Outline

1. Dynamic Games

2. Oligopoly: Stackelberg

3. General Equilibrium: Introduction

4. Edgeworth Box: Pure Exchange
1 Dynamic Games

• Nicholson, Ch. 8, pp. 268-277

• Dynamic games: one player plays after the other

• Decision trees
  – Decision nodes
  – Strategy is a plan of action at each decision node
- Example: battle of the sexes game

<table>
<thead>
<tr>
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<th>She \ He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
<td></td>
</tr>
</tbody>
</table>

- Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy.

• Backward induction. Find optimal action in last period and then work backward.

• Solution
• Example 2: Entry Game

<table>
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<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>10, 0</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>0, 5</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

• Exercise. Dynamic version.

• Coordination games solved if one player plays first
• Can use this to study finitely repeated games

• Suppose we play the prisoner’s dilemma game ten times.

\[
\begin{array}{c|cc}
1 & 2 & | & D & | & ND \\
\hline
D & -4, -4 & | & -1, -5 \\
ND & -5, -1 & | & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
• The result differs if infinite repetition with a probability of terminating

• Can have cooperation

• Strategy of repeated game:
  – Cooperate (ND) as long as opponent always cooperate
  – Defect (D) forever after first defection

• Theory of repeated games: Econ. 104
2 Oligopoly: Stackelberg

- Nicholson, Ch. 15, pp. 552-554

- Setting as in problem set

- 2 Firms

- Cost: $c(y) = cy$, with $c > 0$

- Demand: $p(Y) = a - bY$, with $a > c > 0$ and $b > 0$

- Difference: Firm 1 makes the quantity decision first

- Use subgame perfect equilibrium
• Solution:

• Solve first for Firm 2 decision as function of Firm 1 decision:

\[
\max_{y_2} (a - by_2 - by_1^*) y_2 - cy_2
\]

• F.o.c.: \( a - 2by_2^* - by_1^* - c = 0 \)

• Firm 2 best response function:

\[
y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2}.
\]
• Firm 1 takes this response into account in the maximization:

\[
\max_{y_1} (a - by_1 - by_2^* (y_1)) y_1 - cy_1
\]

or

\[
\max_{y_1} \left( a - by_1 - b \left( \frac{a - c}{2b} - \frac{y_1}{2} \right) \right) y_1 - cy_1
\]

• F.o.c.:

\[a - 2by_1 - \frac{(a - c)}{2} + by_1 - c = 0\]

or

\[y_1^* = \frac{a - c}{2b}\]

and

\[y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} = \frac{a - c}{2b} - \frac{a - c}{4b} = \frac{a - c}{4b} \]
• Total production:

\[ Y_D^* = y_1^* + y_2^* = 3\frac{a - c}{4b} \]

• Price equals

\[ p^* = a - b \left( \frac{3a - c}{4b} \right) = \frac{1}{4}a + \frac{3}{4}c \]

• Compare to monopoly:

\[ y_M^* = \frac{a - c}{2b} \]

and

\[ p_M^* = \frac{a + c}{2} \].

• Compare to Cournot:

\[ Y_D^* = y_1^* + y_2^* = 2\frac{a - c}{3b} \]

and

\[ p_D^* = \frac{1}{3}a + \frac{2}{3}c. \]
• Compare with Cournot outcome

• Firm 2 best response function:

\[ y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} \]

• Firm 1 best response function:

\[ y_1^* = \frac{a - c}{2b} - \frac{y_2^*}{2} \]

• Intersection gives Cournot
• Stackelberg: Equilibrium is point on Best Response of Firm 2 that maximizes profits of Firm 1

• Plot iso-profit curve of Firm 1:

\[ \bar{\Pi}_1 = (a - c) y_1 - by_1 y_2 - b y_1^2 \]

• Solve for \( y_2 \) along iso-profit:

\[ y_2 = \frac{a - c}{b} - y_1 - \frac{\bar{\Pi}_1}{b y_1} \]

• Iso-profit curve is flat for

\[ \frac{dy_2}{dy_1} = -1 + \frac{\bar{\Pi}}{b (y_1)^2} = 0 \]

or

\[ y_1 = \]
3 General Equilibrium: Introduction

- So far, we looked at consumers
  - Demand for goods
  - Choice of leisure and work
  - Choice of risky activities

- We also looked at producers:
  - Production in perfectly competitive firm
  - Production in monopoly
  - Production in oligopoly
• We also combined consumers and producers:
  – Supply
  – Demand
  – Market equilibrium

• Partial equilibrium: one good at a time

• General equilibrium: Demand and supply for all goods!
  – supply of young worker↑ \implies \text{wage of experienced workers?}
  – minimum wage↑ \implies \text{effect on higher earners?}
  – steel tariff↑ \implies \text{effect on car price}
4 Edgeworth Box: Pure Exchange

- Nicholson, Ch. 13, pp. 458-460

- 2 consumers in economy: $i = 1, 2$

- 2 goods, $x_1, x_2$

- Endowment of consumer $i$, good $j$: $\omega^i_j$

- Total endowment: $(\omega_1, \omega_2) = (\omega^1_1 + \omega^2_1, \omega^1_2 + \omega^2_2)$

- No production here. With production (as in book), $(\omega_1, \omega_2)$ are optimally produced
• Edgeworth box

• Draw preferences of agent 1

• Draw preferences of agent 2
• Consumption of consumer $i$, good $j$: $x^i_j$

• Feasible consumption:

$$x^1_i + x^2_i \leq \omega_i \text{ for all } i$$

• If preferences monotonic, $x^1_i + x^2_i = \omega_i \text{ for all } i$

• Can map consumption levels into box
5 Next lecture

- General Equilibrium

- Barter