Economics 101A
(Lecture 5)

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Outline

1. Properties of Preferences II

2. From Preferences to Utility (and vice versa)

3. Common Utility Functions

4. Utility maximization
1 Properties of Preferences II

• Nicholson, Ch. 3, pp. 89-90

• Commodity set $X$ (apples vs. strawberries, work vs. leisure, consume today vs. tomorrow)

• Preference relation $\succeq$ over $X$

• A preference relation $\succeq$ is rational if

  1. It is complete: For all $x$ and $y$ in $X$, either $x \succeq y$, or $y \succeq x$ or both

  2. It is transitive: For all $x$, $y$, and $z$, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$

• Preference relation $\succeq$ is continuous if for all $y$ in $X$, the sets $\{x : x \succeq y\}$ and $\{x : y \succeq x\}$ are closed sets.
• Example 2: choice of combinations of apples and oranges: \( X = \{(1, 0), (0, 1), (1, 1), (0, 0)\} \)

• Example 2: \( X = \mathbb{R}^2 \) with map of indifference curves
• Counterexamples:

  1. Incomplete preferences. Dominance rule.
2. Intransitive preferences. Quasi-discernible differences.
3. Discontinuous preferences. Lexicographic order
• Indifference relation $\sim$: $x \sim y$ if $x \succeq y$ and $y \succeq x$

• Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$

• Exercise. If $\succeq$ is rational,
  
  – $\succ$ is transitive
  
  – $\sim$ is transitive
  
  – Reflexive property of $\succeq$. For all $x$, $x \succeq x$. 
• Other features of preferences

• Preference relation $\succeq$ is:
  
  – *monotonic* if $x \geq y$ implies $x \succeq y$.

  – *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some $j$ implies $x \succ y$.

  – *convex* if for all $x$, $y$, and $z$ in $X$ such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all $t$ in $[0, 1]$.
2 From preferences to utility

• Nicholson, Ch. 3

• Economists like to use utility functions \( u : X \to \mathbb{R} \)

• \( u(x) \) is ‘liking’ of good \( x \)

• \( u(a) > u(b) \) means: I prefer \( a \) to \( b \).

• **Def.** Utility function \( u \) represents preferences \( \geq \) if, for all \( x \) and \( y \) in \( X \), \( x \geq y \) if and only if \( u(x) \geq u(y) \).

• **Theorem.** If preference relation \( \succeq \) is rational and continuous, there exists a continuous utility function \( u : X \to \mathbb{R} \) that represents it.
• [Skip proof]

• Example:

\[(x_1, x_2) \succeq (y_1, y_2) \text{ iff } x_1 + x_2 \geq y_1 + y_2\]

• Draw:

• Utility function that represents it: \( u(x) = x_1 + x_2 \)

• But... Utility function representing \( \succeq \) is not unique

• Take \( 3u(x) \) or \( \exp(u(x)) \)

• \( u(a) > u(b) \iff \exp(u(a)) > \exp(u(b)) \)
• If $u(x)$ represents preferences $\succeq$ and $f$ is a strictly increasing function, then $f(u(x))$ represents $\succeq$ as well.

• If preferences are represented from a utility function, are they rational?
  
  – completeness
  
  – transitivity
• Indifference curves: $u(x_1, x_2) = \bar{u}$

• They are just implicit functions! $u(x_1, x_2) − \bar{u} = 0$

$$\frac{dx_2}{dx_1} = -\frac{U'_x}{U'_x} = MRS$$

• Indifference curves for:

  – monotonic preferences;

  – strictly monotonic preferences;

  – convex preferences
3 Common utility functions

- Nicholson, Ch. 3, pp. 102-105

1. Cobb-Douglas preferences: \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \)

- \( MRS = -\alpha x_1^{\alpha-1} x_2^{1-\alpha} / (1-\alpha) x_1^\alpha x_2^{-\alpha} = \frac{\alpha}{1-\alpha x_1} x_2 \)

2. Perfect substitutes: \( u(x_1, x_2) = \alpha x_1 + \beta x_2 \)

- \( MRS = -\alpha / \beta \)
3. Perfect complements: \( u(x_1, x_2) = \min(\alpha x_1, \beta x_2) \)

- \( MRS \) discontinuous at \( x_2 = \frac{\alpha}{\beta} x_1 \)

4. Constant Elasticity of Substitution: \( u(x_1, x_2) = \left( \alpha x_1^\rho + \beta x_2^\rho \right)^{1/\rho} \)

- \( MRS = -\frac{\alpha}{\beta} \left( \frac{x_1}{x_2} \right)^{\rho - 1} \)

- if \( \rho = 1 \), then...

- if \( \rho = 0 \), then...

- if \( \rho \to -\infty \), then...
4 Utility Maximization

- Nicholson, Ch. 4, pp. 119–128

- $X = R^2_+$ (2 goods)

- Consumers: choose bundle $x = (x_1, x_2)$ in $X$ which yields highest utility.

- Constraint: income = $M$

- Price of good 1 = $p_1$, price of good 2 = $p_2$

- Bundle $x$ is feasible if $p_1 x_1 + p_2 x_2 \leq M$

- Consumer maximizes

$$\max_{x_1, x_2} u(x_1, x_2)$$

$$s.t. \ p_1 x_1 + p_2 x_2 \leq M$$

$$x_1 \geq 0, \ x_2 \geq 0$$
• Maximization subject to inequality. How do we solve that?

• Trick: $u$ strictly increasing in at least one dimension. ($\succeq$ strictly monotonic)

• Budget constraint always satisfied with equality

• Ignore temporarily $x_1 \geq 0$, $x_2 \geq 0$ and check afterwards that they are satisfied for $x_1^*$ and $x_2^*$. 
Problem becomes

\[
\max_{x_1, x_2} u(x_1, x_2) \\
\text{s.t. } p_1 x_1 + p_2 x_2 - M = 0
\]
5 Next Class

• Utility Maximization (ctd)

• Utility Maximization – tricky cases

• Indirect Utility Function