Outline

1. Oligopoly: Cournot

2. Oligopoly: Bertrand

3. Second-price Auction

4. Auctions: eBay Evidence
1 Oligopoly: Cournot

- Nicholson, Ch. 15, pp. 534-540

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:
  \[
  \max_{y_i} p(y_i + y_{-i}) y_i - cy_i.
  \]

- First order condition with respect to $y_i$:
  \[
  p'_{Y}(y^*_i + y^*_{-i}) y^*_i + p - c = 0, \ i = 1, 2.
  \]
• Nash equilibrium:
  
  - $y_1$ optimal given $y_2$;
  
  - $y_2$ optimal given $y_1$.

• Solve equations:

  $$p_Y' (y_1^* + y_2^*) y_1^* + p - c = 0$$
  and

  $$p_Y' (y_2^* + y_1^*) y_2^* + p - c = 0.$$

• Cournot $\rightarrow$ Pricing above marginal cost

• Numerical example $\rightarrow$ Problem set 5


2 Oligopoly: Bertrand

- Nicholson, Ch. 15, pp. 533-534

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

\[
\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y(p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}
\]
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least one firm has a profitable deviation

• Case 1. \( p_1 > p_2 > c \)

• Case 2. \( p_1 = p_2 > c \)

• Case 3. \( p_1 > c \geq p_2 \)
• Case 4. \( c > p_1 \geq p_2 \)

• Case 5. \( p_1 = c > p_2 \)

• Only Case 6 remains: \( p_1 = c = p_2 \), which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium \(\rightarrow\) Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium \(\rightarrow\) Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
3 Second-price Auction

- Nicholson, Ch. 18, pp. 669–676

- Sealed-bid auction

- Highest bidder wins object

- Price paid is second highest price

- Two individuals: \( I = 2 \)

- Strategy \( s_i \) is bid \( b_i \)

- Each individual knows value \( v_i \)
• Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
  v_i - b_i & \text{if } b_i > b_{-i} \\
  (v_i - b_{-i})/2 & \text{if } b_i = b_{-i} \\
  0 & \text{if } b_i < b_{-i}
\end{cases}$$

• Show: weakly dominant to set $b_i^* = v_i$

• To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$

   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i}) / 2 < 0$
   - $u_i(b_i, b_{-i}) = (v_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$
3. Assume now $b_{-i} < v_i$. 
4 Auctions: Evidence from eBay

- In second-price auction, optimal strategy is to bid one’s own value

- Is this true?

- eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder

- eBay is essentially a second-price auction

- Two deviations:
  1. People bid multiple times – they should not in this theory
  2. People may overbid
An example: eBay Bidding for a Board Game

- Bidding environment with clear boundary for rational willingness to pay (“buy-it-now price”).
- Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition “buy-it-now” price).
- Still non-negligible amount ($100-$200).

→ Is there evidence of overbidding?
→ If so, can we detect determinants of overbidding?
The Object
The Data

- Cashflow 101: board game with the purpose of finance/accounting education.
- Retail price: $195 plus shipping cost ($10.75) from manufacturer (www.richdad.com).
- Two ways to purchase Cashflow 101 on eBay
  - Auction (quasi-second price proxy bidding)
  - Buy-it-now
Sample

- **Listings**
  - 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
  - 493 by two retailers (only buy-it-now)

- **Remove non-US$, terminated, unsold items and items without simultaneous *professional* buy-it-now listing.** → 169 auctions

- **Buy-it-now offers of the two retailers**
  - Continuously present for all but six days. (Often individual buy-it-now offers present as well; they are often lower.)
  - 100% and 99.9% positive feedback scores.
  - Same prices $129.95 until 07/31/2004; $139.95 since 08/01/2004.
  - Shipping cost $9.95; other retailer $10.95.
  - New items (with bonus tapes/video).
<table>
<thead>
<tr>
<th>Title</th>
<th>Price</th>
<th>Quantity</th>
<th>Delivery Time</th>
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<td>$12.50</td>
<td>4</td>
<td>1d 00h 14m</td>
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<td>1d 04h 36m</td>
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<td>$207.96</td>
<td></td>
<td>1d 06h 47m</td>
</tr>
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<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
<td>1d 08h 02m</td>
</tr>
<tr>
<td>Your satisfaction is GUARANTEED, 100% $ back</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>MIINT Cashflow 101 *Robert Kiyosaki Game NR!</td>
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<td>13</td>
<td>1d 08h 04m</td>
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<td></td>
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<tr>
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<td></td>
<td>1d 09h 28m</td>
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<tr>
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<td>1</td>
<td>1d 13h 54m</td>
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<td>1d 14h 17m</td>
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<tr>
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<td>$207.96</td>
<td></td>
<td>1d 15h 47m</td>
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<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
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</tbody>
</table>
Listing Example – Magnified

CASHFLOW® 101 202 Robert Kiyosaki Best Pak

TRY IT TODAY, WITH ABSOLUTELY NO RISK.

CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!

Your satisfaction is GUARANTEED, 100% $ back

MINT Cashflow 101 ^Robert Kiyosaki Game NR!

It's easy to be rich. Brand New. Still sealed

Pricing:
[Buy Now] $129.95

Pricing:
$140.00
Hypotheses

Given the information on the listing website:

• (H1) An auction should never end at a price above the concurrently available purchase price.

• (H2) Mentioning of higher outside prices should not affect bidding behavior.
Figure 1. Starting Price (*startprice*)

- 45% below $20; mean=$46; SD=43.88
- only 6 auctions with first bid (not price) above buy-it-now
Figure 2. Final Price (finalprice)

41% are above “buy-it-now” (mean $132; SD 16.83)
Figure 4. Total Price (incl. shipping cost)

- 51% are above “buy-it-now” plus its shipping cost (mean=$144.20; SD=15.00)
5 Next lecture

- Dynamic Games
- Stackelberg duopoly