Outline

1. Example of General Equilibrium

2. Existence and Welfare Theorems

3. Asymmetric Information: Introduction

4. Hidden Action (Moral Hazard)
1 Example

- Consumer 1 has Leontieff preferences:
  \[ u(x_1, x_2) = \min (x_1^1, x_2^1) \]

- Bundle demanded by consumer 1:
  \[
  x_1^{1*} = x_2^{1*} = x^{1*} = \frac{p_1 \omega^1_1 + p_2 \omega^1_2}{p_1 + p_2} = \\
  = \frac{\omega^1_1 + (p_2/p_1) \omega^1_2}{1 + (p_2/p_1)}
  \]

- Graphically
• Comparative statics:

  - increase in $\omega$

  - increase in $p_2/p_1$:

    $$\frac{dx_1^{1*}}{dp_2/p_1} = \frac{\omega_2 \left(1 + \left(p_2/p_1\right)\right)}{(1 + (p_2/p_1))^2} = \frac{\omega_2^1 - \omega_1^1}{(1 + (p_2/p_1))^2}$$

  - Effect depends on income effect through endowments:

    * A lot of good 2 $\rightarrow$ increase in price of good 2 makes richer

    * Little good 2 $\rightarrow$ increase in price of good 2 makes poorer

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5} \]

• Demands of consumer 2:

\[ x_1^{2*} = \frac{.5 \left( p_1 \omega_1^2 + p_2 \omega_2^2 \right)}{p_1} = .5 \left( \frac{\omega_1^2 + \frac{p_2}{p_1} \omega_2^2}{p_1} \right) \]

and

\[ x_2^{2*} = \frac{.5 \left( p_1 \omega_1^2 + p_2 \omega_2^2 \right)}{p_2} = .5 \left( \frac{p_1}{p_2} \omega_1^2 + \omega_2^2 \right) \]
- Comparative statics:
  - increase in $\omega \rightarrow$ Increase in final consumption
  - increase in $p_2/p_1 \rightarrow$ Unambiguous increase in $x_1^{2*}$ and decrease in $x_2^{2*}$
• Impose Walrasian equilibrium in market 1:

\[ x_1^{1\ast} + x_2^{2\ast} = \omega_1 + \omega_2 \]

This implies

\[
\frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} + .5 \left( \omega_1^2 + \frac{p_2}{p_1} \omega_2^2 \right) = \omega_1^1 + \omega_2^1
\]

or

\[
.5 - .5 \frac{(p_2/p_1)}{1 + (p_2/p_1)} \omega_1^1 + \frac{.5 (p_2/p_1) + .5 (p_2/p_1)^2 - 1}{1 + (p_2/p_1)} \omega_2^1 = 0
\]

or

\[
(\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0
\]
• Solution for $p_2/p_1$:

$$
\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\frac{\left(\omega_1^1 + \omega_2^1\right)^2}{-4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}}{2\left(\omega_1^1 - 2\omega_2^1\right)}
$$

• Some complicated solution!

• Problem set has solution that is easier to compute (and interpret)
2 Existence and Welfare Theorems

• Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

• Is Walrasian Equilibrium always unique? Not necessarily

• Is Walrasian Equilibrium efficient? Yes.
• **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

• Figure
• **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation \( ((x_1^1, x_1^1), (x_1^2, x_2^2)) \) there exists some endowment \( (\omega_1, \omega_2) \) such that \( ((x_1^1, x_1^1), (x_1^2, x_2^2)) \) is a Walrasian Equilibrium for endowment \( (\omega_1, \omega_2) \).

• Figure
• Significance of these results:
  
  – First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
  
  – BUT: problems with externalities and public good
  
  – BUT: what about distribution?
  
  – Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
  
  – But redistribution is hard to implement, and distortive.
3 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 641-645

- Common economic relationship

- Contract between two parties:
  - Principal
  - Agent

- Two parties have asymmetric information
  - Principal offers a contract to the agent
  - Agent chooses an action
  - Action of agent (or his type) is not observed by principle
• Example 1: *Manager and worker*
  
  – Manager employs worker and offers wage
  
  – Worker exerts effort (not observed)
  
  – Manager pays worker as function of output
  
• Example 2: *Car Insurance*
  
  – Car insurance company offers insurance contract
  
  – Driver chooses quality of driving (not observed)
  
  – Insurance company pays for accidents
  
• Example 3: *Shareholders and CEO*
  
  – Shareholders choose compensation for CEO
  
  – CEO puts effort
  
  – CEO paid as function of stock price
• In all of these cases (and many more!), common structure

  – Principal would like to observe effort (of worker, of CEO, of driver)

  – Unfortunately, this is not observable

  – Only a related, noisy proxy is observable: output, accident, success

  – Contract offered by principal is function of this proxy

• This means that occasionally an agent that put a lot of effort but has bad luck is ‘punished’

• Also, agents that shirked may instead be compensated

• These principle-agent problems are called *hidden action* or *moral hazard*
• Second category (next lecture): *hidden type* or *adverse selection*

• Example 1: *Manager and worker*
  – Manager employs worker and offers wage
  – Worker can be hard-working or lazy

• Example 2: *Car Insurance*
  – Car insurance company offers insurance contract
  – Drivers ex ante can be careful or careless

• Example 3: *Shareholders and CEO*
  – Shareholders choose compensation for CEO
  – CEO is high-quality or thief
• Problem is similar (action is not observed), but with a twist
  
  – *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives
  
  – *Hidden type*: agent’s behavior is not affected by incentives, but by her type

• Different task for principal:
  
  – *Hidden action*: Principal wants to incentivize agent to work hard
  
  – *Hidden type*: Principal wants to make sure to recruit ‘good’ agent, not ‘bad’ one

• Two look similar, but analysis is different

• Start from *Hidden Action*
4 Hidden Action (Moral Hazard)

- Nicholson, Ch. 18, pp. 645-650

- Example 3: Shareholders and CEO
  - Division of ownership and control

- Shareholders (owners of firm):
  - Have capital, but do not have time to run company themselves
  - Want firm run so as to maximize profits

- CEO (manager)
  - Has time and managerial skill
  - Does not have capital to own the firm
• If CEO owns the company (private enterprises), problem is solved \( \rightarrow \) Infeasible in large companies

• Agent chooses effort \( e \) (unobserved)
  
  – Induces output \( y = e + \varepsilon \), where \( \varepsilon \) is a noise term, with \( E(\varepsilon) = 0 \)

  – Example: Despite putting effort, investment project did not succeed

• Principal pays a salary \( w \) to the agent

  – Salary is a function of output \( y \): \( w = w(y) \)

  – Remember: Salary cannot be function of effort \( e \)
• Principal maximizes expected profits

\[ E [\pi] = E [y - w (y)] = e - E [w (y)] \]

• Agent is risk averse and maximizes

\[ E [U (w (e + \varepsilon))] - c (e) \]

- \( c (e) \) is cost of effort: assume \( c' (e) > 0 \) and \( c'' (e) > 0 \) for all \( e \)

- Utility function \( U \) satisfies \( U' > 0 \) and \( U'' < 0 \)

- Notice: Agent is risk-averse, Principal is risk-neutral

• Assume \( U (w) = -e^{-\gamma w} \) and \( \varepsilon \sim N (0, \sigma^2) \)

• Can solve explicitly for \( EU (w) \):

\[ EU (w) = -\frac{1}{\sqrt{2\pi\sigma^2}} \int e^{-\gamma w} e^{-\frac{1}{2} \frac{(w - \mu_w)^2}{\sigma^2_w}} \, dw = \mu_w - \frac{\gamma}{2} \sigma^2_w \]

[Take this for granted]
- Expected utility of agent is \( EU(w) = \mu_w - \frac{\gamma}{2}\sigma_w^2 \)

- Note: \( \mu_w \) is average salary and \( \sigma_w^2 \) is variance of salary
  - Agent likes high mean salary \( \mu_w \)
  - Agent dislikes variance in salary \( \sigma_w^2 \)
  - Dislike for variance increases in risk aversion \( \gamma \)

- Assume that contract is linear: \( w = a + by = a + be + b\varepsilon \)
  - Compute \( \mu_w = E(w) = E[a + be + b\varepsilon] = a + be + bE[\varepsilon] = a + be \)
  - Compute \( \sigma_w^2 = Var[a + be + b\varepsilon] = b^2\sigma^2 \)

- Rewrite expected utility as
  \[
  EU(w) = a + be - \frac{\gamma}{2}b^2\sigma^2
  \]
• Back to Principal-Agent problem

• Solve problem in three Steps, starting from last stage (backward induction)

  – **Step 1** (*Effort Decision*). Given contract $w(y)$, what effort $e^*$ is agent going to put in?

  – **Step 2.** (*Individual Rationality*) Given contract $w(y)$ and anticipating to put in effort $e^*$, does agent accept the contract?

  – **Step 3.** (*Profit Maximization*) Anticipating that the effort of the agent $e^*$ (and the acceptance of the contract) will depend on the contract, what contract $w(y)$ does principal choose to maximize profits?
5 Next lecture

- Asymmetric Information
- Moral Hazard