

# 219B – Exercise on Present Bias and Retirement Savings

## Question #1

In this Question we consider the impact of self-control problems on investment in retirement savings. Consider a present-biased individual that is considering when (and whether) to undertake an investment activity with immediate costs and delayed benefits. The main example will be calling the Human Resources Department to change the 401(k) allocation. Compared to the alternative activity, which has payoff 0, the investment activity has payoff  $-k < 0$  at time  $t$  (the present) and payoff  $b > 0$  for all periods from  $t+1$  on. ( $t+1$  included). The individual has to choose when to undertake the investment activity, that is, at  $t$ , at  $t+1$ , at  $t+2$ , etc. (The individual can also decide not to do it, which we define as doing at  $t = \infty$ ) Assume that both  $k$  and  $b$  are deterministic.

a) Consider first a time-consistent individual ( $\beta = \hat{\beta} = 1$ ) and solve for the optimal timing of the investment decision. **Show** that the optimal solution takes the form of a threshold rule as a function of  $k$ ,  $\delta$ , and  $b$ .

b) Consider then a sophisticated present-biased individual ( $\beta = \hat{\beta} < 1$ ). Compute the utility for the current self from investing today, at time  $t$ . Compute the utility for the current self from investing  $T$  periods into the future, that is, at  $t + T$ .pw

c) Show that this implies that a sophisticated agent will wait for at most  $T$  days to invest if the cost of investing  $k$  satisfies

$$k \leq \frac{\beta\delta b}{1 - \beta\delta^T} T \quad (1)$$

[You will need a Taylor expansion of  $1 - \delta^T$  for  $\delta$  going to 1:  $1 - \delta^T \simeq (1 - \delta) T$ ]

d) Consider now a fully naive present-biased individual ( $\beta < \hat{\beta} = 1$ ). As of time  $t$ , under what conditions does the individual expect to invest tomorrow (at  $t + 1$ )? Argue that the naive agent compares the utility from investing today and tomorrow.

e) Show that the fully naive present-biased individual invests at time  $t$  (and otherwise never invests) if and only if

$$k \leq \frac{\beta\delta b}{1 - \beta\delta}.$$

f) In particular, discuss what the following sentence means: *The naive agent procrastinates if*

$$\frac{\beta\delta b}{1 - \beta\delta} < k \leq \frac{\delta b}{1 - \delta}.$$

What is the difference between procrastinating ( $\frac{\beta\delta b}{1 - \beta\delta} < k \leq \frac{\delta b}{1 - \delta}$ ) and not investing ( $k > \frac{\delta b}{1 - \delta}$ )? Explain intuitively.

g) Compare the behavior of the naive to the behavior of the sophisticated and exponential individual using the solutions above. Provide intuition.

h) Consider now the case in which the setting is the same as above, but the cost of investment  $k$  is stochastic, with i.i.d. draws in each period  $t$ . Show that the decision rule for an exponential agent will be of the type: Invest today if  $k < k^e$  where  $k^e$  is defined by

$$-k^e + \frac{\delta b}{1 - \delta} = \delta V^e(k^e) \quad (2)$$

and  $V(k^e)$  is the value function of the exponential agent. [To solve out for  $k^e$ , do for uniform distribution]

i) Show that the rule for a fully naive present-biased agent is: Invest today if  $k < k^n$  where  $k^n$  is defined by

$$-k^n + \beta \frac{\delta b}{1 - \delta} = \beta \delta V^e(k^e) \quad (3)$$

and  $V(k^e)$  is the value function of the exponential agent. Compare (2) and (3) and obtain a relationship between  $k^n$  and  $k^e$ .

j) In the model with stochastic  $k$ , do naives delay forever? Assume  $\beta = .5$  and draw a distribution of  $k$  for which naives will delay forever and a distribution for which they will not. (Of course, you cannot determine  $k^e$  just graphically, assume that  $k^e$  is set at the 20th percentile of the cost distribution in both cases).

## Question #2

In this Question we apply the results of Question 1 to default effects in 401(k) choice (Madrian and Shea, 2001; Choi et al., 2006; Carroll et al., 2009—all in the reading list).

a) Consider the attached Table 1 (consider only companies B, C, D, and H) and Figures 1a-1d from the survey paper “Saving for Retirement on The Path of Least Resistance” (Choi et al., 2006). Choi et al. report the result of changes in default for 401(k) investments in 4 companies. The change is of a similar type as in Madrian and Shea (2001) Comment the findings in the Figures. Describe the effect of the change in default.

b) To what extent this evidence goes beyond the evidence in Madrian and Shea (2001)? Cite at least one concern with the evidence in the Madrian and Shea (2001) paper that the evidence in Choi et al. (2006) addresses.

c) Now we go back to the answers in Question 1 and calibrate them to address the evidence in Madrian and Shea (1999) and Choi et al. (2006). As in Question 1, consider a new employee in a company without automatic enrollment (that is, the default is no investment). On any day, the employee can pay an effort cost  $k > 0$  and invest in the 401(k), thereafter reaping benefit  $b$  in every subsequent day. Can you provide reasonable values for  $k$ ,  $\delta$  and  $\beta$  for an individual with average earnings? Justify all the assumptions you make. (Remember:  $b$  and  $\delta$  are on a *daily* scale).

d) In particular, which factors would enter into the determination of  $b$ ? Can it be negative for some employees? Be as precise as you can in deriving an expression for the daily future benefit of investing for retirement,  $b$ . From now on, assume that  $b$  is positive.

e) Go back to the time-consistent employee in point (a) of Question 1. For what value of  $k$  should the individual be indifferent between investing and not (given the calibrated values of  $b$ ,  $\delta$  and  $\beta$ )? What do you expect the individual to do?

f) Move on now to the sophisticated present-biased employee in points (b) and (c) of Question 1. Using equation (1) calibrate the value of  $k$  for which the individual may wait 360 days (that is, one year) to invest, which is about the observed pattern. Are these plausible levels of the parameters, or do you expect that a sophisticated agent will invest earlier? What is a maximal plausible length of delay  $T$ ?

g) Consider now the naive present-biased employee in points (d)-(f). For realistic values of the parameters, is it likely that the employee will rationally delay ( $k > \frac{\delta b}{1-\delta}$ )? Is it likely that the employee will procrastinate ( $\frac{\beta \delta b}{1-\beta \delta} < k \leq \frac{\delta b}{1-\delta}$ )?

h) How do the predictions for the different models (for the calibrated values) vary once the default is shifted to automatic investment? (Assume that this means  $k < 0$  and ‘small’)

i) Conjecture (without explicitly solving the model) what the difference in qualitative predictions in the above points would be if we allowed  $k$  to be stochastic, say, i.i.d. from distribution  $F$ .

- j) In light of points (a)-(i), which calibrated model fits the data better? Why?
- k) Consider now the evidence in the ‘Active Decision’ paper (Carroll et al., 2009) Summarize the evidence in this paper on the impact of the introduction of ‘Active Decision’.
- l) Go back to the calibrated models for the case in which the default is Active Decision. (Assume that this means  $k = 0$ ) Describe the predictions of the different models on the effect of the change in default in Carroll et al. (2007). Which model fits best?
- m) Which problems does your favorite model have? Can you imagine a way to fix them?