Economics 101A
(Lecture 12)

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Outline

1. Mid-Term Feedback

2. Insurance II

3. Investment in Risky Asset

4. Time Consistency

1 Mid-Term Feedback

- Thanks for the feedback!

2 Insurance II

- Individual maximization:

$$\max_{\alpha} (1-p)\, u(w-q\alpha) + pu(w-q\alpha-L+\alpha)$$
$$s.t. \alpha \geq 0$$

- First order conditions:

$$\begin{aligned} 0 &= -q(1-p)\, u'(w-q\alpha) \\ &\quad + (1-q)\, pu'(w-q\alpha-L+\alpha) \end{aligned}$$

or

$$\frac{u'(w-q\alpha)}{u'(w-q\alpha-L+\alpha)} = \frac{1-q}{q}\frac{p}{1-p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* =$?

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all): $\alpha^* < L$

- Exercise: Check second order conditions!

3 Investment in Risky Asset

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability p
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)\, r_- > 0$

- Share of wealth invested in stock S $= \alpha$

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\left(w\left[(1-\alpha)+\alpha\left(1+r_{-}\right)\right]\right)+$$
$$+pu\left(w\left[(1-\alpha)+\alpha\left(1+r_{+}\right)\right]\right)$$
$$s.t. 0 \leq \alpha \leq 1$$

- Case of risk neutrality: $u(x) = a + bx$, $b > 0$

- Assume $a = 0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b\,(1-p)\left(w\left[1+\alpha r_{-}\right]\right)+bp\left(w\left[1+\alpha r_{+}\right]\right)$$

or

$$\max_{\alpha} bw+\alpha bw\left[(1-p)\, r_{-}+pr_{+}\right]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^{*} = 1$

- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$\begin{aligned} 0 &= (1-p)\left(wr_-\right)u'\left(w\left[1+\alpha r_-\right]\right)+ \\ &\quad +p\left(wr_+\right)u'\left(w\left[1+\alpha r_+\right]\right) \end{aligned}$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

4 Time consistency

- Intertemporal choice

- Three periods, $t = 0$, $t = 1$, and $t = 2$

- At each period i, agents:
  - have income $M_i' = M_i+$savings/debts from previous period
  - choose consumption c_i;
  - can save/borrow $M_i' - c_i$
  - no borrowing in last period: at $t = 2$ $M_2' = c_2$

- Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1+\delta} U(c_1) + \frac{1}{(1+\delta)^2} U(c_2)$$

- Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1+\delta} U(c_2)$$

- Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

- $U' > 0$, $U'' < 0$

- Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

- **Period 1**.

- Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- Maximization problem:

$$\max U(c_1) + \frac{1}{1+\delta} U(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{U'(c_2)} = \frac{1+r}{1+\delta}$$

- Back to **period 0**.

- Agent at time 0 can commit to consumption at time 1 as function of uncertain income M_1.

- Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}U(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{U'(c_2)} = \frac{1+r}{1+\delta}$$

- The two conditions coincide!

- **Time consistency.** Plans for future coincide with future actions.

- To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$
\begin{aligned}
& U(c_0) + \frac{1}{1+\delta} U(c_1) + \frac{1}{(1+\delta)^2} U(c_2) \\
= \; & U(c_0) + \frac{1}{1+\delta} \left[U(c_1) + \frac{1}{1+\delta} U(c_2) \right]
\end{aligned}
$$

- Expression in brackets coincides with utility at $t = 1$

- Is time consistency right?
  - addictive products (alcohol, drugs);
  - good actions (exercising, helping friends);
  - immediate gratification (shopping, credit card borrowing)

5 Next lecture and beyond

- Time Inconsistency

- Production Function