Outline

1. Introduction to probability

2. Expected Utility

3. Risk Aversion and Lottery

4. Measures of Risk Aversion

5. Insurance
1 Introduction to Probability

- Nicholson, Ch. 7, p. 209

- So far deterministic world:
  - income given, known $M$
  - interest rate known $r$

- But some variables are unknown at time of decision:
  - future income $M_1$?
  - future interest rate $r_1$?

- Generalize framework to allow for uncertainty
– Events that are truly unpredictable (weather)

– Event that are very hard to predict (future income)
• Probability is the language of uncertainty

• Example:
  
  – Income $M_1$ at $t = 1$ depends on state of the economy
  
  – Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  
  – Three probabilities: $p_1$, $p_2$, $p_3$
  
  – $p_1 = P(M_1) = P($recession$)$

• Properties:
  
  – $0 \leq p_i \leq 1$
  
  – $p_1 + p_2 + p_3 = 1$
• Mean income: \( EM = \sum_{i=1}^{3} p_i M_i \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),
  \[
  EM = \frac{1}{3} 20 + \frac{1}{3} 25 + \frac{1}{3} 30 = \frac{75}{3} = 25
  \]

• Variance of income: \( V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2 \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),
  \[
  V(M) = \frac{1}{3} (20 - 25)^2 + \frac{1}{3} (25 - 25)^2 + \frac{1}{3} (30 - 25)^2 \\
  = \frac{1}{3} 5^2 + \frac{1}{3} 5^2 = 2/3 \times 25
  \]

• Mean and variance if \((p_1, p_2, p_3) = (1/4, 1/2, 1/4)\)?
2 Expected Utility

- Nicholson, Ch. 7, pp. 210-217

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\max U(c^1)$$

$$s.t. \ c_i^1 \leq M_i^1 + (1 + r)(M^0 - c^0)$$

with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U(M_i^1)$

- This is uncertain, depends on which $i$ is realized!
• How do we evaluate future uncertain utility?

• **Expected utility**

\[ EU = \sum_{i=1}^{3} p_i U(M_i^1) \]

• In example:

\[ EU = \frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30) \]

• Compare with \( U(EC) = U(25) \).

• Agents prefer riskless outcome \( EM \) to uncertain outcome \( M \) if

\[ \frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30) < U(25) \text{ or} \]
\[ \frac{1}{3} U(20) + \frac{1}{3} U(30) < \frac{2}{3} U(25) \text{ or} \]
\[ \frac{1}{2} U(20) + \frac{1}{2} U(30) < U(25) \]
• Picture
• Depends on sign of $U''$, on concavity/convexity

• Three cases:

  - $U''(x) = 0$ for all $x$. (linearity of $U$)
    * $U(x) = a + bx$
    * $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all $x$. (concavity of $U$)
    * $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all $x$. (convexity of $U$)
    * $1/2U(20) + 1/2U(30) > U(25)$
• If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

• If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

• If $U''(x) > 0$ (convexity), consumer likes uncertainty

• Do consumers like uncertainty?
Theorem. (Jensen’s inequality) If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where $E$ indicates expectation. If $f$ is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function $U$.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen’s inequality then implies $U$ concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income
3 Risk aversion and Lottery

- Risk aversion:
  - individuals dislike uncertainty
  - \( u \) concave, \( u'' < 0 \)

- Implications?
  - purchase of insurance (possible accident)
  - investment in risky asset (risky investment)
  - choice over time (future income uncertain)
• Experiment — Are you risk-averse?
4 Measures of Risk Aversion

• Nicholson, Ch. 7, pp. 217-221

• How risk averse is an individual?

• Two measures:
  – Absolute Risk Aversion $r_A$:
    \[ r_A = -\frac{u''(x)}{u'(x)} \]
  – Relative Risk Aversion $r_R$:
    \[ r_R = -\frac{u''(x)}{u'(x)} x \]

• Examples in the Problem Set
5 Insurance

• Individual has:
  – wealth $w$
  – utility function $u$, with $u' > 0$, $u'' < 0$

• Probability $p$ of accident with loss $L$

• Insurance offers coverage:
  – premium $q$ for each $1$ paid in case of accident
  – units of coverage purchased $\alpha$

• Individual maximization:

$$\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha)$$

s.t. $\alpha \geq 0$
• Assume $\alpha^* \geq 0$, check later

• First order conditions:

$$0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)$$

or

$$\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.$$ 

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* =$?
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
6 Next Lectures

- Risk aversion

- Applications:
  - Portfolio choice
  - Consumption choice II