Economics 101A
(Lecture 15)

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Outline

1. Cost Minimization: Example

2. Cost Curves and Supply Function
1 Cost Minimization: Example

- Continue example above: $y = f(L, K) = AK^\alpha L^\beta$

- Cost minimization:

$$\begin{align*}
\min wL + rK \\
s.t. AK^\alpha L^\beta = y
\end{align*}$$
• Solutions:

  – Optimal amount of labor:

    \[ L^* (r, w, y) = \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}} \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha+\beta}} \]

  – Optimal amount of capital:

    \[ K^* (r, w, y) = \frac{w \alpha}{r \beta} \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}} \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha+\beta}} = \]

    \[ = \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}} \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha+\beta}} \]

• Check various comparative statics:

  – \( \frac{\partial L^*}{\partial A} < 0 \) (technological progress and unemployment)

  – \( \frac{\partial L^*}{\partial y} > 0 \) (more workers needed to produce more output)
- \frac{\partial L^*}{\partial w} < 0, \frac{\partial L^*}{\partial r} > 0 \text{ (substitute away from more expensive inputs)}

- Parallel comparative statics for $K^*$
• Cost function

\[ c(w, r, y) = wL^*(r, w, y) + rK^*(r, w, y) = \]

\[ = \left( \frac{y}{A} \right) \frac{1}{\alpha + \beta} \left[ w \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha + \beta}} + r \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha + \beta}} \right] \]

• Define \( B := w \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha + \beta}} + r \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha + \beta}} \)

• Cost-minimizing output choice:

\[ \max_{py - B \left( \frac{y}{A} \right)^{\frac{1}{\alpha + \beta}}} \]
• First order condition:
\[ p - \frac{1}{\alpha + \beta} \frac{B}{A} \left( \frac{y}{A} \right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0 \]

• Second order condition:
\[ -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]

• When is the second order condition satisfied?
Solution:

- $\alpha + \beta = 1$ (CRS):
  * S.o.c. equal to 0
  * Solution depends on $p$
    * For $p > \frac{B}{A}$, produce $y^* \to \infty$
    * For $p = \frac{B}{A}$, produce any $y^* \in [0, \infty)$
    * For $p < \frac{B}{A}$, produce $y^* = 0$
- $\alpha + \beta > 1$ (IRS):
  * S.o.c. positive
  * Solution of f.o.c. is a minimum!
  * Solution is $y^* \to \infty$.
  * Keep increasing production since higher production is associated with higher returns
\(- \alpha + \beta < 1 \) (DRS):

* s.o.c. negative. OK!

* Solution of f.o.c. is an interior optimum

* This is the only "well-behaved" case under perfect competition

* Here can define a supply function
2 Cost Curves

• Nicholson, Ch. 10, pp. 341-349; Ch. 11, pp. 380-383

• Marginal costs $MC = \partial c/\partial y$ → Cost minimization
  
  \[ p = MC = \partial c(w, r, y) / \partial y \]

• Average costs $AC = c/y$ → Does firm break even?
  
  \[ \pi = py - c(w, r, y) > 0 \text{ iff } \]
  
  \[ \pi/y = p - c(w, r, y) / y > 0 \text{ iff } \]
  
  \[ c(w, r, y) / y = AC < p \]

• Supply function. Portion of marginal cost $MC$ above average costs. (price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• **Case 1.** Production function. $y = L^\alpha$
  
  – Cost function? (cost of input is $w$):
  
  \[
  c(w, y) = wL^*(w, y) = wy^{1/\alpha}
  \]

  – Marginal cost?
  
  \[
  \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{(1-\alpha)/\alpha}
  \]

  – Average cost $c(w, y)/y$?
  
  \[
  \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}
  \]
• **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2.1 Supply Function

• Supply function: \( y^* = y^* (w, r, p) \)

• What happens to \( y^* \) as \( p \) increases?

• Is the supply function upward sloping?

• Remember f.o.c:

\[
 p - c_y'(w, r, y) = 0
\]

• Implicit function:

\[
 \frac{\partial y^*}{\partial p} = -\frac{1}{-c_{y,y}''(w, r, y)} > 0
\]

as long as s.o.c. is satisfied.

• Yes! Supply function is upward sloping.
3 Next Lectures

- Profit Maximization
- Aggregation
- Market Equilibrium