Outline

1. Mid-Term Feedback

2. Insurance II

3. Investment in Risky Asset

4. Time Consistency
1 Mid-Term Feedback

• Thanks for the feedback!
2 Insurance II

• Individual maximization:

$$\max_{\alpha} (1 - p) u (w - q\alpha) + p u (w - q\alpha - L + \alpha)$$

s.t. $\alpha \geq 0$

• First order conditions:

$$0 = -q (1 - p) u' (w - q\alpha) + (1 - q) p u' (w - q\alpha - L + \alpha)$$

or

$$\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.$$  

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* = ?$
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
3 Investment in Risky Asset

● Individual has:
  – wealth $w$
  – utility function $u$, with $u' > 0$

● Two possible investments:
  – Asset B (bond) yields return 1 for each dollar
  – Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability $p$
    * $r = r_- < 0$ with probability $1 - p$
      * $Er = pr_+ + (1 - p)r_- > 0$

● Share of wealth invested in stock $S = \alpha$
• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w [(1 - \alpha) + \alpha (1 + r_-)]) + 
+ pu (w [(1 - \alpha) + \alpha (1 + r_+)]) \\
\text{s.t.} \ 0 \leq \alpha \leq 1
\]

• Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

• Assume \( a = 0 \) (no loss of generality)

• Maximization becomes

\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+]) \\
or \\
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

• Sign of term in square brackets? Positive!

• Set \( \alpha^* = 1 \)
• Case of risk aversion: $u'' < 0$

• Assume $0 \leq \alpha^* \leq 1$, check later

• First order conditions:

\[
0 = (1 - p) (wr_-) u' (w [1 + \alpha r_-]) + \\
+ p (wr_+) u' (w [1 + \alpha r_+])
\]

• Can $\alpha^* = 0$ be solution?

• Solution is $\alpha^* > 0$ (positive investment in stock)

• Exercise: Check s.o.c.
4 Time consistency

• Intertemporal choice

• Three periods, \( t = 0, t = 1, \) and \( t = 2 \)

• At each period \( i \), agents:
  
  – have income \( M'_i = M_i + \text{savings/debts from previous period} \)
  
  – choose consumption \( c_i \);
  
  – can save/borrow \( M'_i - c_i \)
  
  – no borrowing in last period: at \( t = 2 \) \( M'_2 = c_2 \)
• Utility function at $t = 0$

\[ u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2) \]

• Utility function at $t = 1$

\[ u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} U(c_2) \]

• Utility function at $t = 2$

\[ u(c_2) = U(c_2) \]

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1+\delta} U(c_2)$$

$$s.t. \ c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to period 0.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}U(c_2)$$

s.t. $c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1+r}{1+\delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}U(c_2)$$

$$= U(c_0) + \frac{1}{1+\delta} \left[ U(c_1) + \frac{1}{1+\delta}U(c_2) \right]$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?

  – addictive products (alcohol, drugs);

  – good actions (exercising, helping friends);

  – immediate gratification (shopping, credit card borrowing)
5 Next lecture and beyond

- Time Inconsistency

- Production Function