Economics 101A
(Lecture 11)

Stefano DellaVigna

February 26, 2015

Outline

1. Introduction to probability
2. Expected Utility
3. Risk Aversion and Lottery
4. Measures of Risk Aversion
5. Insurance

1 Introduction to Probability

- Nicholson, Ch. 7, p. 209

- So far deterministic world:
  - income given, known M
  - interest rate known r

- But some variables are unknown at time of decision:
  - future income M_1?
  - future interest rate r_1?

- Generalize framework to allow for uncertainty

- Events that are truly unpredictable (weather)
- Event that are very hard to predict (future income)

- Probability is the language of uncertainty

- Example:

  - Income M_1 at $t = 1$ depends on state of the economy

  - Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)

  - Three probabilities: p_1, p_2, p_3

  - $p_1 = P(M_1) = P(\text{recession})$

- Properties:

  - $0 \leq p_i \leq 1$

  - $p_1 + p_2 + p_3 = 1$

- Mean income: $EM = \sum_{i=1}^{3} p_i M_i$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25$$

- Variance of income: $V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$\begin{aligned} V(M) &= \frac{1}{3}(20-25)^2 + \frac{1}{3}(25-25)^2 + \frac{1}{3}(30-25)^2 \\ &= \frac{1}{3}5^2 + \frac{1}{3}5^2 = 2/3 * 25 \end{aligned}$$

- Mean and variance if $(p_1, p_2, p_3) = (1/4, 1/2, 1/4)$?

2 Expected Utility

- Nicholson, Ch. 7, pp. 210-217

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\max U(c^1)$$
$$s.t.\ c_i^1 \leq M_i^1 + (1+r)(M^0 - c^0)$$

  with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U\left(M_i^1\right)$

- This is uncertain, depends on which i is realized!

- How do we evaluate future uncertain utility?

- **Expected utility**

$$EU = \sum_{i=1}^{3} p_i U\left(M_i^1\right)$$

- In example:

$$EU = 1/3U(20) + 1/3U(25) + 1/3U(30)$$

- Compare with $U(EC) = U(25)$.

- Agents prefer riskless outcome EM to uncertain outcome M if

$$\begin{aligned} 1/3U(20) + 1/3U(25) + 1/3U(30) &< U(25) \text{ or} \\ 1/3U(20) + 1/3U(30) &< 2/3U(25) \text{ or} \\ 1/2U(20) + 1/2U(30) &< U(25) \end{aligned}$$

- Picture

- Depends on sign of U'', on concavity/convexity

- Three cases:

  - $U''(x) = 0$ for all x. (linearity of U)

    - $U(x) = a + bx$

    - $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all x. (concavity of U)

    - $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all x. (convexity of U)

    - $1/2U(20) + 1/2U(30) > U(25)$

- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- **Theorem. (Jensen's inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where E indicates expectation. If f is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function U.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen's inequality then implies U concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income

3 Risk aversion and Lottery

- Risk aversion:
  - individuals dislike uncertainty
  - u concave, $u'' < 0$
- Implications?
  - purchase of insurance (possible accident)
  - investment in risky asset (risky investment)
  - choice over time (future income uncertain)

- Experiment — Are you risk-averse?

4 Measures of Risk Aversion

- Nicholson, Ch. 7, pp. 217-221

- How risk averse is an individual?

- Two measures:

  – Absolute Risk Aversion r_A:

$$r_A = -\frac{u''(x)}{u'(x)}$$

  – Relative Risk Aversion r_R:

$$r_R = -\frac{u''(x)}{u'(x)}x$$

- Examples in the Problem Set

5 Insurance

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$, $u'' < 0$
- Probability p of accident with loss L
- Insurance offers coverage:
  - premium \$$q$ for each \$1 paid in case of accident
  - units of coverage purchased α
- Individual maximization:

$$\max_{\alpha} (1-p)\, u\,(w - q\alpha) + pu\,(w - q\alpha - L + \alpha)$$
$$s.t. \alpha \geq 0$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$\begin{aligned} 0 &= -q(1-p)u'(w-q\alpha) \\ &\quad + (1-q)pu'(w-q\alpha-L+\alpha) \end{aligned}$$

or

$$\frac{u'(w-q\alpha)}{u'(w-q\alpha-L+\alpha)} = \frac{1-q}{q}\frac{p}{1-p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* =$?

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all): $\alpha^* < L$

- Exercise: Check second order conditions!

6 Next Lectures

- Risk aversion

- Applications:

  - Portfolio choice

  - Consumption choice II