Economics 101A
(Lecture 16)

Stefano DellaVigna

March 18, 2014
Outline

1. Cost Curves

2. One-step Profit Maximization

3. Second-Order Conditions

4. Introduction to Market Equilibrium

5. Aggregation

6. Market Equilibrium in the Short-Run
1 Cost Curves

- Nicholson, Ch. 10, pp. 341-349; Ch. 11, pp. 380-383

- Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization
  \[ p = MC = \partial c (w, r, y) / \partial y \]

- Average costs $AC = c / y \rightarrow$ Does firm break even?
  \[ \pi = py - c(w, r, y) > 0 \text{ iff } \]
  \[ \pi / y = p - c(w, r, y) / y > 0 \text{ iff } \]
  \[ c(w, r, y) / y = AC < p \]

- **Supply function.** Portion of marginal cost $MC$
  above average costs.(price equals marginal cost)
• Assume only 1 input (expenditure minimization is trivial)

• **Case 1.** Production function. \( y = L^\alpha \)
  
  – Cost function? (cost of input is \( w \)):
  
  \[
  c(w, y) = wL^*(w, y) = wy^{1/\alpha}
  \]

  – Marginal cost?
  
  \[
  \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{(1-\alpha)/\alpha}
  \]

  – Average cost \( c(w, y) / y \)?
  
  \[
  \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}
  \]
• **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 383-393

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  
  - Firms are small relative to market
  
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  
  - Will firm produce at $p < p_M$?

  - $\implies p = p_M$
• Revenue: $py = pf(L, K)$

• Cost: $wL + rK$

• Profit $pf(L, K) - wL - rK$
• Agent optimization:

\[
\max_{L,K} pf (L, K) - wL - rK
\]

• First order conditions:

\[
pf'_L (L, K) - w = 0
\]

and

\[
pf'_K (L, K) - r = 0
\]

• Second order conditions? \( pf''_{L,L} (L, K) < 0 \) and

\[
|H| = \left| \begin{array}{cc}
pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\
pf''_{L,K} (L, K) & pf''_{K,K} (L, K) \\
\end{array} \right| = p^2 \left[ f''_{L,L}f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0
\]

• Need \( f''_{L,K} \) not too large for maximum
• Comparative statics with respect to to $p$, $w$, and $r$.

• What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p_{L,K}''(L, K) \\ 0 & p_{K,K}''(L, K) \end{vmatrix}}{p_{L,L}''(L, K) p_{K,K}''(L, K) - p_{L,K}''(L, K) p_{L,K}''(L, K)} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

• Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$. 
3 Second Order Conditions in P-Max: Cobb-Douglas

• How do the second order conditions relate for:
  – Cost Minimization
  – Profit Maximization?

• Check for Cobb-Douglas production function
  \[ y = AK^\alpha L^\beta \]

• **Cost Minimization.** S.o.c.:
  \[ c''_y (y^*, w, r) > 0 \]

• As we showed, for CD prod. ftn.,
  \[ c''_y (y^*, w, r) = -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{A^2} \frac{B}{\alpha + \beta} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]
  which is > 0 as long as \( \alpha + \beta < 1 \) (DRS)
• **Profit Maximization.** S.o.c.: \( p f''_{L,L} (L, K) < 0 \) and

\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0
\]

• As long as \( \beta < 1 \),

\[
p f''_{L,L} = p \beta (\beta - 1) AK^\alpha L^{\beta - 2} < 0
\]

• Then,

\[
|H| = \frac{\beta (\beta - 1) AK^\alpha L^{\beta - 2}}{\alpha (\alpha - 1) AK^{\alpha - 2} L^{\beta - 1}} \left( \alpha \beta AK^{\alpha - 1} L^{\beta - 1} \right)^2 = p^2 A^2 K^{2\alpha - 2} L^{2\beta - 2} \alpha \beta [1 - \alpha - \beta]
\]

• Therefore, \( |H| > 0 \) iff \( \alpha + \beta < 1 \) (DRS)

• The two conditions coincide
4 Introduction to Market Equilibrium

• Nicholson, Ch. 12, pp. 409–419

• Two ways to analyze firm behavior:
  – Two-Step Cost Minimization
  – One-Step Profit Maximization

• What did we learn?
  – Optimal demand for inputs $L^*, K^*$ (see above)
  – Optimal quantity produced $y^*$
• **Supply function.** \( y = y^* (p, w, r) \)
  
  – From profit maximization:
    
    \[ y = f \left( L^* (p, w, r), K^* (p, w, r) \right) \]
  
  – From cost minimization:
    
    \[ MC \text{ curve above } AC \]
  
  – Supply function is increasing in \( p \)

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5 Aggregation

5.1 Producers aggregation

• $J$ companies, $j = 1, \ldots, J$, producing good $i$

• Company $j$ has supply function

$$y_i^j = y_i^j*(p_i, w, r)$$

• Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^j*(p_i, w, r)$$

• Graphically,
5.2 Consumer aggregation

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

\[
x_1^* = x_1^*(p_1, \ldots, p_n, M),
\]
\[
\vdots
\]
\[
x_n^* = x_n^*(p_1, \ldots, p_n, M).
\]
Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$.

**Single-consumer demand function:**

$$x_i^* = x_i^* (p_i \mid p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

What is sign of $\partial x_i^* / \partial p_i$?

- Negative if good $i$ is normal
- Negative or positive if good $i$ is inferior
• Aggregation: $J$ consumers, $j = 1, \ldots, J$

• Demand for good $i$ by consumer $j$:

$$x_{i}^{j*} = x_{i}^{j*} (p_1, \ldots, p_n, M^j)$$

• Market demand $X_i$:

$$X_i (p_1, \ldots, p_n, M^1, \ldots, M^J) = \sum_{j=1}^{J} x_{i}^{j*} (p_1, \ldots, p_n, M^j)$$

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
6 Market Equilibrium in the Short-Run

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

  $$ Y^* = Y_i^S (p_i^*, w, r) = X_i^D (p_1^*, ..., p_n^*, M^1, ..., M^J) $$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:

  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
7 Next Lecture

- Market Equilibrium

- Comparative Statics of Equilibrium

- Elasticities

- Taxes and Subsidies