Outline

1. Mid-Term Feedback

2. Investment in Risky Asset

3. Time Consistency

4. Time Inconsistency
1 Mid-Term Feedback

- Thanks for the feedback!
2 Investment in Risky Asset

• Individual has:
  – wealth \( w \)
  – utility function \( u \), with \( u' > 0 \)

• Two possible investments:
  – Asset B (bond) yields return 1 for each dollar
  – Asset S (stock) yields uncertain return \((1 + r)\):
    * \( r = r_+ > 0 \) with probability \( p \)
    * \( r = r_- < 0 \) with probability \( 1 - p \)
    * \( Er = pr_+ + (1 - p)r_- > 0 \)

• Share of wealth invested in stock \( S = \alpha \)
• Individual maximization:

$$\max_\alpha (1 - p) u (w [(1 - \alpha) + \alpha (1 + r_-)]) + +pu (w [(1 - \alpha) + \alpha (1 + r_+)])$$

s.t. \(0 \leq \alpha \leq 1\)

• Case of risk neutrality: \(u(x) = a + bx, b > 0\)

• Assume \(a = 0\) (no loss of generality)

• Maximization becomes

$$\max_\alpha b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+])$$
or

$$\max_\alpha bw + \alpha bw [(1 - p) r_- + pr_+]$$

• Sign of term in square brackets? Positive!

• Set \(\alpha^* = 1\)
- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

  $$0 = (1 - p)(wr_-)u'(w[1 + \alpha r_-]) + p(wr_+)u'(w[1 + \alpha r_+])$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.
3 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M_i' = M_i + \text{savings/debts from previous period}$
  
  – choose consumption $c_i$;
  
  – can save/borrow $M_i' - c_i$
  
  – no borrowing in last period: at $t = 2$ $M_2' = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}U(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta}U(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at \( t = 1 \):

\[
c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2
\]

• Maximization problem:

\[
\max U(c_1) + \frac{1}{1 + \delta}U(c_2)
\]

\[
s.t. c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U''(c_1)}{U''(c_2)} = \frac{1 + r}{1 + \delta}
\]
• Back to **period 0**.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}U(c_2)$$

s.t. $c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1 + r}{1 + \delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2)$$

$$= U(c_0) + \frac{1}{1 + \delta} \left[ U(c_1) + \frac{1}{1 + \delta} U(c_2) \right]$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?

  – addictive products (alcohol, drugs);
  
  – good actions (exercising, helping friends);
  
  – immediate gratification (shopping, credit card borrowing)
4 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

- Utility at time $t$ is $u(c_t, c_{t+1}, c_{t+2})$:
\[ u(c_t) + \frac{\beta}{1+\delta} u(c_{t+1}) + \frac{\beta}{(1+\delta)^2} u(c_{t+2}) + ... \]

- Discount factor is
\[ 1, \frac{\beta}{1+\delta'}, \frac{\beta}{(1+\delta)^2}, \frac{\beta}{(1+\delta)^3}, ... \]

  instead of
\[ 1, \frac{1}{1+\delta'}, \frac{1}{(1+\delta)^2}, \frac{1}{(1+\delta)^3}, ... \]

- What is the difference?

- *Immediate gratification*: $\beta < 1$
• Back to our problem: **Period 1**.

• Maximization problem:

\[
\max U(c_1) + \frac{\beta}{1 + \delta} U(c_2)
\]

\[
s.t. \, c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*)}{U'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}
\]
• Now, period 0 with commitment.

• Maximization problem:

\[
\max_U c_0 + \frac{\beta}{1 + \delta} c_1 + \frac{\beta}{(1 + \delta)^2} c_2 \\
\text{s.t. } c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2
\]

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U'(c_1^*, c)}{U'(c_2^*, c)} = \frac{1 + r}{1 + \delta}
\]

• The two conditions differ!

• Time inconsistency: \( c_1^{*, c} < c_1^* \) and \( c_2^{*, c} > c_2^* \)

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!
  – One trillion dollars in credit card debt;
  – Most debt is in teaser rates;
  – Two thirds of Americans are overweight or obese;
  – $10bn health-club industry

• Is this testable?
  – In the laboratory?
  – In the field?
5 Next lecture and beyond

- Time Inconsistency

- Production Function