# Economics 101A
# (Lecture 16)

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Outline

1. Cost Curves II
2. One-step Profit Maximization
3. Second-Order Conditions
4. Introduction to Market Equilibrium
5. Aggregation
6. Market Equilibrium in the Short-Run

# 1 Cost Curves II

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?

# 2 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 383-393

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  - Firms are small relative to market
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Longrightarrow p = p_M$

- Revenue: $py = pf\left(L, K\right)$

- Cost: $wL + rK$

- Profit $pf\left(L, K\right) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$ and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Need $f''_{L,K}$ not too large for maximum

- Comparative statics with respect to to $p$, $w$, and $r$.

- What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.

# 3 Second Order Conditions in P-Max: Cobb-Douglas

- How do the second order conditions relate for:
  - Cost Minimization
  - Profit Maximization?

- Check for Cobb-Douglas production function
$$y = AK^{\alpha}L^{\beta}$$

- **Cost Minimization.** S.o.c.:
$$c_y''(y^*, w, r) > 0$$

- As we showed, for CD prod. ftn.,
$$c_y''(y^*, w, r) = -\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$
which is $> 0$ as long as $\alpha + \beta < 1$ (DRS)

- **Profit Maximization.** S.o.c.: $pf''_{L,L}(L,K) < 0$ and

$$|H| = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0$$

- As long as $\beta < 1$,

$$pf''_{L,L} = p\beta (\beta - 1) AK^{\alpha} L^{\beta-2} < 0$$

- Then,

$$\begin{aligned} |H| &= p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] = \\ &= p^2 \left[ \begin{array}{c} \beta (\beta - 1) AK^{\alpha} L^{\beta-2} * \\ \alpha (\alpha - 1) AK^{\alpha-2} L^{\beta} - \\ \left( \alpha\beta AK^{\alpha-1} L^{\beta-1} \right)^2 \end{array} \right] = \\ &= p^2 A^2 K^{2\alpha-2} L^{2\beta-2} \alpha\beta \left[ 1 - \alpha - \beta \right] \end{aligned}$$

- Therefore, $|H| > 0$ iff $\alpha + \beta < 1$ (DRS)

- The two conditions coincide

# 4 Introduction to Market Equilibrium

- Nicholson, Ch. 12, pp. 409–419

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*$, $K^*$ (see above)
  - Optimal quantity produced $y^*$

- **Supply function**. $y = y^*(p, w, r)$
  - From profit maximization:

$$y = f\left(L^*(p, w, r), K^*(p, w, r)\right)$$

  - From cost minimization:

$$MC \text{ curve above } AC$$

  - Supply function is increasing in $p$

- Market Equilibrium. Equate demand and supply.

- Aggregation?

- Industry supply function!

# 5 Aggregation

## 5.1 Producers aggregation

- $J$ companies, $j = 1, ..., J$, producing good $i$

- Company $j$ has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r)$$

- Graphically,

## 5.2 Consumer aggregation

- *One-consumer economy*

- Utility function $u(x_1, ..., x_n)$

- prices $p_1, ..., p_n$

- Maximization $\Longrightarrow$

$$\begin{aligned} x_1^* &= x_1^*(p_1, ..., p_n, M), \\ &\vdots \\ x_n^* &= x_n^*(p_1, ..., p_n, M). \end{aligned}$$

- Focus on good $i$. Fix prices $p_1, ..., p_{i-1}, p_{i+1}, ..., p_n$ and $M$

- **Single-consumer demand function**:

$$x_i^* = x_i^* \left(p_i | p_1, ..., p_{i-1}, p_{i+1}, ..., p_n, M\right)$$

- What is sign of $\partial x_i^* / \partial p_i$?

- Negative if good $i$ is normal

- Negative or positive if good $i$ is inferior

- *Aggregation:* $J$ consumers, $j = 1, ..., J$

- Demand for good $i$ by consumer $j$ :

$$x_i^{j*} = x_i^{j*}\left(p_1, ..., p_n, M^j\right)$$

- Market demand $X_i$:

$$\begin{aligned} & X_i\left(p_1, ..., p_n, M^1, ..., M^J\right) \\ = & \sum_{j=1}^{J} x_i^{j*}\left(p_1, ..., p_n, M^j\right) \end{aligned}$$

- Graphically,

- Notice: market demand function depends on distribution of income $M^J$

- Market demand function $X_i$:

  - Consumption of good $i$ as function of prices $\mathbf{p}$

  - Consumption of good $i$ as function of income distribution $M^j$

# 6 Market Equilibrium in the Short-Run

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $\mathbf{p}^*$ equates demand and supply of good $i$:

  $$Y^* = Y_i^S\left(p_i^*, w, r\right) = X_i^D\left(p_1^*, ..., p_n^*, M^1, ..., M^J\right)$$

- Graphically,

- Notice: in short-run firms can make positive profits

- Comparative statics exercises with endogenous price $p_i$ :

  – increase in wage $w$ or interest rate $r$:

  – change in income distribution

# 7 Next Lecture

- Market Equilibrium
- Comparative Statics of Equilibrium
- Elasticities
- Taxes and Subsidies