftn.,

\[ c_y''(y^*, w, r) = -\frac{1}{\alpha + \beta} \left( \frac{1 - (\alpha + \beta)}{\alpha + \beta} \right) \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]

which is \( > 0 \) as long as \( \alpha + \beta < 1 \) (DRS)
• **Profit Maximization.** S.o.c.: \( p f''_{L,L}(L, K) < 0 \) and

\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0
\]

• As long as \( \beta < 1 \),

\[ p f''_{L,L} = p \beta (\beta - 1) AK^\alpha L^{\beta-2} < 0 \]

• Then,

\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] =
\]

\[
= p^2 \left[ \beta (\beta - 1) AK^\alpha L^{\beta-2} - (\alpha (\alpha - 1) AK^{\alpha-2} L^{\beta-1}) \right] =
\]

\[
= p^2 A^2 K^{2\alpha-2} L^{2\beta-2} \alpha \beta [1 - \alpha - \beta]
\]

• Therefore, \( |H| > 0 \) iff \( \alpha + \beta < 1 \) (DRS)

• The two conditions coincide
4 Introduction to Market Equilibrium

- Nicholson, Ch. 12, pp. 409–419

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*, K^*$ (see above)
  - Optimal quantity produced $y^*$
• **Supply function.** $y = y^* (p, w, r)$

  – From profit maximization:
    
    $$y = f (L^*(p, w, r), K^*(p, w, r))$$

  – From cost minimization:

    $MC$ curve above $AC$

  – Supply function is increasing in $p$

• **Market Equilibrium.** Equate demand and supply.

• **Aggregation?**

• **Industry supply function!**
5 Aggregation

5.1 Producers aggregation

- \( J \) companies, \( j = 1, ..., J \), producing good \( i \)

- Company \( j \) has supply function

\[
y_i^j = y_i^{j*} (p_i, w, r)
\]

- Industry supply function:

\[
Y_i (p_i, w, r) = \sum_{j=1}^{J} y_i^{j*} (p_i, w, r)
\]

- Graphically,
5.2 Consumer aggregation

- One-consumer economy
- Utility function $u(x_1, \ldots, x_n)$
- prices $p_1, \ldots, p_n$

- Maximization $\implies$

\[ x_1^* = x_1^* (p_1, \ldots, p_n, M), \]
\[ \vdots \]
\[ x_n^* = x_n^* (p_1, \ldots, p_n, M). \]
Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

Single-consumer demand function:

$$x_i^* = x_i^*(p_i|p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

What is sign of $\frac{\partial x_i^*}{\partial p_i}$?

Negative if good $i$ is normal

Negative or positive if good $i$ is inferior
• Aggregation: $J$ consumers, $j = 1, \ldots, J$

• Demand for good $i$ by consumer $j$:

$$x_i^{j*} = x_i^{j*}(p_1, \ldots, p_n, M^j)$$

• Market demand $X_i$:

$$X_i \left( p_1, \ldots, p_n, M^1, \ldots, M^J \right) = \sum_{j=1}^{J} x_i^{j*} \left( p_1, \ldots, p_n, M^j \right)$$

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$: 
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
6 Market Equilibrium in the Short-Run

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y_i^S (p_i^*, w, r) = X_i^D (p_1^*, ..., p_n^*, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
7 Next Lecture

- Market Equilibrium

- Comparative Statics of Equilibrium

- Elasticities

- Taxes and Subsidies