Economics 101A
(Lecture 1)

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Outline

1. Who are we?
2. Prerequisites for the course
3. A test in maths
4. Optimization with 1 variable

1 Who are we?

Stefano DellaVigna – call me 'Stefano'

- Professor of Economics

- Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

- Psychology and Economics, Economics of Media, Behavioral Finance

- Evans 515, Office Hours: TBA

Jonas Tungodden (2 Sections)

- Graduate Student, Department of Economics
- Psychology and Economics
- Office Hours: TBA

Katalin Springel (1 Section)

- Graduate Student, Department of Economics
- Industrial Organization
- Office Hours: TBA

2 Prerequisites

- Mathematics
  - Good knowledge of multivariate calculus – Maths 1A or 1B and 53
  - Basic knowledge of probability theory and matrix algebra
- Economics
  - Knowledge of fundamentals – Ec1 or 2 or 3
  - High interest!

- Go over syllabus

3 A Test in Maths

1. Can you differentiate the following functions with respect to x?

   (a) $y = \exp(x)$

   (b) $y = a + bx + cx^2$

   (c) $y = \frac{\exp(x)}{b^x}$

2. Can you partially differentiate these functions with respect to x and w?

   (a) $y = axw + bx - c\frac{x}{w} + d\sqrt{xw}$

   (b) $y = \exp(x/w)$

   (c) $y = \int_0^1 (x + aw^2 + xs)ds$

3. Can you plot the following functions of one variable?

   (a) $y = \exp(x)$

   (b) $y = -x^2$

   (c) $y = \exp(-x^2)$

4. Are the following functions concave, convex or neither?

   (a) $y = x^3$

   (b) $y = -\exp(x)$

   (c) $y = x^{.5}y^{.5}$ for $x > 0$, $y > 0$

5. Consider an urn with 20 red and 40 black balls?

   (a) What is the probability of drawing a red ball?

   (b) What is the probability of drawing a black ball?

6. What is the determinant of the following matrices?

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 10 & 10 \\ 20 & 20 \end{bmatrix}$

4 Optimization with 1 variable

- Nicholson, Ch.2, pp. 20-23

- Example. Function $y = -x^2$ – Graph it

- What is the maximum?

- Maximum is at 0

- General method?

- Sure! Use derivatives

- Derivative is slope of the function at a point:

$$\frac{\partial f(x)}{\partial x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Necessary condition for maximum** x^* is

$$\frac{\partial f(x^*)}{\partial x} = 0 \tag{1}$$

- Try with $y = -x^2$.

- $\frac{\partial f(x)}{\partial x} =$ $\qquad\qquad = 0 \Longrightarrow x^* =$

- Does this guarantee a maximum? No!

- Consider the function $y = x^3$

- $\frac{\partial f(x)}{\partial x} = \qquad\qquad\qquad = 0 \Longrightarrow x^* =$

- Plot $y = x^3$.

- **Sufficient condition for a (local) maximum**:

$$\frac{\partial f(x^*)}{\partial x} = 0 \text{ and } \left.\frac{\partial^2 f(x)}{\partial^2 x}\right|_{x^*} < 0 \qquad (2)$$

- Proof: At a maximum, $f(x^* + h) - f(x^*) < 0$ for all h.

- Taylor Rule: $f(x^*+h)-f(x^*) = \frac{\partial f(x^*)}{\partial x}h+\frac{1}{2}\frac{\partial^2 f(x^*)}{\partial^2 x}h^2+$ higher order terms.

- Notice: $\frac{\partial f(x^*)}{\partial x} = 0$.

- $f(x^* + h) - f(x^*) < 0$ for all $h \Longrightarrow \frac{\partial^2 f(x^*)}{\partial^2 x}h^2 < 0 \Longrightarrow \frac{\partial^2 f(x^*)}{\partial^2 x} < 0$

- Careful: Maximum may not exist: $y = \exp(x)$

- Tricky examples:
  - *Minimum.* $y = x^2$
  - *No maximum.* $y = \exp(x)$ for $x \in (-\infty, +\infty)$
  - *Corner solution.* $y = x$ for $x \in [0, 1]$

5 Multivariate optimization I

- Nicholson, Ch.2, pp. 26-31 and 33-35

- Function from R^n to R: $y = f(x_1, x_2, ..., x_n)$

- Partial derivative with respect to x_i:

$$\frac{\partial f(x_1, ..., x_n)}{\partial x_i} = \lim_{h \to 0} \frac{f(x_1,..., x_i + h, ...x_n) - f(x_1,..., x_i, ...x_n)}{h}$$

- Slope along dimension i

- Total differential:

$$df = \frac{\partial f(x)}{\partial x_1} dx_1 + \frac{\partial f(x)}{\partial x_2} dx_2 + ... + \frac{\partial f(x)}{\partial x_n} dx_n$$

- One important economic example

- Example 1: Partial derivatives of $y = f(L, K) = L^{.5}K^{.5}$

- $f'_L =$
  (marginal productivity of labor)

- $f'_K =$
  (marginal productivity of capital)

- $f''_{L,K} =$

Maximization over an open set (like R)

- **Necessary condition for maximum** x^* is

$$\frac{\partial f(x^*)}{\partial x_i} = 0 \ \forall i \tag{3}$$

or in vectorial form

$$\nabla f(x) = 0$$

- These are commonly referred to as first order conditions (f.o.c.)

- Sufficient conditions? Next lecture

6 Next Class

- Example: Economics of Discrimination
- Multivariate Maximization II
- Comparative Statics
- Implicit Function Theorem
- Envelope Theorem

- Going toward:
  - Preferences
  - Utility Maximization (where we get to apply maximization techniques the first time)