# Economics 101A (Lecture 24)

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#### Outline

- 1. Example of General Equilibrium
- 2. Existence and Welfare Theorems
- 3. Asymmetric Information: Introduction
- 4. Hidden Action (Moral Hazard)

#### 1 Example

• Consumer 1 has Leontieff preferences:

$$u(x_{1,}x_{2}) = \min\left(x_{1}^{1}, x_{2}^{1}\right)$$

• Bundle demanded by consumer 1:

$$x_1^{1*} = x_2^{1*} = x^{1*} = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)}$$

• Graphically

- Comparative statics:
  - increase in  $\omega$
  - increase in  $p_2/p_1$ :

$$\frac{dx_1^{1*}}{dp_2/p_1} = \frac{-\left(\omega_1^1 + (p_2/p_1)\right)}{\left(1 + (p_2/p_1)\omega_2^1\right)} = \frac{\omega_2^1 - \omega_1^1}{\left(1 + (p_2/p_1)\right)^2} =$$

- Effect depends on income effect through endowments:
  - \* A lot of good 2 -> increase in price of good
    2 makes richer
  - Little good 2 -> increase in price of good 2 makes poorer
- Notice: Only ratio of prices matters (general feature)

• Consumer 2 has Cobb-Douglas preferences:

$$u(x_{1,x_{2}}) = (x_{1}^{2})^{.5} (x_{2}^{2})^{.5}$$

• Demands of consumer 2:

$$x_1^{2*} = \frac{.5\left(p_1\omega_1^2 + p_2\omega_2^2\right)}{p_1} = .5\left(\omega_1^2 + \frac{p_2}{p_1}\omega_2^2\right)$$

 $\mathsf{and}$ 

$$x_2^{2*} = \frac{.5\left(p_1\omega_1^2 + p_2\omega_2^2\right)}{p_2} = .5\left(\frac{p_1}{p_2}\omega_1^2 + \omega_2^2\right)$$

- Comparative statics:
  - increase in  $\omega$  –> Increase in final consumption
  - increase in  $p_2/p_1$  –> Unambiguous increase in  $x_1^{2\ast}$  and decrease in  $x_2^{2\ast}$

• Impose Walrasian equilibrium in market 1:

$$x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$$

This implies

$$\frac{\omega_1^1 + (p_2/p_1)\omega_2^1}{1 + (p_2/p_1)} + .5\left(\omega_1^2 + \frac{p_2}{p_1}\omega_2^2\right) = \omega_1^1 + \omega_1^2$$

or

$$\frac{.5 - .5 \left(p_2/p_1\right)}{1 + \left(p_2/p_1\right)} \omega_1^1 + \frac{.5 \left(p_2/p_1\right) + .5 \left(p_2/p_1\right)^2 - 1}{1 + \left(p_2/p_1\right)} \omega_2^1 = 0$$
 or

$$(\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0$$

• Solution for  $p_2/p_1$ :

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\begin{array}{c} \left(\omega_1^1 + \omega_2^1\right)^2 \\ -4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1 \\ 2\left(\omega_1^1 - 2\omega_2^1\right) \end{array}}$$

• Some complicated solution!

• Problem set has solution that is easier to compute (and interpret)

#### 2 Existence and Welfare Theorems

• Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

• Is Walrasian Equilibrium always unique? Not necessarily

• Is Walrasian Equilibrium efficient? Yes.

• First Fundamental Welfare Theorem. All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

• Figure

• Second Fundamental Welfare theorem. Given convex preferences, for every Pareto efficient allocation  $((x_1^1, x_1^1), (x_1^2, x_2^2))$  there exists some endowment  $(\omega_1, \omega_2)$  such that  $((x_1^1, x_1^1), (x_1^2, x_2^2))$  is a Walrasian Equilibrium for endowment  $(\omega_1, \omega_2)$ .



- Significance of these results:
  - First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
  - BUT: problems with externalities and public good
  - BUT: what about distribution?

- Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
- But redistribution is hard to implement, and distortive.

## 3 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 641-645
- Common economic relationship
- Contract between two parties:
  - Principal
  - Agent
- Two parties have asymmetric information
  - Principal offers a contract to the agent
  - Agent chooses an action
  - Action of agent (or his type) is not observed by principle

- Example 1: Manager and worker
  - Manager employs worker and offers wage
  - Worker exerts effort (not observed)
  - Manager pays worker as function of output
- Example 2: Car Insurance
  - Car insurance company offers insurance contract
  - Driver chooses quality of driving (not observed)
  - Insurance company pays for accidents
- Example 3: Shareholders and CEO
  - Shareholders choose compensation for CEO
  - CEO puts effort
  - CEO paid as function of stock price

- In all of these cases (and many more!), common structure
  - Principal would like to observe effort (of worker, of CEO, of driver)
  - Unfortunately, this is not observable
  - Only a related, noisy proxy is observable: output, accident, success
  - Contract offered by principal is function of this proxy
- This means that occasionally an agent that put a lot of effort but has bad luck is 'punished'
- Also, agents that shirked may instead be compensated
- These principle-agent problems are called *hidden action* or *moral hazard*

- Second category (next lecture): *hidden type* or *adverse selection*
- Example 1: Manager and worker
  - Manager employs worker and offers wage
  - Worker can be hard-working or lazy
- Example 2: Car Insurance
  - Car insurance company offers insurance contract
  - Drivers ex ante can be careful or careless
- Example 3: Shareholders and CEO
  - Shareholders choose compensation for CEO
  - CEO is high-quality or thief

- Problem is similar (action is not observed), but with a twist
  - Hidden action: principal can convince agent to exert high effort with the appropriate incentives
  - Hidden type: agent's behavior is not affected by incentives, but by her type
- Different task for principal:
  - Hidden action: Principal wants to incentivize agent to work hard
  - Hidden type: Principal wants to make sure to recruit 'good' agent, not 'bad' one
- Two look similar, but analysis is different
- Start from *Hidden Action*

### 4 Hidden Action (Moral Hazard)

- Nicholson, Ch. 18, pp. 645-650
- Example 3: Shareholders and CEO
  - Division of ownership and control
- Shareholders (owners of firm):
  - Have capital, but do not have time to run company themselves
  - Want firm run so as to maximize profits
- CEO (manager)
  - Has time and managerial skill
  - Does not have capital to own the firm

- If CEO owns the company (private enterprises), problem is solved -> Infeasible in large companies
- Agent chooses effort *e* (unobserved)
  - Induces output  $y = e + \varepsilon$ , where  $\varepsilon$  is a noise term, with  $E(\varepsilon) = 0$
  - Example: Despite putting effort, investment project did not succeed
- $\bullet\,$  Principal pays a salary w to the agent
  - Salary is a function of output y: w = w(y)
  - Remember: Salary cannot be function of effort e

• Principal maximizes expected profits

$$E[\pi] = E[y - w(y)] = e - E[w(y)]$$

• Agent is risk averse and maximizes

$$E\left[U\left(w\left(e+\varepsilon\right)\right)\right]-c\left(e\right)$$

- c(e) is cost of effort: assume c'(e) > 0 and c''(e) > 0 for all e
- Utility function U satisfies  $U^{\prime}>0$  and  $U^{\prime\prime}<0$
- Notice: Agent is risk-averse, Principal is riskneutral
- Assume  $U(w) = -e^{-\gamma w}$  and  $\varepsilon \sim N(\mathbf{0}, \sigma^2)$
- Can solve explicitly for EU(w):

$$EU(w) = -\frac{1}{\sqrt{2\pi\sigma^2}} \int e^{-\gamma w} e^{-\frac{1}{2}\frac{(w-\mu_w)^2}{\sigma_w^2}} dw = \mu_w -\frac{\gamma}{2}\sigma_w^2$$
  
[Take this for granted]

- Expected utility of agent is  $EU(w) = \mu_w \frac{\gamma}{2}\sigma_w^2$
- Note:  $\mu_w$  is average salary and  $\sigma_w^2$  is variance of salary
  - Agent likes high mean salary  $\mu_w$
  - Agent dislikes variance in salary  $\sigma_w^2$
  - Dislike for variance increses in risk aversion  $\gamma$
- Assume that contract is linear:  $w = a + by = a + be + b\varepsilon$ 
  - Compute  $\mu_w = E(w) = E[a + be + b\varepsilon] = a + be + bE[\varepsilon] = a + be$
  - Compute  $\sigma_w^2 = Var \left[ a + be + b\varepsilon \right] = b^2 \sigma^2$
- Rewrite expected utility as

$$EU(w) = a + be - \frac{\gamma}{2}b^2\sigma^2$$

- Back to Principal-Agent problem
- Solve problem in three Steps, starting from last stage (backward induction)
  - Step 1 (Effort Decision). Given contract w(y), what effort  $e^*$  is agent going to put in?
  - Step 2. (Individual Rationality) Given contract w(y) and anticipating to put in effort  $e^*$ , does agent accept the contract?
  - Step 3. (Profit Maximization) Anticipating that the effort of the agent  $e^*$  (and the acceptance of the contract) will depend on the contract, what contract w(y) does principal choose to maximize profits?

#### 5 Next lecture

- Asymmetric Information
- Moral Hazard