Economics 101A
(Lecture 8)

Stefano DellaVigna

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Outline

1. Expenditure Minimization

2. Slutsky Equation

1 Expenditure minimization

- Nicholson, Ch. 4, pp. 131-135; Ch. 5, pp. 155-157

- Solve problem **EMIN** (minimize expenditure):

$$\begin{array}{l} \min p_1 x_1 + p_2 x_2 \\ s.t.\ u(x_1, x_2) \geq \bar{u} \end{array}$$

- Choose bundle that attains utility $\bar{u}$ with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility u strictly increasing in x_i, can maximize s.t. equality

- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem

- $h_i(p_1, p_2, \bar{u})$ is *Hicksian or compensated demand*

- Graphically:
  - Fix indifference curve at level $\bar{u}$
  - Consider budget sets with different M
  - Pick budget set which is tangent to indifference curve

- Optimum coincides with optimum of Utility Maximization!

- Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$

- Expenditure function is expenditure at optimum

- $e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2 (p_1, p_2, \bar{u})$

- $h_i(p_i)$ is *Hicksian or compensated demand* function

- Is h_i always decreasing in p_i? Yes!

- Graphical proof: moving along a convex indifference curve

- (For non-convex indifferent curves, still true)

- Using first order conditions:

$$L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda \left(u(x_1, x_2) - \bar{u}\right)$$

$$\frac{\partial L}{\partial x_i} = p_i - \lambda u_i'(x_1, x_2) = 0$$

- Write as ratios:

$$\frac{u_1'(x_1, x_2)}{u_2'(x_1, x_2)} = \frac{p_1}{p_2}$$

- $MRS =$ ratio of prices as in utility maximization!

- However: different constraint $\Longrightarrow \lambda$ is different

- Example 1: Cobb-Douglas utility

$$\min p_1 x_1 + p_2 x_2$$
$$s.t.\ x_1^{\alpha} x_2^{1-\alpha} \geq \bar{u}$$

- Lagrangean $=$

- F.o.c.:

- Solution: $h_1^* =$ $\qquad\qquad\qquad$ $, h_2^* =$

- $\partial h_i^* / \partial p_i < 0$, $\partial h_i^* / \partial p_j > 0$, $j \neq i$

2 Slutsky Equation

- Nicholson, Ch. 5, pp. 160-163

- Now: go back to Utility Max. in case where p_2 increases to $p_2' > p_2$

- What is $\partial x_2^* / \partial p_2$? Decompose effect:

  1. Substitution effect of an increase in p_i

     – $\partial h_2^* / \partial p_2$, that is change in EMIN point as p_2 descreases

     – Moving along an indifference curve

     – Certainly $\partial h_2^* / \partial p_2 < 0$

2. Income effect of an increase in p_i

   – $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

   – Shift out a budget line

   – $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$

- How does the Hicksian demand change if price p_i changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^* (\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^* (\mathbf{p}, e)}{\partial M} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is $\frac{\partial e(\mathbf{p},\bar{u})}{\partial p_i}$? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i} \left[p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})\right] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e\left(p, \bar{u}\right)) \end{aligned}$$

- Therefore

$$\frac{\partial h_i\left(\mathbf{p},\bar{u}\right)}{\partial p_i}=\frac{\partial x_i^*\left(\mathbf{p},e\right)}{\partial p_i}+\frac{\partial x_i^*\left(\mathbf{p},e\right)}{\partial M}x_1^*(p_1,p_2,e)$$

- Rewrite as

$$\begin{aligned}\frac{\partial x_i^*\left(\mathbf{p},M\right)}{\partial p_i} &= \frac{\partial h_i\left(\mathbf{p},v(\mathbf{p},M)\right)}{\partial p_i}\\ &\quad -x_1^*(p_1,p_2,M)\frac{\partial x_i^*\left(\mathbf{p},M\right)}{\partial M}\end{aligned}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

  1. Substitution effect negative: $\frac{\partial h_i(\mathbf{p},v(\mathbf{p},M))}{\partial p_i} < 0$

  2. Income effect: $-x_1^*(p_1,p_2,M)\frac{\partial x_i^*(\mathbf{p},M)}{\partial M}$

     – negative if good i is normal ($\frac{\partial x_i^*(\mathbf{p},M)}{\partial M} > 0$)

     – positive if good i is inferior ($\frac{\partial x_i^*(\mathbf{p},M)}{\partial M} < 0$)

- Overall, sign of $\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_i}$?

  – negative if good i is normal

  – it depends if good i is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M / p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i \left(\mathbf{p}, \overline{u}\right)}{\partial p_i} =$$

- Rewrite h_i^* as function of m: $h_i \left(\mathbf{p}, v(\mathbf{p}, M)\right)$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M)\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial p_i} =$$

- It works!

3 Next Lectures

- Complements and Substitutes

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism