Economics 101A
(Lecture 14)

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Outline

1. Production Function II
2. Returns to Scale
3. Two-step Cost Minimization

1 Production Function II

- Convex production function if convex isoquants

- Mathematically, convex isoquants if $d^2K/d^2L > 0$

- Solution:

$$\frac{d^2K}{d^2L} = -\frac{f''_{L,L}f'_K - 2f''_{L,K}f'_L + f''_{K,K}\left(f'_L\right)^2/f'_K}{\left(f'_K\right)^2}$$

- Hence, $d^2K/d^2L > 0$ if $f''_{L,K} > 0$ (inputs are complements in production)

2 Returns to Scale

- Nicholson, Ch. 9, pp. 310-313

- Effect of increase in labor: f'_L

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$

- How much does output increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

– Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

– Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Example: $y = f(K, L) = AK^{\alpha}L^{\beta}$

- Marginal product of labor: $f'_L =$

- Decreasing marginal product of labor: $f''_{L,L} =$

- $MRTS =$

- Convex isoquant?

- Returns to scale: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

3 Two-step Cost minimization

- Nicholson, Ch. 10, pp. 333-341

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level y, choose cost-minimizing combinations of inputs
  - Choose optimal level of y.

- *First step.* Cost-Minimizing choice of inputs

- Two-input case: Labor, Capital

- Input prices:
  - Wage w is price of L
  - Interest rate r is rental price of capital K

- Expenditure on inputs: $wL + rK$

- Firm objective function:

$$\min_{L,K} wL + rK$$
$$s.t. f(L,K) \geq y$$

- Equality in constraint holds if:

  1. $w > 0$, $r > 0$;

  2. f strictly increasing in at least L or K.

- Counterexample if ass. 1 is not satisfied

- Counterexample if ass. 2 is not satisfied

- Compare with expenditure minimization for consumers

- First order conditions:

$$w - \lambda f'_L = 0$$

and

$$r - \lambda f'_K = 0$$

- Rewrite as

$$\frac{f'_L(L^*, K^*)}{f'_K(L^*, K^*)} = \frac{w}{r}$$

- MRTS (slope of isoquant) equals ratio of input prices

- Graphical interpretation

- Derived demand for inputs:
  - $L = L^*(w, r, y)$
  - $K = K^*(w, r, y)$
- Value function at optimum is **cost function:**

$$c(w, r, y) = wL^*(r, w, y) + rK^*(r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well

- Price of output is p.

- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c_y'(w, r, y) = 0$$

- Price equals marginal cost – very important!

- Second order condition:

$$-c''_{y,y}(w, r, y^*) < 0$$

- For maximum, need increasing marginal cost curve.

4 Cost Minimization: Example I

- Continue example above: $y = f(L, K) = AK^{\alpha}L^{\beta}$

- Cost minimization:

$$\min wL + rK$$
$$s.t. AK^{\alpha}L^{\beta} = y$$

- What is the return to scale for this example?

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$

- How much does input increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

- Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

- Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Returns to scale depend on $\alpha + \beta \lesseqgtr 1$: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

- Solutions:
  - Optimal amount of labor:

$$L^*(r,w,y) = \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}}$$

  - Optimal amount of capital:

$$\begin{aligned} K^*(r,w,y) &= \frac{w}{r}\frac{\alpha}{\beta}\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{aligned}$$

- Check various comparative statics:
  - $\partial L^*/\partial A < 0$ (technological progress and unemployment)
  - $\partial L^*/\partial y > 0$ (more workers needed to produce more output)

- $\partial L^*/\partial w < 0$, $\partial L^*/\partial r > 0$ (substitute away from more expensive inputs)

- Parallel comparative statics for K^*

- Cost function

$$\begin{aligned} c(w,r,y) &= wL^*(r,w,y) + rK^*(r,w,y) = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left[\begin{array}{c} w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \\ +r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{array} \right] \end{aligned}$$

- Define $B := w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

- Cost-minimizing output choice:

$$\max py - B\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}}$$

5 Next Lecture

- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization