Economics 101A
(Lecture 8)

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Outline

1. Price Changes II

2. Expenditure Minimization

3. Slutsky Equation
1 Price changes II

• Price of good $i$ decreases from $p_i$ to $p_i' > p_i$

• For example, decrease in price of good 2, $p_2' < p_2$

• Budget line tilts:

\[ x_2 = \frac{M}{p_2'} - x_1 \frac{p_1}{p_2'} \]

• New optimum?
• Demand curve: $x_i^*(p_i)$: demand for good $i$ as function of own price holding fixed $p_j$ and $M$

• Odd convention of economists: plot price $p_i$ on vertical axis and quantity $x_i$ on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?
  
  – Yes. Most cases

  – No. Good $i$ is *Giffen*

  – Ex.: Potatoes in Ireland

  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
2 Expenditure minimization

- Nicholson, Ch. 4, pp. 131-135; Ch. 5, pp. 155-157

- Solve problem EMIN (minimize expenditure):
  \[
  \min p_1 x_1 + p_2 x_2 \\
  \text{s.t. } u(x_1, x_2) \geq \bar{u}
  \]

- Choose bundle that attains utility \( \bar{u} \) with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility \( u \) strictly increasing in \( x_i \), can maximize s.t. equality

- Denote by \( h_i(p_1, p_2, \bar{u}) \) solution to EMIN problem

- \( h_i(p_1, p_2, \bar{u}) \) is Hicksian or compensated demand
• Graphically:
  
  – Fix indifference curve at level \( \bar{u} \)
  
  – Consider budget sets with different \( M \)
  
  – Pick budget set which is tangent to indifference curve

• Optimum coincides with optimum of Utility Maximization!

• Formally:

\[
h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))
\]
- Expenditure function is expenditure at optimum

- \[ e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u}) \]

- \( h_i(p_i) \) is *Hicksian or compensated demand* function

- Is \( h_i \) always decreasing in \( p_i \)? Yes!

- Graphical proof: moving along a convex indifference curve

- (For non-convex indifferent curves, still true)
• Using first order conditions:

\[ L(x_1, x_2, \lambda) = p_1x_1 + p_2x_2 - \lambda (u(x_1, x_2) - \bar{u}) \]

\[ \frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0 \]

• Write as ratios:

\[ \frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2} \]

• \( MRS \) = ratio of prices as in utility maximization!

• However: different constraint \( \implies \lambda \) is different
• Example 1: Cobb-Douglas utility

\[
\min p_1 x_1 + p_2 x_2 \\
\text{s.t. } x_1^\alpha x_2^{1-\alpha} \geq \bar{u}
\]

• Lagrangean =

• F.o.c.:

• Solution: \( h_1^* = \), \( h_2^* = \)

• \( \partial h_i^*/\partial p_i < 0, \partial h_i^*/\partial p_j > 0, j \neq i \)
3 Slutsky Equation

• Nicholson, Ch. 5, pp. 160-163

• Now: go back to Utility Max. in case where \( p_2 \) increases to \( p'_2 > p_2 \)

• What is \( \partial x^*_2 / \partial p_2 \)? Decompose effect:

  1. Substitution effect of an increase in \( p_i \)
     - \( \partial h^*_2 / \partial p_2 \), that is change in EMIN point as \( p_2 \) decreases
     - Moving along an indifference curve
     - Certainly \( \partial h^*_2 / \partial p_2 < 0 \)
2. Income effect of an increase in $p_i$

- $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods
• \( h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u})) \)

• How does the Hicksian demand change if price \( p_i \) changes?

\[
\frac{dh_i}{dp_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} \frac{\partial e(p, \bar{u})}{\partial p_i}
\]

• What is \( \frac{\partial e(p, \bar{u})}{\partial p_i} \)? Envelope theorem:

\[
\frac{\partial e(p, \bar{u})}{\partial p_i} = \frac{\partial}{\partial p_i} [p_1h_1^* + p_2h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})]
\]

\[
= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u}))
\]
Therefore

$$\frac{\partial h_i(p, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} x_1^*(p_1, p_2, e)$$

Rewrite as

$$\frac{\partial x_i^*(p, M)}{\partial p_i} = \frac{\partial h_i(p, v(p, M))}{\partial p_i} - x_1^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M}$$

Important result! Allows decomposition into substitution and income effect
- Two effects of change in price:

1. Substitution effect negative: \( \frac{\partial h_i(p,v(p,M))}{\partial p_i} < 0 \)

2. Income effect: \( -x_1^*(p_1, p_2, M) \frac{\partial x_i^*(p,M)}{\partial M} \)
   - negative if good \( i \) is normal \( (\frac{\partial x_i^*(p,M)}{\partial M} > 0) \)
   - positive if good \( i \) is inferior \( (\frac{\partial x_i^*(p,M)}{\partial M} < 0) \)

- Overall, sign of \( \frac{\partial x_i^*(p,M)}{\partial p_i} \)?
  - negative if good \( i \) is normal
  - it depends if good \( i \) is inferior
• Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

• $x_i^* = \alpha M/p_i$

• $h_i^* =$

• Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i (p, \overline{u})}{\partial p_i} =$$

• Rewrite $h_i^*$ as function of $m$: $h_i (p, v(p, M))$

• Compute $v(p, M) =$
• Substitution effect:

\[
\frac{\partial h_i(p, v(p, M))}{\partial p_i} = 
\]

• Income effect:

\[
-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} = 
\]

• Sum them up to get

\[
\frac{\partial x_i^*(p, M)}{\partial p_i} = 
\]

• It works!
4 Next Lectures

- Complements and Substitutes

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism