# Economics 101A (Lecture 9)

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#### Outline

- 1. Complements and substitutes
- 2. Do utility functions exist?
- 3. Application 1: Labor Supply
- 4. Application 2: Intertemporal choice I

#### **1** Complements and substitutes

- Nicholson, Ch. 6, pp. 187-192
- How about if price of another good changes?
- Generalize Slutsky equation

• Slutsky Equation:

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j}$$
$$-x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

• Substitution effect

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} > \mathbf{0}$$

for n = 2 (two goods). Ambiguous for n > 2.

• Income effect:

$$-x_j^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- negative if good i is normal

- positive if good i is inferior

• How do we define complements and substitutes?

Def. 1. Goods *i* and *j* are gross substitutes at price
 p and income *M* if

$$rac{\partial x_{i}^{*}\left(\mathbf{p},M
ight)}{\partial p_{j}}>$$
 0

• Def. 2. Goods *i* and *j* are **gross complements** at price **p** and income *M* if

$$\frac{\partial x_{i}^{*}\left(\mathbf{p},M\right)}{\partial p_{j}}<\mathbf{0}$$

- Example 1 (ctd.):  $x_1^* = \alpha M/p_1, x_2^* = \beta M/p_2.$
- Gross complements or gross substitutes? Neither!

• Notice: 
$$\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j}$$
 is usually different from  $\frac{\partial x_j^*(\mathbf{p},M)}{\partial p_i}$ 

- Better definition.
- Def. 3. Goods i and j are net substitutes at price
   p and income M if

$$\frac{\partial h_i^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} = \frac{\partial h_j^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} > 0$$

• Def. 4. Goods *i* and *j* are **net complements** at price **p** and income *M* if

$$\frac{\partial h_i^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^*(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$$

- Example 1 (ctd.):  $h_1^* = \overline{u} \left( \frac{\alpha}{1-\alpha} \frac{p_2}{p_1} \right)^{1-\alpha}$
- Net complements or net substitutes? Net substitutes!

# 2 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them

- How do we tie them to the world?
- Use actual choices revealed preferences approach

- Typical economists' approach. Compromise of:
  - realism
  - simplicity

- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters
- Estimate the parameters using the data

## 3 Labor Supply

- Nicholson Ch. 16, pp. 581-589
- Labor supply decision: how much to work in a day.

- $\bullet$  Goods: consumption good c, hours worked h
- Price of good p, hourly wage w
- Consumer spends 24 h = l hours in units of leisure

• Utilify function: u(c, l)

- Budget constraint?
- Income of consumer: M + wh = M + w(24 l)
- Budget constraint:  $pc \leq M + w(24 l)$  or

$$pc + wl \le M + 24w$$

- Notice: leisure *l* is a consumption good with price *w*. Why?
- General category: **opportunity cost**
- Instead of enjoying one hour of TV, I could have worked one hour and gained wage w.
- You should value the marginal hour of TV w!

• Opportunity costs are very important!

- Example 2. CostCo has a warehouse in SoMa
- SoMa used to have low cost land, adequate for warehouses
- Price of land in SoMa triples in 10 years.

• Should firm relocate the warehouse?

• Did costs of staying in SoMa go up?

- No.
- Did the opportunity cost of staying in SoMa go up?



• Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply
- Maximization problem is

$$\max u(c, l)$$
  
s.t.  $pc + wl \le M + 24w$ 

- Standard problem (except for 24w)
- First order conditions

• Assume utility function Cobb-Douglas:

$$u(c,l) = c^{\alpha} l^{1-\alpha}$$

• Solution is

$$c^* = \alpha \frac{M + 24w}{p}$$
$$l^* = (1 - \alpha) \left(24 + \frac{M}{w}\right)$$

- Both c and l are normal goods
- Unlike in standard Cobb-Douglas problems,  $c^{\ast}$  depends on price of other good w
- Why? Agents are endowed with *M* AND 24 hours of *l* in this economy
- $\bullet\,$  Normally, agents are only endowed with M

#### 4 Intertemporal choice

- Nicholson Ch. 17, pp. 609-613
- So far, we assumed people live for one period only
- Now assume that people live for two periods:
  - t = 0 people are young
  - -t = 1 people are old
- t = 0: income  $M_0$ , consumption  $c_0$  at price  $p_0 = 1$
- t = 1: income  $M_1 > M_0$ , consumption  $c_1$  at price  $p_1 = 1$
- Credit market available: can lend or borrow at interest rate  $\boldsymbol{r}$

- Budget constraint in period 1?
- Sources of income:

- 
$$M_1$$
  
-  $(M_0 - c_0) * (1 + r)$  (this can be negative)

• Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1+r}c_1 \le M_0 + \frac{1}{1+r}M_1$$

- Utility function?
- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1+\delta}U(c_1)$$

- U' > 0, U'' < 0
- $\delta$  is the discount rate
- Higher  $\delta$  means higher impatience

- Elicitation of  $\delta$  through hypothetical questions
- Person is indifferent between 1 hour of TV today and  $1+\delta$  hours of TV next period

• Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1)$$
  
s.t.  $c_0 + \frac{1}{1+r}c_1 \le M_0 + \frac{1}{1+r}M_1$ 

### **5** Next Lectures

- Applications:
  - Intertemporal Choice II
  - Economics of Altruism