Economics 101A
(Lecture 1)

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Outline

1. Who are we?

2. Prerequisites for the course

3. A test in maths

4. Optimization with 1 variable
1 Who are we?

Stefano DellaVigna – call me ‘Stefano’

- Professor of Economics

- Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

- Psychology and Economics, Economics of Media, Behavioral Finance

- Evans 515, Office Hours: TBA
Jonas Tungodden (2 Sections)

- Graduate Student, Department of Economics
- Psychology and Economics
- Office Hours: TBA

Katalin Springel (1 Section)

- Graduate Student, Department of Economics
- Industrial Organization
- Office Hours: TBA
2 Prerequisites

• Mathematics
  – Good knowledge of multivariate calculus – Maths 1A or 1B and 53
  – Basic knowledge of probability theory and matrix algebra

• Economics
  – Knowledge of fundamentals – Ec1 or 2 or 3
  – High interest!
• Go over syllabus
3  A Test in Maths

1. Can you differentiate the following functions with respect to \( x \)?

   (a) \( y = \exp(x) \)

   (b) \( y = a + bx + cx^2 \)

   (c) \( y = \frac{\exp(x)}{b^x} \)

2. Can you partially differentiate these functions with respect to \( x \) and \( w \)?

   (a) \( y = axw + bx - c\frac{x}{w} + d\sqrt{xw} \)

   (b) \( y = \exp(x/w) \)

   (c) \( y = \int_0^1 (x + aw^2 + xs) \, ds \)
3. Can you plot the following functions of one variable?

(a) \( y = \exp(x) \)

(b) \( y = -x^2 \)

(c) \( y = \exp(-x^2) \)

4. Are the following functions concave, convex or neither?

(a) \( y = x^3 \)

(b) \( y = -\exp(x) \)

(c) \( y = x^5 y^5 \) for \( x > 0, y > 0 \)
5. Consider an urn with 20 red and 40 black balls?

(a) What is the probability of drawing a red ball?

(b) What is the probability of drawing a black ball?

6. What is the determinant of the following matrices?

   (a) \[ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

   (b) \[ A = \begin{bmatrix} 10 & 10 \\ 20 & 20 \end{bmatrix} \]
4 Optimization with 1 variable

- Nicholson, Ch.2, pp. 20-23

- Example. Function $y = -x^2$ – Graph it

- What is the maximum?

  - Maximum is at 0

- General method?
• Sure! Use derivatives

• Derivative is slope of the function at a point:

\[ \frac{\partial f(x)}{\partial x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

• Necessary condition for maximum \( x^* \) is

\[ \frac{\partial f(x^*)}{\partial x} = 0 \]  \hspace{1cm} (1)

• Try with \( y = -x^2 \).

\[ \frac{\partial f(x)}{\partial x} = \hspace{1cm} = 0 \implies x^* = \]
• Does this guarantee a maximum? No!

• Consider the function $y = x^3$

  \[ \frac{\partial f(x)}{\partial x} =\quad \quad = 0 \rightarrow x^* = \]

• Plot $y = x^3$. 
• Sufficient condition for a (local) maximum:

\[
\frac{\partial f(x^*)}{\partial x} = 0 \quad \text{and} \quad \frac{\partial^2 f(x)}{\partial^2 x} \bigg|_{x^*} < 0 \quad (2)
\]

• Proof: At a maximum, \( f(x^* + h) - f(x^*) < 0 \) for all \( h \).

• Taylor Rule: \( f(x^* + h) - f(x^*) = \frac{\partial f(x^*)}{\partial x} h + \frac{1}{2} \frac{\partial^2 f(x^*)}{\partial^2 x} h^2 + \) higher order terms.

• Notice: \( \frac{\partial f(x^*)}{\partial x} = 0 \).

• \( f(x^* + h) - f(x^*) < 0 \) for all \( h \) \( \Rightarrow \) \( \frac{\partial^2 f(x^*)}{\partial^2 x} h^2 < 0 \)

\( 0 \Rightarrow \frac{\partial^2 f(x^*)}{\partial^2 x} < 0 \)

• Careful: Maximum may not exist: \( y = \exp(x) \)
• Tricky examples:

  – *Minimum.* $y = x^2$

  – *No maximum.* $y = \exp(x)$ for $x \in (-\infty, +\infty)$

  – *Corner solution.* $y = x$ for $x \in [0, 1]$
5 Multivariate optimization I

- Nicholson, Ch.2, pp. 26-31 and 33-35

- Function from $\mathbb{R}^n$ to $\mathbb{R}$: $y = f(x_1, x_2, \ldots, x_n)$

- Partial derivative with respect to $x_i$:
  \[
  \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i} = \lim_{h \to 0} \frac{f(x_1, \ldots, x_i + h, \ldots x_n) - f(x_1, \ldots, x_i, \ldots x_n)}{h}
  \]

  - Slope along dimension $i$

- Total differential:
  \[
  df = \frac{\partial f(x)}{\partial x_1}dx_1 + \frac{\partial f(x)}{\partial x_2}dx_2 + \ldots + \frac{\partial f(x)}{\partial x_n}dx_n
  \]
• One important economic example

• Example 1: Partial derivatives of \( y = f(L, K) = L^{.5}K^{.5} \)

• \( f'_{L} = \)
  (marginal productivity of labor)

• \( f'_{K} = \)
  (marginal productivity of capital)

• \( f''_{L,K} = \)
Maximization over an open set (like $\mathbb{R}$)

- **Necessary condition for maximum** $x^*$ is
  \[
  \frac{\partial f(x^*)}{\partial x_i} = 0 \quad \forall i
  \]  
  (3)

  or in vectorial form
  \[
  \nabla f(x) = 0
  \]

- These are commonly referred to as first order conditions (f.o.c.)

- Sufficient conditions? Next lecture
6 Next Class

• Example: Economics of Discrimination

• Multivariate Maximization II

• Comparative Statics

• Implicit Function Theorem

• Envelope Theorem

• Going toward:
  – Preferences
  – Utility Maximization (where we get to apply maximization techniques the first time)