Economics 101A
(Lecture 9)

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Outline

1. Slutsky Equation II
2. Complements and substitutes
3. Do utility functions exist?
4. Application 1: Labor Supply
5. Application 2: Intertemporal choice I

1 Slutsky Equation II

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M / p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i\left(\mathbf{p}, \overline{u}\right)}{\partial p_i} =$$

- Rewrite h_i^* as function of m: $h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M)\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial p_i} =$$

- It works!

2 Complements and substitutes

- Nicholson, Ch. 6, pp. 187-192

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

$$\begin{aligned} \frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} &= \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} \\ &\quad - x_j^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} \end{aligned}$$

- Substitution effect

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} > 0$$

for $n = 2$ (two goods). Ambiguous for $n > 2$.

- Income effect:

$$-x_j^*(p_1, p_2, M)\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial M}$$

  - negative if good i is normal
  - positive if good i is inferior

- How do we define complements and substitutes?

- Def. 1. Goods i and j are **gross substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods i and j are **gross complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

- Gross complements or gross substitutes? Neither!

- Notice: $\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(\mathbf{p},M)}{\partial p_i}$

- Better definition.

- Def. 3. Goods i and j are **net substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} > 0$$

- Def. 4. Goods i and j are **net complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$$

- Example 1 (ctd.): $h_1^* = \overline{u}\left(\frac{\alpha}{1-\alpha}\frac{p_2}{p_1}\right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

3 Do utility functions exist?

- Preferences and utilities are theoretical objects

- Many different ways to write them

- How do we tie them to the world?

- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
  - realism
  - simplicity

- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

- Estimate the parameters using the data

4 Labor Supply

- Nicholson Ch. 16, pp. 581-589

- Labor supply decision: how much to work in a day.

- Goods: consumption good c, hours worked h

- Price of good p, hourly wage w

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$

- Budget constraint?

- Income of consumer: $M + wh = M + w(24 - l)$

- Budget constraint: $pc \leq M + w(24 - l)$ or

$$pc + wl \leq M + 24w$$

- Notice: leisure l is a consumption good with price w. Why?

- General category: **opportunity cost**

- Instead of enjoying one hour of TV, I could have worked one hour and gained wage w.

- You should value the marginal hour of TV w!

- Opportunity costs are very important!

- Example 2. CostCo has a warehouse in SoMa

- SoMa used to have low cost land, adequate for warehouses

- Price of land in SoMa triples in 10 years.

- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?

- No.

- Did the opportunity cost of staying in SoMa go up?

- Yes!

- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\max u(c, l)$$
$$s.t.\ pc + wl \leq M + 24w$$

- Standard problem (except for $24w$)

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c, l) = c^{\alpha} l^{1-\alpha}$$

- Solution is

$$
\begin{aligned}
c^* &= \alpha \frac{M + 24w}{p} \\
l^* &= (1-\alpha)\left(24 + \frac{M}{w}\right)
\end{aligned}
$$

- Both c and l are normal goods

- Unlike in standard Cobb-Douglas problems, c^* depends on price of other good w

- Why? Agents are endowed with M AND 24 hours of l in this economy

- Normally, agents are only endowed with M

5 Intertemporal choice

- Nicholson Ch. 17, pp. 609-613

- So far, we assumed people live for one period only

- Now assume that people live for two periods:
  - $t = 0$ – people are young
  - $t = 1$ – people are old

- $t = 0$: income M_0, consumption c_0 at price $p_0 = 1$

- $t = 1$: income $M_1 > M_0$, consumption c_1 at price $p_1 = 1$

- Credit market available: can lend or borrow at interest rate r

- Budget constraint in period 1?

- Sources of income:
  - M_1
  - $(M_0 - c_0) * (1 + r)$ (this can be negative)

- Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1+r} c_1 \leq M_0 + \frac{1}{1+r} M_1$$

- Utility function?

- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1+\delta} U(c_1)$$

- $U' > 0$, $U'' < 0$

- δ is the discount rate

- Higher δ means higher impatience

- Elicitation of δ through hypothetical questions

- Person is indifferent between 1 hour of TV today and $1 + \delta$ hours of TV next period

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta} U(c_1)$$
$$s.t.\ c_0 + \frac{1}{1+r} c_1 \leq M_0 + \frac{1}{1+r} M_1$$

6 Next Lectures

- Applications:
  - Intertemporal Choice II
  - Economics of Altruism