

Economics 101A

(Lecture 7)

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Outline

1. Utility Maximization II
2. Utility maximization – Tricky Cases
3. Indirect Utility Function
4. Comparative Statics (Introduction)
5. Income Changes
6. Price Changes

1 Utility maximization II

- Example with CES utility function.

$$\begin{aligned} \max_{x_1, x_2} & (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho} \\ \text{s.t.} & p_1 x_1 + p_2 x_2 - M = 0 \end{aligned}$$

- Solution:

$$\begin{aligned} x_1^* &= \frac{M}{p_1 \left(1 + \left(\frac{\alpha}{\beta} \right)^{\frac{1}{\rho-1}} \left(\frac{p_2}{p_1} \right)^{\frac{\rho}{\rho-1}} \right)} \\ x_2^* &= \frac{M}{p_2 \left(1 + \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho-1}} \left(\frac{p_1}{p_2} \right)^{\frac{\rho}{\rho-1}} \right)} \end{aligned}$$

- Special case 1: $\rho \rightarrow -\infty$ (Perfect Complements)

$$x_1^* = \frac{M}{p_1 + p_2} = x_2^*$$

- Parameter ρ indicates substitution pattern between goods:
 - $\rho > 0 \rightarrow$ Goods are (net) substitutes
 - $\rho < 0 \rightarrow$ Goods are (net) complements

2 Utility maximization – tricky cases

1. Non-convex preferences. Example:

2. Example with CES utility function.

$$\begin{aligned} \max_{x_1, x_2} & (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho} \\ \text{s.t.} & p_1 x_1 + p_2 x_2 - M = 0 \end{aligned}$$

- With $\rho > 1$ the interior solution is a minimum!
- Draw indifference curves for $\rho = 1$ (boundary case) and $\rho = 2$
- Can also check using second order conditions

2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\begin{aligned} \max x_1 * (x_2 + 5) \\ \text{s.t. } p_1 x_1 + p_2 x_2 = M \end{aligned}$$

- In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

3. Multiplicity of solutions.

- Example 1: Perfect Substitutes with $p_1/p_2 = \alpha/\beta$
- Example 2: Non-convex preferences with two optima

3 Indirect utility function

- Nicholson, Ch. 4, pp. 128-130
- Define the indirect utility $v(\mathbf{p}, M) \equiv u(\mathbf{x}^*(\mathbf{p}, M))$, with \mathbf{p} vector of prices and \mathbf{x}^* vector of optimal solutions.
- $v(\mathbf{p}, M)$ is the utility at the optimum for prices \mathbf{p} and income M
- Some comparative statics: $\partial v(\mathbf{p}, M) / \partial M = ?$
- Hint: Use Envelope Theorem on Lagrangean function

- What is the sign of λ ?
- $\lambda = u'_{x_i}/p > 0$
- $\partial v(\mathbf{p}, M)/\partial p_i = ?$
- Properties:
 - Indirect utility is always increasing in income M
 - Indirect utility is always decreasing in the price p_i

4 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 145-155
- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$
- Quantity consumed as a function of income and price

- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM, p'_1 = tp_1, p'_2 = tp_2.$

- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2).$

- What happens?

- Write budget line: $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in \mathbf{p} and M :

$$x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).$$

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is $\partial x_1^*/\partial M$?

- What is $\partial x_1^*/\partial p_1$?

- What is $\partial x_1^*/\partial p_2$?

- General results?

5 Income changes

- Income increases from M to $M' > M$.
- Budget line ($p_1x_1 + p_2x_2 = M$) shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M ?

- Yes. Good i is *normal*

- No. Good i is *inferior*

6 Price changes

- Price of good i decreases from p_i to $p'_i > p_i$
- For example, decrease in price of good 2, $p'_2 < p_2$
- Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

- New optimum?

- Does x_i^* decrease with p_i ?

- Yes. Most cases

- No. Good i is *Giffen*

- Ex.: Potatoes in Ireland

- Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model

7 Next Class

- More comparative statics:
 - More on Price Effects
 - Slutsky Equation
- Then moving on to applications:
 - Labor Supply
 - Intertemporal choice
 - Economics of Altruism