Economics 101A
(Lecture 15)

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Outline

1. Two-step Cost Minimization II

2. Cost Minimization: Example

3. Cost Curves and Supply Function
1 Two-step Cost minimization II

- *First Step.* Minimize input costs for given production

- Firm objective function:

\[
\begin{align*}
\min_{L,K} & \quad wL + rK \\
\text{s.t.} & \quad f(L, K) \geq y
\end{align*}
\]

- Derived demand for inputs:

- \( L = L^* (w, r, y) \)

- \( K = K^* (w, r, y) \)

- Value function at optimum is cost function:

\[
c (w, r, y) = wL^* (r, w, y) + rK^* (r, w, y)
\]
• **Second step.** Given cost function, choose optimal quantity of $y$ as well

• Price of output is $p$.

• Firm’s objective:

$$\max px - c(w, r, y)$$

• First order condition:

$$p - c'_y(w, r, y) = 0$$

• Price equals marginal cost – very important!
• Second order condition:

$$-c''_{y,y}(w, r, y^*) < 0$$

• For maximum, need increasing marginal cost curve.
2 Cost Minimization: Example

- Continue example above: \( y = f(L, K) = AK^\alpha L^\beta \)

- Cost minimization:

\[
\begin{align*}
\min & \quad wL + rK \\
\text{s.t.} & \quad AK^\alpha L^\beta = y
\end{align*}
\]

- What is the return to scale for this example?

- Increase of all inputs: \( f(tz) \) with \( t \) scalar, \( t > 1 \)

- How much does input increase?
  - Decreasing returns to scale: for all \( z \) and \( t > 1 \),
    \[
    f(tz) < tf(z)
    \]
– Constant returns to scale: for all \( z \) and \( t > 1 \),
\[
f(tz) = tf(z)
\]

– Increasing returns to scale: for all \( z \) and \( t > 1 \),
\[
f(tz) > tf(z)
\]

• Returns to scale depend on \( \alpha + \beta \leq 1 \):
\[
f(tK, tL) = A(tK)^{\alpha} (tL)^{\beta} = t^{\alpha + \beta} AK^{\alpha} L^{\beta} = t^{\alpha + \beta} f(K, L)
\]
• Solutions:

– Optimal amount of labor:

\[
L^* (r, w, y) = \left( \frac{y}{A} \right)^{\frac{1}{\alpha + \beta}} \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha + \beta}}
\]

– Optimal amount of capital:

\[
K^* (r, w, y) = \frac{w \alpha}{r \beta} \left( \frac{y}{A} \right)^{\frac{1}{\alpha + \beta}} \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha + \beta}}
\]

\[
= \left( \frac{y}{A} \right)^{\frac{1}{\alpha + \beta}} \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha + \beta}}
\]

• Check various comparative statics:

– \( \partial L^*/\partial A < 0 \) (technological progress and unemployment)

– \( \partial L^*/\partial y > 0 \) (more workers needed to produce more output)
- $\partial L^*/\partial w < 0$, $\partial L^*/\partial r > 0$ (substitute away from more expensive inputs)

- Parallel comparative statics for $K^*$
• Cost function

\[ c(w, r, y) = wL^*(r, w, y) + rK^*(r, w, y) = \]
\[ = \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}} \left[ w \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha+\beta}} + r \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha+\beta}} \right] \]

• Define \( B := w \left( \frac{w \alpha}{r \beta} \right)^{-\frac{\alpha}{\alpha+\beta}} + r \left( \frac{w \alpha}{r \beta} \right)^{\frac{\beta}{\alpha+\beta}} \)

• Cost-minimizing output choice:

\[ \max_{y} py - B \left( \frac{y}{A} \right)^{\frac{1}{\alpha+\beta}} \]
• First order condition:

\[ p - \frac{1}{\alpha + \beta} \frac{B}{A} \left( \frac{y}{A} \right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0 \]

• Second order condition:

\[ -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]

• When is the second order condition satisfied?
• Solution:

- \( \alpha + \beta = 1 \) (CRS):
  
  * S.o.c. equal to 0

  * Solution depends on \( p \)

  * For \( p > \frac{B}{A} \), produce \( y^* \to \infty \)

  * For \( p = \frac{B}{A} \), produce any \( y^* \in [0, \infty) \)

  * For \( p < \frac{B}{A} \), produce \( y^* = 0 \)
- \( \alpha + \beta > 1 \) (IRS):

* S.o.c. positive

* Solution of f.o.c. is a minimum!

* Solution is \( y^* \rightarrow \infty \).

* Keep increasing production since higher production is associated with higher returns
− \( \alpha + \beta < 1 \) (DRS):

* s.o.c. negative. OK!

* Solution of f.o.c. is an interior optimum

* This is the only "well-behaved" case under perfect competition

* Here can define a supply function
3 Cost Curves

- Nicholson, Ch. 10, pp. 341-349; Ch. 11, pp. 380-383

- Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization
  
  $p = MC = \partial c (w, r, y) / \partial y$

- Average costs $AC = c / y \rightarrow$ Does firm break even?
  
  $\pi = py - c (w, r, y) > 0 \text{ iff }$

  $\pi / y = p - c (w, r, y) / y > 0 \text{ iff }$

  $c (w, r, y) / y = AC < p$

- **Supply function.** Portion of marginal cost $MC$ above average costs.(price equals marginal cost)
- Assume only 1 input (expenditure minimization is trivial)

- **Case 1.** Production function. \( y = L^\alpha \)
  
  - Cost function? (cost of input is \( w \)):
    \[
    c(w, y) = wL^*(w, y) = wy^{1/\alpha}
    \]
  
  - Marginal cost?
    \[
    \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy^{1-\alpha}/\alpha
    \]
  
  - Average cost \( c(w, y)/y \)?
    \[
    \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}
    \]
• **Case 1a.** \( \alpha > 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** \( \alpha = 1 \). Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** \( \alpha < 1 \). Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
3.1 Supply Function

- Supply function: \( y^* = y^*(w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[
  p - c'_y(w, r, y) = 0
  \]

- Implicit function:
  \[
  \frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y}(w, r, y)} > 0
  \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
4 Next Lectures

- Profit Maximization
- Aggregation
- Market Equilibrium