Outline

1. Introduction to probability

2. Expected Utility

3. Risk Aversion and Lottery

4. Measures of Risk Aversion

5. Insurance
1 Introduction to Probability

- Nicholson, Ch. 7, p. 209

- So far deterministic world:
  - income given, known $M$
  - interest rate known $r$

- But some variables are unknown at time of decision:
  - future income $M_1$?
  - future interest rate $r_1$?

- Generalize framework to allow for uncertainty
– Events that are truly unpredictable (weather)

– Event that are very hard to predict (future income)
• Probability is the language of uncertainty

• Example:
  - Income $M_1$ at $t = 1$ depends on state of the economy
  - Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  - Three probabilities: $p_1$, $p_2$, $p_3$
  - $p_1 = P(M_1) = P($recession$)$

• Properties:
  - $0 \leq p_i \leq 1$
  - $p_1 + p_2 + p_3 = 1$
• Mean income: $EM = \sum_{i=1}^{3} p_i M_i$

• If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3),$ 
  \[ EM = \frac{1}{3} 20 + \frac{1}{3} 25 + \frac{1}{3} 30 = \frac{75}{3} = 25 \]

• Variance of income: $V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2$

• If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3),$ 
  \[ V(M) = \frac{1}{3} (20 - 25)^2 + \frac{1}{3} (25 - 25)^2 + \frac{1}{3} (30 - 25)^2 = \frac{1}{3} 5^2 + \frac{1}{3} 5^2 = 2/3 \times 25 \]

• Mean and variance if $(p_1, p_2, p_3) = (1/4, 1/2, 1/4)$?
2 Expected Utility

• Nicholson, Ch. 7, pp. 210-217

• Consumer at time 0 asks: what is utility in time 1?

• At $t = 1$ consumer maximizes

$$\max U(c^1)$$

$$s.t. \ c^1_i \leq M^1_i + (1 + r)(M^0 - c^0)$$

with $i = 1, 2, 3$.

• What is utility at optimum at $t = 1$ if $U' > 0$?

• Assume for now $M^0 - c^0 = 0$

• Utility $U(M^1_i)$

• This is uncertain, depends on which $i$ is realized!
• How do we evaluate future uncertain utility?

• **Expected utility**

\[
EU = \sum_{i=1}^{3} p_i U(M_i^1)
\]

• In example:

\[
EU = \frac{1}{3}U(20) + \frac{1}{3}U(25) + \frac{1}{3}U(30)
\]

• Compare with \( U(EC) = U(25) \).

• Agents prefer riskless outcome \( EM \) to uncertain outcome \( M \) if

\[
\frac{1}{3}U(20) + \frac{1}{3}U(25) + \frac{1}{3}U(30) < U(25) \quad \text{or} \\
\frac{1}{3}U(20) + \frac{1}{3}U(30) < \frac{2}{3}U(25) \quad \text{or} \\
\frac{1}{2}U(20) + \frac{1}{2}U(30) < U(25)
\]
• Picture
• Depends on sign of $U''$, on concavity/convexity

• Three cases:

  - $U''(x) = 0$ for all $x$. (linearity of $U$)
    \[ U(x) = a + bx \]
    \[ 1/2U(20) + 1/2U(30) = U(25) \]

  - $U''(x) < 0$ for all $x$. (concavity of $U$)
    \[ 1/2U(20) + 1/2U(30) < U(25) \]

  - $U''(x) > 0$ for all $x$. (convexity of $U$)
    \[ 1/2U(20) + 1/2U(30) > U(25) \]
• If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

• If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

• If $U''(x) > 0$ (convexity), consumer likes uncertainty

• Do consumers like uncertainty?
• **Theorem. (Jensen’s inequality)** If a function \( f (x) \) is concave, the following inequality holds:

\[ f (E x) \geq E f (x) \]

where \( E \) indicates expectation. If \( f \) is strictly concave, we obtain

\[ f (E x) > E f (x) \]

• Apply to utility function \( U \).

• Individuals dislike uncertainty:

\[ U (E x) \geq E U (x) \]

• Jensen’s inequality then implies \( U \) concave (\( U'' \leq 0 \))

• Relate to diminishing marginal utility of income
3 Risk aversion and Lottery

• Risk aversion:
  – individuals dislike uncertainty
  – $u$ concave, $u'' < 0$

• Implications?
  – purchase of insurance (possible accident)
  
  – investment in risky asset (risky investment)
  
  – choice over time (future income uncertain)
• Experiment — Are you risk-averse?
4 Measures of Risk Aversion

• Nicholson, Ch. 7, pp. 217-221

• How risk averse is an individual?

• Two measures:
  - Absolute Risk Aversion $r_A$:
    \[ r_A = -\frac{u''(x)}{u'(x)} \]
  - Relative Risk Aversion $r_R$:
    \[ r_R = -\frac{u''(x)}{u'(x)} x \]

• Examples in the Problem Set
5 Insurance

• Individual has:
  
  – wealth $w$
  
  – utility function $u$, with $u' > 0$, $u'' < 0$

• Probability $p$ of accident with loss $L$

• Insurance offers coverage:

  – premium $q$ for each $1$ paid in case of accident
  
  – units of coverage purchased $\alpha$

• Individual maximization:

$$\max_{\alpha} (1 - p)u(w - q\alpha) + pu(w - q\alpha - L + \alpha)$$

$$s.t. \alpha \geq 0$$
• Assume $\alpha^* \geq 0$, check later

• First order conditions:

\[
0 = -q(1-p)u'(w-q\alpha) \\
+ (1-q)p u'(w-q\alpha - L + \alpha)
\]

or

\[
\frac{u'(w-q\alpha)}{u'(w-q\alpha-L+\alpha)} = \frac{1-q}{q} \frac{p}{1-p}.
\]

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* =$?
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
6 Next Lectures

- Risk aversion

- Applications:
  - Portfolio choice
  - Consumption choice II