Outline

1. Utility Maximization II

2. Utility maximization – Tricky Cases

3. Indirect Utility Function

4. Comparative Statics (Introduction)

5. Income Changes

6. Price Changes
1 Utility maximization II

• Example with CES utility function.

$$\max_{x_1, x_2} \left( \alpha x_1^\rho + \beta x_2^\rho \right)^{1/\rho}$$

s.t. $p_1 x_1 + p_2 x_2 - M = 0$

• Solution:

$$x_1^* = \frac{M}{p_1 \left( 1 + \left( \frac{\alpha}{\beta} \right)^{1/\rho_1} \left( \frac{p_2}{p_1} \right)^{\rho/\rho_1} \right)}$$

$$x_2^* = \frac{M}{p_2 \left( 1 + \left( \frac{\beta}{\alpha} \right)^{1/\rho_2} \left( \frac{p_1}{p_2} \right)^{\rho/\rho_2} \right)}$$

• Special case 1: $\rho \to -\infty$ (Perfect Complements)

$$x_1^* = \frac{M}{p_1 + p_2} = x_2^*$$
• Parameter $\rho$ indicates substitution pattern between goods:

  - $\rho > 0 \rightarrow$ Goods are (net) substitutes
  
  - $\rho < 0 \rightarrow$ Goods are (net) complements
2 Utility maximization – tricky cases

1. Non-convex preferences. Example:
2. Example with CES utility function.

\[
\max_{x_1, x_2} \left( \alpha x_1^\rho + \beta x_2^\rho \right)^{1/\rho}
\]
\[
s.t. \ p_1 x_1 + p_2 x_2 - M = 0
\]

• With \( \rho > 1 \) the interior solution is a minimum!

• Draw indifference curves for \( \rho = 1 \) (boundary case) and \( \rho = 2 \)

• Can also check using second order conditions
2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\max x_1 \ast (x_2 + 5)$$

$$s.t. \ p_1x_1 + p_2x_2 = M$$

• In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

- **Example 1**: Perfect Substitutes with \( p_1/p_2 = \alpha/\beta \)

- **Example 2**: Non-convex preferences with two optima
3 Indirect utility function

- Nicholson, Ch. 4, pp. 128-130

- Define the indirect utility $v(p, M) \equiv u(x^*(p, M))$, with $p$ vector of prices and $x^*$ vector of optimal solutions.

- $v(p, M)$ is the utility at the optimum for prices $p$ and income $M$

- Some comparative statics: $\partial v(p, M) / \partial M = ?$

- Hint: Use Envelope Theorem on Lagrangean function
• What is the sign of $\lambda$?

• $\lambda = \frac{u'_x}{p} > 0$

• $\partial u(p, M)/\partial p_i =$?

• Properties:
  
  – Indirect utility is always increasing in income $M$
  
  – Indirect utility is always decreasing in the price $p_i$
4 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 145-155

- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed $x_i^*$ as prices or income varies?
• Simple case: Equal increase in prices and income.

\[ M' = tM, \ p'_1 = tp_1, \ p'_2 = tp_2. \]

• Compare \( x^*(tM, tp_1, tp_2) \) and \( x^*(M, p_1, p_2) \).

• What happens?

• Write budget line: \( tp_1 x_1 + tp_2 x_2 = tM \)

• Demand is homogeneous of degree 0 in \( p \) and \( M \):

\[
x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).
\]
• Consider Cobb-Douglas Case:

\[ x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, \quad x_2^* = \frac{\beta}{\alpha + \beta} M/p_2 \]

• What is \( \partial x_1^*/\partial M? \)

• What is \( \partial x_1^*/\partial p_1? \)

• What is \( \partial x_1^*/\partial p_2? \)

• General results?
5 Income changes

- Income increases from $M$ to $M' > M$.

- Budget line $(p_1x_1 + p_2x_2 = M)$ shifts out:

  $$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?
• Engel curve: \( x_i^*(M) \): demand for good \( i \) as function of income \( M \) holding fixed prices \( p_1, p_2 \)

• Does \( x_i^* \) increase with \( M \)?
  
  – Yes. Good \( i \) is normal
  
  – No. Good \( i \) is inferior
6 Price changes

- Price of good $i$ decreases from $p_i$ to $p_i' > p_i$

- For example, decrease in price of good 2, $p_2' < p_2$

- Budget line tilts:

\[
  x_2 = \frac{M}{p_2'} - x_1 \frac{p_1}{p_2'}
\]

- New optimum?
Demand curve: $x_i^*(p_i)$: demand for good $i$ as function of own price holding fixed $p_j$ and $M$.

Odd convention of economists: plot price $p_i$ on vertical axis and quantity $x_i$ on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?

  – Yes. Most cases

  – No. Good $i$ is *Giffen*

  – Ex.: Potatoes in Ireland

  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
7 Next Class

- More comparative statics:
  - More on Price Effects
  - Slutzky Equation

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism