Economics 101A
(Lecture 7)

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Outline

1. Utility maximization – Tricky Cases
2. Indirect Utility Function
3. Comparative Statics (Introduction)
4. Income Changes
5. Price Changes

1 Utility maximization – tricky cases

- Tricky Cases (ctd)

2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\max x_1 * (x_2 + 5)$$
$$s.t.\ p_1 x_1 + p_2 x_2 = M$$

- In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

3. Multiplicity of solutions.

   - Example 1: Perfect Substitutes with $p_1/p_2 = \alpha/\beta$

   - Example 2: Non-convex preferences with two optima

2 Indirect utility function

- Nicholson, Ch. 4, pp. 128-130

- Define the indirect utility $v(\mathbf{p}, M) \equiv u(\mathbf{x}^*(\mathbf{p}, M))$, with $\mathbf{p}$ vector of prices and $\mathbf{x}^*$ vector of optimal solutions.

- $v(\mathbf{p}, M)$ is the utility at the optimum for prices $\mathbf{p}$ and income M

- Some comparative statics: $\partial v(\mathbf{p}, M)/\partial M =$?

- Hint: Use Envelope Theorem on Lagrangean function

- What is the sign of λ?

- $\lambda = u'_{x_i}/p > 0$

- $\partial v(\mathbf{p}, M)/\partial p_i =$?

- Properties:
  - Indirect utility is always increasing in income M
  - Indirect utility is always decreasing in the price p_i

3 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 145-155
- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$
- Quantity consumed as a function of income and price
- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM$, $p_1' = tp_1$, $p_2' = tp_2$.

- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

- What happens?

- Write budget line: $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in $\mathbf{p}$ and M:

  $x^*(tM, tp_1, tp_2) = t^0x^*(M, p_1, p_2) = x^*(M, p_1, p_2)$.

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha+\beta} M/p_1, x_2^* = \frac{\beta}{\alpha+\beta} M/p_2$$

- What is $\partial x_1^*/\partial M$?

- What is $\partial x_1^*/\partial p_1$?

- What is $\partial x_1^*/\partial p_2$?

- General results?

4 Income changes

- Income increases from M to to $M' > M$.

- Budget line ($p_1 x_1 + p_2 x_2 = M$) shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M?
  - Yes. Good i is *normal*
  - No. Good i is *inferior*

5 Price changes

- Price of good i decreases from p_i to to $p_i' > p_i$

- For example, decrease in price of good 2, $p_2' < p_2$

- Budget line tilts:

$$x_2 = \frac{M}{p_2'} - x_1 \frac{p_1}{p_2'}$$

- New optimum?

- Demand curve: $x_i^*(p_i)$: demand for good i as function of own price holding fixed p_j and M

- Odd convention of economists: plot price p_i on vertical axis and quantity x_i on horizontal axis. Better get used to it!

- Does x_i^* decrease with p_i?
  - Yes. Most cases
  - No. Good i is *Giffen*
  - Ex.: Potatoes in Ireland
  - Do not confuse with Veblen effect for luxury goods or informational asimmetries: these effects are real, but not included in current model

6 Next Class

- More comparative statics:
  - More on Price Effects
  - Slutzky Equation
- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism