Age Effects, Irrationality and Excessive Risk-Taking in Supposedly Expert Agents

William Aldred
Department of Economics, UC Berkeley, Berkeley, CA 94720
waldred@berkeley.edu
(510)326-0410

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Abstract

This paper investigates the behavior of "expert" agents, using the context of PGA Tour golfers. We find two main results. Firstly, we find risk aversion increases linearly with age. Second, while the standard economic model predicts expert agents should be rational, we find this is not the case. PGA Tour players do not strategize optimally on the golf course: the average player takes excessive risk and fails to maximize expected performance, costing him $60000 per year. Additionally, we find evidence for loss aversion and diminishing sensitivity, two features of prospect theory, where the reference point is backward-looking and updated on a one-year cycle.

Keywords: risk, behavior, age, experience, PGA Tour

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1. Introduction

An inconsistency in the literature motivates this paper. On the one hand, there is a body of work on the relationship between risk attitude and age, with several papers (e.g. Morin and Suarez, 1983; Sung and Hanna, 1996) arguing that risk aversion increases with age. A separate literature is concerned with experienced actors competing at high stakes (for a review see Al-Ubaydli and List, 2015).[1] The key takeaway here is how professional experience leads actors to strategize optimally, i.e. without behavioral biases. Now, in a market where acting with risk-neutrality is rational, risk aversion would be considered a non-optimal behavioral anomaly. However, what if the actors in said market have a wide range in ages? If we see risk aversion varying with age, then they cannot all be acting optimally. And if they all play risk-neutral strategies, then clearly risk aversion can't be increasing with age. In this way, we have identified an academic conflict.

This paper is an attempt at finding which effect dominates, so as to resolve the conflict. We investigate the effect of age on risk attitude, on a stage of expert actors competing at high stakes, where risk attitude is quantified by the "risk index" formulated in this paper and the given stage is the PGA Tour. The null hypothesis is that age has no effect on risk index. The alternative hypothesis is that risk index decreases with age, where we note that a more negative risk index corresponds to greater risk aversion. The PGA Tour is the professional league which brings together the best golfers in the world, and it is a good setting for a few reasons. Firstly, in the context of a golf tournament, all PGA Tour players are expert actors. Also, to carry out this study requires individual level data on both age and risk appetite. Accordingly, golf is an individual

[1] Note "experienced actors" here means people with a high level of expertise in their field, i.e. expert agents. Experience in this sense is unrelated to age, since all experts count as experienced.

game, and the PGA Tour maintains a vast statistical database on player performance. Further, there is a wide age range amongst professional golfers – wider than in virtually any other sport. Finally, it might seem more natural to carry out our study using a setting of, say, day traders or hedge funds. However, in settings like these we would face the sizeable problem of having to untangle apparently risky investment decisions from those which leverage private information (see Kyle, 1985). In fact, while the theory behind risk-aversion is well-established,[2] the constraints of most datasets mean it is rarely tractable empirically. The golf setting provides a fortunate exception.

Our empirical strategy will follow two distinct stages. First, we identify a measure of risk attitude – generating the residuals from a regression of standard deviation in golfer scoring distribution on relevant skill attributes yields our risk index values. Then, we explore causality between age and risk. We will graph the data to get a sense for what type of relation, if any, exists, and finally we'll regress risk index on (appropriate functions of) age, controlling for other factors. We arrive at the finding that older golfers, on average, are more risk averse than younger golfers. This lends support to the literature arguing risk aversion increases with age. Seeing this variation in risk behavior, we extend our investigation to look at how rational professional golfers are with their risk decisions. We find evidence against the standard economic model: golfers are distinctly irrational agents. This also contravenes a number of papers in the behavioral economics literature, which argue expert agents behave without biases.

2. Background & Literature Review

[2] The mathematical formulation of the two leading measures of risk behavior, absolute risk aversion and relative risk aversion, are taught in undergraduate-level microeconomic theory.

The aim of this paper is to make progress toward resolving a conflict in the literature. On one side of the conflict are scholars who allege risk attitude varies with age, specifically that risk aversion increases with age, or that risk tolerance decreases with age. Note risk aversion and risk tolerance are almost perfectly negatively related (Faff, Mulino and Chai, 2007), so these are equivalent statements. On the other side are the anti-behavioral scholars. They maintain that behavioral biases, including risk aversion, while sometimes present in the laboratory and the field, disappear on a stage of experienced actors competing at high stakes. In other words, they say experienced actors always play optimal strategies. For ease of reference, we shall henceforth refer to these two sides of the divide as the risk aversion camp and the optimal strategies camp.

Before moving on, it is crucial to recognize the connection between risk attitudes and optimal strategies. An optimal financial planning strategy, for example, should be risk averse to some degree, since returns are by their nature compounding, not additive, and so losses hurt more than equal and opposite percentage gains. Also, the goal in retirement saving is often not to accumulate as much money as possible, but rather to meet some threshold amount for comfortable living. However, an optimal strategy in golf must be risk neutral. Intuitively, this is because one's score in golf is simply the addition of shots taken, where the aim of the game is to take as few as possible, and so there is symmetry between strokes gained and strokes lost. A rigorous proof can be found in appendix A.

Returning to the contention at hand, early to join the risk aversion camp were Morin and Suarez, who, in their 1983 paper *Risk Aversion Revisited,* looked at households' demand for risky assets. They concluded that "The investor's life-cycle plays a prominent role in portfolio selection behavior, with risk aversion increasing uniformly with age" (p. 2). To find this, they used data on the asset holdings of Canadian individual households, taken from the Survey of

Consumer Finances which was carried out in 1970 by Statistics Canada. The analysis is an OLS regression of the ratio of risky assets to net worth for a household, on age and controlling for net worth. While risky asset to net worth ratio is a decent measure of risk, there are problems. Firstly, the survey is a questionnaire completed by each household, so it's likely there are substantial errors in reporting, both intentional and unintentional. Further, many people do not invest in the stock market or in high-yield bonds simply because they are unaware of these options, and not because they are risk averse.

Also members of the risk aversion camp are Sung and Hanna, who analyzed the effects of financial and demographic variables in their 1996 paper *Factors Related to Risk Tolerance*. They used data on the asset holdings of US households, taken from the Federal Reserve Board's Survey of Consumer Finances (this is the US equivalent of the Canadian survey Morin and Suarez used the decade before). Based on the survey responses, Sung and Hanna categorized households as either risk tolerant or not risk tolerant – their dependent variable is binary, taking value 1 for above average risk tolerance and 0 for below average. Clearly this measure of risk tolerance is not precise; my paper stands out in the literature for its accurate and quantitative measure of risk. They then ran a multivariate logistic regression of this binary on several explanatory variables, including age. Interesting to us is the negative and statistically significant coefficient they found on age.

Many academics, however, are in the optimal strategies camp. Al-Ubaydli and List (2015) summarize their stance in the review paper *Field Experiments in Markets* – "(1) Generally speaking, markets organize the efficient exchange of commodities; (2) There are some behavioral anomalies that impede efficient exchange; (3) Many behavioral anomalies disappear when traders are experienced" (p. 1). List, in *Does Market Experience Eliminate Market*

Anomalies (2003), carried out a study on sporting memorabilia traders. He found that inexperienced agents exhibit the endowment effect and loss aversion, but these same behavioral biases are not present with experienced agents. List and Millimet (2008) studied GARP violations amongst children participating in sports card markets, and found these violations were significantly lower for experienced traders. Harrison and List (2008) found inexperienced traders suffered from the winner's curse, but experienced traders did not. The argument against risk aversion and other behavioral biases, and for optimal strategies, is expressed best by Al-Ubaydli and List:

> The conclusion that the majority of studies support—that professional experience diminishes the incidence of (and sometimes eliminates) behavioral anomalies that impede efficient exchange—should not come as a surprise. After all, impeding efficient exchange is tantamount to leaving money on the table, either at the individual or collective level. As such, all parties have an intrinsic incentive to actively seek to rid themselves of their behavioral anomalies. One imagines that a stock trader afflicted by the endowment effect will suffer below average earnings until they remedy the situation.
>
> Even if people are ignorant of their biases, or find that they are incapable of doing anything about them, the market imposes a selection force that will tend to weed out the sufferers since they will have inferior performance. There is a rich mainstream (Witt, 1986; Blume and Easley, 2002) and heterodox (Hayek, 1945) literature that argues this point. (p. 29)

Other scholars in the optimal strategies camp are Broadie and Mack. Broadie, in his 2011 paper *Assessing Golfer Performance*, asserts, “PGA TOUR golfers are among the best golfers in the world, so it is not unreasonable to assume that they play optimal or nearly optimal strategies” (p. 7). Mack built on the work of Broadie with his entirely theoretical paper, *A Model of Score Minimization and Rational Behavior in Golf* (2015). Mack’s contribution was finding that a model of rational choice and perfect information, in which expert golfers act as expected score-minimizing agents, is one which explains well several observable phenomena in golf strategy. He shows how tour players, almost all of whom are unaware of the formal dynamic programming problem that is golf strategy, are nevertheless playing optimal strategies as if they have perfect information and the ability to compute for a score-minimizing solution. Despite the success of Mack’s model, we expect it might not predict reality entirely accurately. We say this because we know professional golfers do not really follow a rigorous, homo economicus-like thought process. My research puts Mack’s model to the test: if we find evidence of idiosyncratic risk aversion amongst PGA Tour players, then we have observed something it cannot account for.

A handful more papers impact our research question. The first is Jennifer Brown’s *Quitters Never Win: The (Adverse) Incentive Effects of Competing with Superstars* (2011). Brown was interested in how internal competition motivates worker effort. She looked at PGA Tour scoring data from 1999 to 2010, the premise being workplace employees compete for promotion in a rank-order contest, much like how golfers on the PGA Tour compete against one another for prize money. She found that, on average and at a statistically significant level, pro golfers play worse when Tiger Woods is present in a tournament. In other words, their performance declines when the competition is tougher. *Risk-Taking Dynamics in Tournaments: Evidence from Professional Golf* (2016) by McFall and Rotthoff is another paper of importance.

They set out to build on the work of Brown, and conclude that the drop in field performance when Tiger Woods plays is due to golfers changing their strategies and taking on less risk. Concerns over their methodology are highlighted in appendix B. Nonetheless, we make sure to control for the so-called Tiger Woods effect or superstar effect in our regression analysis, discussed in the next section.

A well cited paper in behavioral economics is Pope and Schweitzer's *Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes* (2011). Pope and Schweitzer look specifically at PGA Tour putting data (a putt is a type of golf shot), from the 2004 through 2008 seasons inclusive. Applying a prospect theory framework, they find that virtually every player on tour, including Tiger Woods, is loss averse with respect to the salient reference point of par. One oversight in their methodology is highlighted in appendix C. Notwithstanding this, we are cognizant of how loss aversion might confound risk aversion, and take appropriate measures in our regressions. What we end up finding as we control for loss aversion – that professional golfers are loss averse with respect to their money rank the previous year – is a noteworthy tangent from this paper's main line of inquiry.

Finally, we consider Malmendier and Nagle's paper *Despression Babies: Do Macroeconomic Experiences Affect Risk Taking?* (2011). Malmendier and Nagle analyze financial risk-taking between different generations, and find that experiences have a causal effect on risk. In particular, they find the generation who experienced the Great Depression as young adults became very risk-averse, and remained this way for decades. They are the so-called depression babies. The difference between two generations is an example of a cohort effect. The mechanism driving a cohort effect is time-specific experience and learning. Of interest to us is how Malmendier and Nagle acknowledge how the age mechanism might also influence risk, and thus control for age. They wanted to find the cohort effect. We want the opposite. We want to tackle the time series question of how risk behavior changes within a player as he ages. We want to find the causal effect of age on risk, controlling for cohort effects. In our context, a plausible cohort effect could be that older golfers grew up in an era where they were taught to play safe, whereas younger golfers grew up being told to take risks. Unaccounted for, this would negatively bias our estimate on age. Accordingly, our final regression will include player fixed effects.

Mine is the first research effort that looks at whether risk attitudes vary with age amongst expert agents. The Morin and Suarez and Sung and Hanna papers, both focusing on household risk, consider only a linear relationship between risk and age. However, a priori, there is no good reason to expect linearity. Further, these papers use cross-sectional datasets and OLS regressions, where omitted variable bias is always a concern. My panel dataset accommodates the more powerful fixed effects regression methodology, so we have more confidence that endogeneity problems are dealt with. Mine is also the first paper that looks at the rationality of risk decisions in experienced agents. For some time scholars have had difficulty measuring people's risk attitudes accurately, on account of confounding factors. I develop an objective and quantitative measure which is superior to previous empirical risk measures.

3. Model, Methods and Data

This paper sets out to investigate the effect of age on risk attitude, on a stage of experienced actors competing at high stakes, where risk attitude is quantified by the risk index formulated in this paper and the given stage is the PGA Tour. The null hypothesis is that age has no effect on risk index. The one-sided alternative hypothesis is that risk index decreases with age, where we note that a more negative risk index corresponds to greater risk aversion.

We use a panel dataset with observations on all PGA Tour players in each PGA Tour season from 2004 to 2018 inclusive. With 3,870 observations across 42 variables for a total of 162,540 data points, we possess one of the largest clean datasets on PGA Tour statistics in the world. Further, since each golfer plays 70 rounds (about 5000 shots) on average in a season, the data reflects measurements on approximately 20 million golf shots. This feat is made possible by the PGA Tour's ShotLink system. Descriptions of the variables, as well as collection methods, can be found in table 1. Table 2 displays summary statistics. This is a paper on risk and age, so our analysis will culminate in a regression where risk index is the dependent variable, and age is the independent variable. All other variables listed are used as controls, some in this final regression, and others in the preliminary regressions from which we obtain our risk index.

The data analysis follows two distinct stages. First, we identify a measure of risk attitude. Then, we explore causality between age and risk. Before outlining our identification strategy for risk, we make note of the facts about golf given in appendix D. We shall consider a golfer's scores on par 5 holes as a discrete probability distribution, a concept new to the literature. The distribution is dependent on the difficulty of the holes played, the physical skill of the golfer, and the golfer's risk attitude. Since PGA Tour players compete on the same course each week, the difficulty of holes factor does not vary between players. Our identification strategy thus hinges on the insight that par 5 scoring distribution of PGA Tour golfers is part explained by physical skill, and part explained by risk attitude.

In finance, portfolio risk is measured simply as the standard deviation of a return distribution. Things are less straightforward here, because the spread in par 5 scoring is due to both skill and risk. Our solution is to identify risk – our risk index – as the residuals generated from the following OLS regression in each year:

$$par5sdinscore_i = \gamma_0 + \gamma_1 clubheadspeed_i{}^* + \gamma_2 fairwayshitfraction_i + \gamma_3 interactchsfwh_i + \gamma_4 sgapproach_i + \gamma_5 interactsgapprchs_i + \gamma_6 sgshortgame_i + \gamma_7 sgputting_i + \varepsilon_i$$

(* club head speed data collection began in 2007, so for the three prior years $drivingdistance_i$ was used here in place of $clubheadspeed_i$)

Please look to appendix E for a thorough explanation of the above specification. And for a more intuitive walk-through of why this method works, see appendix F.

Having obtained our measure of risk, we move onto the second stage in our analysis, where we examine the possible causal effect of age on risk. Before diving into regressions, we graph risk index against age, shown as figure 4. The quadratic fit lies right on top of the linear fit, demonstrating that the relation, if there is one at all, is linear. Thus we specify the following linear model:

$$risk\_index_{it} = \beta_0 + \beta_1 age_{it} + \beta_2 events_{it} + \beta_3 owgr50_{it} + \beta_4 rookie_{it} + \beta_5 superstar_{it} + \beta_6 lossdomain_{it} + \beta_7 Z_i + u_{it}$$

Note we include events as a control in light of how figure 5 indicates risk is (weakly) positively correlated with events. This makes intuitive sense because players who play tournaments less frequently may be less sure of their skill, and so play more cautiously. Owgr50, another control, is an indicator, taking value 1 for top 50 world-ranked players and 0 otherwise. We include it to allow for the possibility that the best golfers in the world systematically take more risk, because they might care more about winning a few times than about having consistently high finishes.

The next control is rookie. The rookie indicator variable has value 1 if given player in given season is in their first year on tour, and has value 0 otherwise. A plausible mechanism through which rookie status could affect risk is that rookie players, competing on PGA Tour courses for the first time, might be unaware of some of the dangers lurking on the golf holes. Hence they might systematically play more aggressively than players with at least one year of experience. This rookie mechanism has not appeared in the golf-related literature before; we are concerned about it since rookie players tend to be much younger than the average age on tour. So not including the rookie variable could lead to omitted variable bias. Without it, what might appear to be the effect of age on risk could actually be the effect of rookie status on risk.

The two final controls are superstar and lossdomain. The reasons for believing these play a part in risk strategy are perhaps less concrete than for the other factors, so we run our regressions both with and without them. Superstar refers to the superstar effect, which we pay heed to in light of the Brown and McFall and Rotthoff papers. Tiger Woods was supremely dominant from 1997 through 2009; at the end of 2009 came his infamous sex scandal, and in the aftermath he lost his golfing superiority. Our dataset runs from 2004 to 2018, hence we construct a binary which takes value 1 (for all players other than Tiger Woods; for Woods it always takes value 0) in 2009 and all prior years. It takes value 0 in 2010 and subsequent years.

Lossdomain refers to the behavioral phenomenon of loss aversion. The prevalence of loss averse behavior amongst expert agents, and specifically amongst PGA Tour golfers, was brought to our attention by Pope and Schweitzer. However, there are large differences in how loss aversion applies to their context versus how it applies to ours. Firstly, they look at loss aversion in players' putting, considering par as the reference point. They consider all golf holes. Par is indeed a good reference point for par 3 and par 4 holes, which together make up 14 of the 18 holes on an average golf course. This is because PGA Tour scoring average is very close to par on these holes. However, par is not a good reference point on par 5 holes. Professionals often view these as "birdie holes", and the scoring average on par 5 holes is not far off half a shot under par. Since we look exclusively at par 5 holes, following Pope and Schweitzer's reference point of par will not be informative. Furthermore, where the Pope and Schweitzer research was focused on finding evidence for loss aversion, our research is focused on finding a causal relation between age and risk. Owing to this, we look at players' risk behavior over entire seasons, rather than on individual shots. Ergo we consider a reference point of a different nature. There is one statistic that PGA Tour players are made acutely aware of – their money list ranking. This is what appears next to their names on tournament leaderboards each week.[3] Further, of the roughly 260 golfers on tour each year, only the top 125 on the season ending money list get to retain their PGA Tour playing privileges (popularly known as "tour cards") for the following year. Most interesting to us is how in post-season interviews, it's not uncommon for players to reference their rank on the money list this year versus last year, as they explain their satisfaction or disappointment with their season. From this, we consider the possibility of players internalizing their previous year's money list ranking as a backward-looking reference point. Accordingly, we create the lossdomain binary, which takes value 1 if the player's money list ranking is worse this year than last year, i.e. they are in the loss domain relative to their reference point, and 0 value otherwise. Figure 6 illustrates the idea here. Note the exact slopes and curvature of the reference-dependent utility function should not be taken too seriously. However, the prospect theory model makes the important prediction that as a player falls into the loss domain, they suddenly employ a more risk-seeking strategy. This is because their utility function over money list ranking is locally concave in the gain domain, but locally convex in the loss domain.

The entity and time dimensions of our panel are player and year, respectively. Player fixed effects are captured with the categorical Z variable. Year fixed effects are not necessary

[3] Technically, since 2013, FedEx Cup points have taken the place of money list rankings, but the two are equivalent for regular season events.

since our risk index values are generated as residuals, one year at a time. So by definition the mean risk index in each year is zero.

Of course, finding a statistically significant estimator for β_1 does not on its own imply causation. One method for strengthening the claim of causality is to employ an instrumental variable, however, due to the nature of our data, there is no instrument available. Instead we establish how all four ordinary least squares (OLS)/fixed effects (FE) assumptions hold for our multivariate linear regression, and that there are no violations of internal validity. Please turn to appendix G for this discussion.

4. Results

This paper set out to investigate 1) the effect of age on risk attitude, and 2) whether expert agents are as rational as the standard economic model claims. The underlying goal is to help settle an inconsistency in the literature. First, within each year, we ran the regression:

$$par5sdinscore_i = \gamma_0 + \gamma_1 clubheadspeed_i{}^* + \gamma_2 fairwayshitfraction_i + \gamma_3 interactchsfwh_i + \gamma_4 sgapproach_i + \gamma_5 interactsgapprchs_i + \gamma_6 sgshortgame_i + \gamma_7 sgputting_i + \varepsilon_i$$

(* $drivingdistance_i$ for years prior to 2007)

The residuals give us a risk measure, for each player in each year. We can now proceed with the second part of the analysis, where we examine the possible causal effect of age on risk. We specify the model below, which is linear in age:

$$risk\_index_{it} = \beta_0 + \beta_1 age_{it} + \beta_2 events_{it} + \beta_3 owgr50_{it} + \beta_4 rookie_{it} + \beta_5 superstar_{it} + \beta_6 lossdomain_{it} + \beta_7 Z_i + u_{it}$$

The regression results are shown in table 3. In all the regressions, whether OLS or FE, and whether or not the last two controls were included, the estimator on age is negative and statistically significant at either the 0.1% or 1% level. From our earlier visual inspection, we are confident the linear specification is correct. We also believe the OLS/FE assumptions are justified (see appendix D). Thus we are reasonably confident we have found a significant causal relation: older golfers, on average, are more risk averse than younger golfers. This finding lends support to the literature arguing risk aversion increases with age. Moreover, it is evidence against the literature claiming expert agents play optimal strategies.

We have rejected the null hypothesis, that age has no effect on risk index, in favor of the one-sided alternative that risk index decreases with age. But what is the economic magnitude of this result? Further, we've found older golfers are relatively more risk averse, but where are all our golfers located on the absolute scale of risk aversion? To answer these questions, we start out by defining a new variable for players' par 5 scoring relative to their skill levels. The thought process here is that each player's actual par 5 scoring should depend on two things – their physical skill attributes and their strategy. By normalizing a player's actual par 5 scoring for skill, we get to see how good or bad their strategy is. A more positive par-5-scoring-relative-to-skill corresponds to a worse strategy, since this means the player has a worse scoring average than most other players of the same skill level. The new variable is generated, for one year's data at a time, as the residuals from the OLS regression:

$$\text{par5scoringrelativetoskill}_i = \pi_0 + \pi_1\text{sgofftheteee}_i + \pi_2\text{sgapproach}_i + \pi_3\text{sgshortgame}_i + \pi_4\text{sgputting}_i + v_i$$

We now look to figure 7, which is a plot of this against risk index. The plot's signal to noise ratio is disappointingly low, but this was to be expected since physical skill is much more important than strategy. Nonetheless, we do see that, on average, if we take two players with the exact same physical skill in all areas of the game, then the player with the lower risk index tends to score better on par 5 holes. To professional golfers and their coaches, this is a huge discovery!

Most PGA Tour pros appear to be playing with too much risk. And we have even more to say. We've already established how the only optimal risk level in golf is the knife edge case of risk-neutrality. We have figure 7, an empirical plot. We theoretically map out, in figure 8, how varying risk should cause par-5-scoring-relative-to-skill to change. For an explanation of how this theoretical graph was created, please turn to appendix H. At this point, through comparing our theoretical graph with the empirical plot, we can attempt to locate, in an absolute sense, where on the risk scale our tour pros fall.

What we find is that the overwhelming majority of PGA Tour pros are too risk-seeking. While, as aforementioned, the signal to noise ratio in figure 7 is low, there is a clear trend amongst the scatter points that higher risk index values are associated with higher skill-normalized-scoring-averages. The positive slope of the linear fit line, which is statistically significant at the 0.1% level (see regression table 4), confirms this. Nonetheless, the thing we're most interested in is the minimum point of the quadratic fit curve, since this in theory marks risk-neutral (as explained in appendix H). Regression table 4 shows the squared term in risk index is positive and significant at the 5% level, which suggests the quadratic specification better fits the data than the linear specification. This aligns with our theory. The minimum point occurs at a risk index of around -10, which is at the extreme low end of the empirical distribution in risk index values. It would appear that -10 translates to risk-neutral. However, we have to question the accuracy of this finding. Since the minimum is so far toward the low end of observed risk index values, there are scarce data points for the left half of the parabola to fit to. Specifically, of the 3562 scatter points in figure 7, just 34 of them are found to the left of -10. We would like to add more data points to remedy this, but we are already at the limit of our dataset. Thus we will treat with caution any further results which build on this.

Ambiguity over the specifics does not take away from our discovery. Our analysis suggests more than 99% of tour pros play damagingly risk-seeking strategies. That is remarkable. And we can go further still. With our quadratic fit, and our earlier regressions of risk index on age, we can estimate the economic magnitude of these findings in more familiar units. The calculations behind the estimates can be found in appendix I. From an ageing point of view, we arrive at the result that a ten year increase in age, for a typical PGA Tour pro, is associated with a less risk seeking, and better, strategy, which over a season of play earns them an additional $6000. This is not a nontrivial amount of money. This tells us the effect of age on risk is economically as well as statistically significant. Further, from an optimal strategies viewpoint, if we take three average-skilled tour pros – one who's risk-neutral (-10 risk index), one who's averagely risky (0 risk index), and one who's 90th percentile for riskiness (risk index +5) – the risk-neutral player is expected to earn, in a season, $60000 more than the averagely risky player and $110000 more than the 90th percentile risky player. These are enormous differences! The accuracy of these last two numbers does hinge on the accuracy of the quadratic fit curve, especially the minimum point, in figure 7. We noted uncertainties with the fit, hence there is some uncertainty to these numbers. Acknowledging the uncertainties, it is still very clear that pro golfers are leaving significant sums of money on the table through inefficient risk behavior. Most of them, our evidence suggests, play strategies which are a fair ways off optimal.[4] This is direct counterevidence to an assertion made by Mark Broadie, strokes-gained pioneer and world-leading golf statistician: "PGA TOUR golfers are among the best golfers in the world, so it is not unreasonable to assume that they play optimal or nearly optimal strategies."

[4] Please excuse the golfing pun. I had to putt one in.

It is interesting how the optimal strategies effect we find is much greater in magnitude than the age effect. This would imply there is more heterogeneity in risk within players of the same age than within a single player as he ages. And this is consistent with the low R-squared coefficients we observe in our regressions of risk on age (table 3), as well as what common sense tells us: a person's risk appetite may well be influenced by their age, but it's mainly determined by other individual, psychological factors. Through the lens of optimal strategies, we see how the huge majority of our expert agents are too risk-seeking. It's worth considering what mechanisms might be behind this. Perhaps the most obvious candidate is overconfidence. Overconfidence is a well-established psychological bias where a person's subjective confidence in his or her judgments is reliably above the objective accuracy of those judgments. So in our context, overconfidence means a golfer overestimating his skill and acting accordingly. This would certainly explain why golfers are overly risk-seeking. If they believe they are more skilled than they really are, then inevitably they will take on risky shots following decision-making which is unjustified by their actual skill. Overconfidence has been widely studied in behavioral finance, and a great summary is given by Muthukrishna et al. (2017) –

> Overconfidence has been described as "one of the most consistent, powerful and widespread [psychological biases]" (Johnson & Fowler, 2011), with "no problem… more prevalent and more potentially catastrophic" (Plous, 1993). Overconfident CEOs make poorer investment and merger decisions (Malmendier & Tate, 2005, 2008), overconfident traders increase trade volume and lead markets to underreact to relevant information and overreact to anecdotal information (Odean, 1998), overconfident leaders are more likely to go to war even when the odds are stacked against them (Johnson, 2009), and overconfident people are more likely to start a

> business, even though most businesses fail (Camerer & Lovallo, 1999)… the common assumption underlying all these claims is that overconfidence is universal. (p. 3)

The literature widely agrees on the universality of overconfidence. If overconfidence is exhibited by laypeople and professional traders alike, albeit to differing extents, then us finding PGA Tour players aren't immune is maybe not all that surprising. Nevertheless, there are alternative explanations. Decisions concerning risk in golf are not straightforward. A player must weigh up their expected score for a safe play versus a risky play, which involves estimating likely probabilities of eagle, birdie, par and bogey in each case. Given the associated effort cost, players may instead simply follow what other players are doing. This could potentially set a herd behavior effect in motion, with players going for the risky play because they see other players going risky, and so on.

Another viable explanation is that professional golfers are not actually trying to minimize their expected score. Expected score minimization is a necessary condition for risk-neutral being optimal, yet it could be that professional golfers feel more satisfaction (i.e. gain more utility) at making a birdie than they lose when they make a bogey. This kind of golfing psychology, where birdies are remembered and bad holes are forgotten, does in fact describe a lot of recreational golfers. However, it seems unlikely that professionals would operate in this manner, since their livelihoods depend on their accruing low total scores.

Another alternative explanation revolves around machismo. Note machismo is not the same thing as overconfidence. Overconfidence comes from a genuine misunderstanding of one's skill level, whereas a macho golfer realizes the smart play might be conservative, yet goes for the risky, gung-ho play anyway. Given the strong fraternity culture on the PGA Tour,[5] it seems possible machismo might contribute toward the excessive on-course risk-taking we observe. A thorough exploration into why pro golfers, or more broadly why expert agents, strategize incorrectly is beyond the scope of this paper. However, we have found they are overly risk-seeking, with economically significant consequences. We possess robust evidence that PGA Tour golfers, who are certainly expert agents in their domain, do not act rationally in the field of play. Moreover, we claim most aren't even close to rational, and estimate 50% give up more than $50000 per year through overzealous risk-taking. We speculate behavioral psychology, and overconfidence in particular, might offer an explanation.

5. Discussion

As a first sanity check on our results, we draw from mainstream golfing commentary, where some well-known players are repeatedly referred to as "aggressive" while others are seen as "conservative". Tiger Woods, who did not play at all in 2016 or 2017 due to injury, was noted for his cautious play in 2018, and we see his risk index then was -3. Jordan Spieth and Jon Rahm are young, exciting players who draw crowds for their audacious shotmaking. Both have a positive risk index for every season they've played. Charles Howell III, known for his relentless greens in regulation and bogey avoidance, has competed through the full time span of our dataset and displays a risk index distribution tightly centered on -4. Another tour veteran, Phil Mickelson, nicknamed "Phil the Thrill" for his borderline recklessness, we find has a highly positive risk index in almost all seasons. This matching up between conventional wisdom and our risk index is strong evidence for it being an accurate measure of risk.

[5] If interested, see https://totalfratmove.com/a-comprehensive-list-of-every-greek-on-the-pga-tour-past-and-present/ and https://www.thedailybeast.com/the-secret-world-of-golf-groupies. Note these are popular press articles.

Next, we note how in golf, decisions involving risk are largely exclusive to par 5 holes. Therefore our risk index should not have a causal effect on relative-to-skill-scoring on either par 4 or par 3 holes. Figures 10 and 11, through their near-horizontal fit lines, appear to confirm this. More quantitatively, regression tables 5 and 6, which refer to par 4 and par 3 scoring respectively, both show zero and non-significant coefficients on risk index. Thus we have further evidence our risk index is measuring risk as intended. Note there's actually a little more going on in these figures and tables – please turn to appendix J if interested.

As a robustness check, we rerun our regressions excluding the most extreme observations on risk index (see table 8). Again we obtain estimators on age which are significant – at the 5% level in all four specifications, and additionally at 0.1% in three of the four. This gives us confidence our result is not driven by a few outliers. We also test the sensitivity of our risk measure. We generate different risk indices as residuals from the two preliminary regressions shown below:

$$par5sdinscore_i = \theta_0 + \theta_1 clubheadspeed_i^* + \theta_2 fairwayshitfraction_i + \theta_3 sgapproach_i + \theta_4 sgshortgame_i + \theta_5 sgputting_i + \omega_i$$

$$par5sdinscore_i = \varphi_0 + \varphi_1 clubheadspeed_i^* + \varphi_2 fairwayshitfraction_i + \varphi_3 sgapproach_i + \zeta_i$$

(* $drivingdistance_i$ for years prior to 2007)

The regression results for these first and second alternate risk indices are shown in tables 9 and 10. With our original risk index, the FE regression with all controls estimates the age coefficient as -0.085 (see table 3). With the first and second alternative risk indices, the estimators are, respectively, -0.084 (table 9) and -0.076 (table 10). Since the standard error is 0.03 in all cases, the three estimators are, to high significance, not statistically different. Consequently, though it could be argued the exact formulation of each risk index type, including the original, is somewhat arbitrary, we have evidence that risk index is robust to variation in its construction. This gives us increased confidence our risk index is indeed doing its job of accurately measuring risk.

Another observation we make is, for all three risk indices, the FE regression yields a more negative estimator on age than the OLS regression (see tables 3, 9 and 10). Thinking back to the cohort effect discussion with the Malmendier and Nagle paper, we notice this is the opposite of what we expected. If anything, it appears older golfers grew up in an era where they were taught to take risks, whereas younger golfers grew up being told to play more safe.

A final observation is that the estimator on lossdomain is positive and significant at at least the 5% level with all three risk indices, in both the OLS and FE regressions. This is worth interpreting. PGA Tour golfers, who are all expert agents, are systematically more risk-seeking when in the loss domain. They act like the gambler who digs himself into a deeper and deeper hole! Two aspects of prospect theory are loss aversion and diminishing sensitivity. While there is a rich and coherent literature on loss aversion (e.g. Genesove and Mayer, 2001; Allen et al., 2016; Rees-Jones, 2018), the evidence for diminishing sensitivity is not nearly as consistent (see Odean, 2002; Imas, 2016; Barberis, 2018). Pope and Schweitzer previously found how PGA Tour players are loss averse relative to the reference point of par, on each hole. Note how the reference point of par is short-lived: it resets at each hole. Note also how they found strong evidence for loss aversion, but not diminishing sensitivity.[6] Like them, we follow a prospect theory framework (see figure 6). However, our reference point is different in nature. We set the reference point for a given golfer in a given year as their money rank the previous year. This

[6] Since pro golfers only commonly make par, birdie, or bogey on par 5 holes, Pope and Schweitzer could not properly test diminishing sensitivity.

reference is longer term. Our analysis suggests that when a golfer is on track for a worse money rank finish this year than last year, they spend the whole season chasing down that reference point with increased risk-taking. As well as loss aversion, this is evidence for diminishing sensitivity. Since real-world evidence for diminishing sensitivity is scarce, our result may be of interest. Our result also suggests reference points are rigid – agents do not update them during the year. This contrasts with a recent study on New York cab drivers showing they update their reference points regularly (Thakral and Tô, 2018). It seems reference point flexibility might be heavily dependent on context.

6. Conclusion

The primary aim of this paper was to help resolve the risk-aversion-with-age versus optimal strategies conflict for expert agents. This conflict had been overlooked in the literature until now. Our findings support the argument that risk aversion increases with age. Moreover, we show this is indeed an age effect and not a cohort effect – an important distinction. The literature before us, notably the Morin and Suarez, and Sung and Hanna papers, could not achieve this. We also find evidence that expert agents do not behave optimally. In our context optimal is risk-neutral, and we find the vast majority of our expert agents are too risk-seeking. One possible explanation is overconfidence. Additionally, we find evidence for prospect theory; we find agents in their loss domain act more risk-seeking.

To reach these conclusions, we first invented a new measure of risk. Then we carried out a fixed effects regression of this risk index on age, and saw a negative and highly significant coefficient on age. We made sure to control for all the mechanisms the existing literature highlights, plus additional mechanisms specific to our context. We subsequently examined the magnitudes of our findings, and saw the variation in risk level exhibited by professional golfers leads to performance differences worth tens of thousands of dollars. These findings, while they should be of interest to behavioral economists, may well receive more attention amongst PGA Tour players and their coaches: a player who learns to strategize more efficiently from this paper could save shots and raise their earnings substantially. Our results here add to the critique of the standard economic model, and extend this position to include expert agent contexts. In light of this, we encourage researchers to now go after more mainstream-economic expert agents, for example pension fund managers, where overly risky decisions could have hefty economic consequences. Though as noted in our introduction, such settings pose the formidable challenge of having to disentangle risk from private information (Kyle, 1985).

The first major limitation of this paper is how all our expert agents are male. The literature (e.g. Halek and Eisenhauer, 2001; Eckel and Grossman, 2008; Charness and Gneezy, 2012) agrees there are differences in risk behavior between men and women. We would like to carry out all the same analysis on a dataset from the LPGA Tour (Ladies' PGA Tour). Unfortunately, the LPGA does not keep statistics in the same way as the PGA. Another limitation is how the age range in our dataset is 19 to 55 years old. Hence the linear age-causes-risk relation we found is something we can only be confident of within this range. We cannot say much about how a person's risk behavior changes as they approach and enter retirement age. Therefore, building on the risk index approach to measuring risk developed here, we encourage further inquiry into the risk-age relation.

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Table 1. Descriptions of Variables

β	playername	Name of given PGA Tour player.
β	year	Given PGA Tour season.
β	age	Age of given player on July 1st (traditional mid-point of season) of the given year.
β	events	Total number of events a given player competes in during a given season.
γ	owgr50	Binary variable taking value 1 if player holds a top 50 ranking in the official world golf rankings on July 1st of given year, and 0 else.
δ	rookie	Binary variable taking value 1 if player is in his first year as a full PGA Tour member, and 0 else.
φ	superstar	Binary variable taking value 1 for years 2009 and prior, when Tiger Woods dominated the PGA Tour, and 0 for all subsequent years.
β	moneyrank	A given player's ranking on the end of season money list.
φ	lossdomain	Binary variable taking value 1 when a player has a worse money rank for the year in question compared to the previous year, and 0 else.
β	money	Total prize money, in USD, a given player earns in a given season.
φ	moneyin2018	Total prize money, in USD, a given player earns in a given season, normalized to 2018 dollars. This assumes the increase in PGA Tour purses over the years has equaled inflation, which may not be completely accurate but is a sound approximation.
β	clubheadspeed	Average driver club head speed, measured by radar and quoted in miles per hour, for a given player in a given season. A measure of power; can be taken as a proxy for maximum range off the tee and with approach shots.
β	drivingdistance	Average driving distance, in yards, for a given player in a given season. Measured only on the two longest holes per course. A less good measure of power than club head speed, but radar technology was not in use prior to 2007.
β	fairwayshitfraction	The fraction of all tee shots on par 4 holes and par 5 holes which finish on the fairway, quoted as a decimal, for a given player in a given season. A measure of driving accuracy.
β	sgoffthetee	Strokes gained off the tee statistic for a given player in a given season. This is a measure of player's skill off the tee, in units of shots per round, where 0 is PGA Tour average and more positive means more skilled.
β	sgapproach	Strokes gained approach statistic for a given player in a given season. This is a measure of player's approach shot skill, in units of shots per round, where 0 is PGA Tour average and more positive means more skilled.

β	sgshortgame	Strokes gained short game statistic for a given player in a given season. This is a measure of player's short game skill, in units of shots per round, where 0 is PGA Tour average and more positive means more skilled.
β	sgputting	Strokes gained putting statistic for a given player in a given season. This is a measure of player's putting skill, in units of shots per round, where 0 is PGA Tour average and more positive means more skilled.
β	sgtotal	Strokes gained total statistic for a given player in a given season. This is a measure of player's total skill, in units of shots per round, where 0 is PGA Tour average and more positive means more skilled. Strokes gained total equal to the sum of the above four strokes gained categories.
β	par3scoring	Average score on par 3 holes for a given player in a given season.
χ	par3scoringrelativetoskill	Residual for given player from the regression within each season of par3scoring on sgapproach, sgshortgame and sgputting.
β	par4scoring	Average score on par 4 holes for a given player in a given season.
χ	par4scoringrelativetoskill	Residual for given player from the regression within each season of par4scoring on sgoffthetee, sgapproach, sgshortgame and sgputting.
β	par5scoring	Average score on par 5 holes for a given player in a given season.
χ	par5scoringrelativetoskill	Residual for given player from the regression within each season of par5scoring on sgoffthetee, sgapproach, sgshortgame and sgputting.
δ	par5meanscoretopar	Calculated as (par5scoring – 5) for each player in each year.
δ	par5meanscoreunderpar	Calculated as –(par5meanscoretopar) for each player in each year.
β	holespereagle	Total number of tournament holes a given player plays in a given season, divided by their total number of eagles.
β	par5birdieandbetterrate	Total number of birdies and eagles a given player scores on par 5 holes in a given season, divided by number of par 5 holes played.
δ	par5eaglerate	Calculated as (holespereagle*4/18)^(-1). Implicit are the assumptions that on average a PGA Tour course has four par 5 holes, which is true, and that all eagles occur on par 5 holes, which is true for all intents and purposes.
δ	par5birdierate	Calculated as (par5birdieandbetterrate – par5eaglerate).
δ	par5parrate	Calculated as (1 – (par5eaglerate + par5birdierate + par5bogeyrate)).
δ	par5bogeyrate	Calculated as (par5meanscoretopar + 2*par5eaglerate + par5birdierate). Implicit is the assumption that no double-bogeys or worse are scored on par 5 holes, which is true for all intents and purposes for PGA Tour players.

χ	par5varianceinscore	Calculated variance in a given player's par 5 scoring distribution in a given season, where scoring distribution is composed of the above eagle, birdie, par and bogey rates.
χ	par5sdinscore	Standard deviation in par 5 scoring distribution, in units of shots, calculated as square root of the above variance.
χ	resid	Residual for given player from the robust regression within each season of par5sdinscore on clubheadspeed (drivingdistance for years prior to 2007), fairwayshitfraction, their interaction, sgapproach, the interaction of sgapproach and clubheadspeed, sgshortgame, and sgputting.
χ	risk_index	resid multiplied by 100, such that most values fall between -10 and 10. Wherever "risk index" is written in this paper, we are referring to this risk_index variable.
χ	Risk Index	risk_index rounded to the nearest whole number. Risk Index is a neat one-digit statistic, although risk_index is still used in all further regressions.
χ	alt_resid	Residual for given player from the robust regression within each season of par5sdinscore on clubheadspeed (drivingdistance for years prior to 2007), fairwayshitfraction, sgapproach, sgshortgame, and sgputting.
χ	alt_risk_index	alt_resid multiplied by 100, such that most values fall between -10 and 10.
χ	alt_resid_2	Residual for given player from the robust regression within each season of par5sdinscore on just clubheadspeed (drivingdistance for years prior to 2007), fairwayshitfraction and sgapproach.
χ	alt_risk_index_2	alt_resid_2 multiplied by 100, such that most values fall between -10 and 10.

[β] Obtained from https://www.pgatour.com/stats.html and https://www.pgatour.com/players.html. Each of these 73,530 data points was manually picked out from the PGA Tour website, which displays most statistics only on player profile pages and not in tabular format, and then manually entered into the author's dataset (the PGA Tour does not currently give researchers access to its raw datasets, and the site is not amenable to web scraping techniques).

[γ] Obtained from the official world golf ranking past rankings archives http://www.owgr.com/about?tabID={BBE32113-EBCB-4AD1-82AA-E3FE9741E2D9}. 3,870 data points of this type.

[δ] These are recognized golfing statistics, however no data was available. The rookie data, totaling 3,870 points, were generated through (tedious!) manual inspection. The other variables, accounted for by 23,220 points, were algorithmically generated by the author.

[φ] The superstar and lossdomain data, totaling 7,740 points, were generated through manual inspection. The 3,870 moneyin2018 data points were generated algorithmically. These three variables represent ideas in economics which have not previously been applied to golf (respectively the superstar effect, prospect theory, and inflation).

[χ] These are new golfing statistics, theorized by the author. The 46,440 data points were generated by the regression techniques described in this paper.

Table 2. Summary Statistics

	# obs	mean	median	min	max
age	3870	36.19	36	19	65
events	3870	20.62	23	1	36
owgr50	3870	0.1607	0	0	1
rookie	3870	0.1121	0	0	1
superstar	3870	0.4041	0	0	1
moneyrank	3870	129.55	129.5	1	268
lossdomain	3870	0.4623	0	0	1
money	3870	1057200	675800	5520	12000000
moneyin2018	3870	1191200	765300	5520	14500000
clubheadspeed	3015	112.46	112.31	91.22	128.18
drivingdistance	790	287.2	287.0	245.5	320.5
fairwayshitfraction	3850	0.6246	0.6242	0.3929	0.8750
sgoffthetee	3801	-0.048	-0.005	-3.533	1.485
sgapproach	3801	-0.028	0.016	-3.194	2.168
sgshortgame	3800	0.001	0.014	-1.543	1.202
sgputting	3803	-0.026	-0.004	-3.348	2.444
sgtotal	3801	-0.101	0.000	-9.189	3.815
par3scoring	3850	3.077	3.07	2.63	3.56
par3scoringrelativetoskill	3800	0.000	-0.001	-0.375	0.372
par4scoring	3850	4.066	4.06	3.78	4.48
par4scoringrelativetoskill	3800	0.000	-0.002	-0.303	0.228
par5scoring	3850	4.697	4.69	4.00	5.25
par5scoringrelativetoskill	3800	0.000	-0.001	-0.677	0.402
par5meanscoretopar	3850	-0.303	-0.31	-1.00	0.25
par5meanscoreunderpar	3850	0.303	0.31	-0.25	1.00
par5birdieandbetterrate	3850	0.3984	0.4015	0.0000	0.8333
par5eaglerrate	3850	0.0188	0.0179	0.0000	0.1875
par5birdierate	3850	0.3797	0.3829	0.0000	0.7500
par5parrate	3850	0.4876	0.4859	0.1292	1.0000
par5bogeyrate	3850	0.1139	0.1090	-0.0834	0.4550
par5varianceinscore	3850	0.4678	0.4672	-0.0001	0.9233
par5sdinscore	3849	0.6818	0.6835	0.0134	0.9609
resid	3562	0.00000	-0.00029	-0.22534	0.19189
risk_index	3562	0.000	-0.029	-22.534	19.189
Risk Index	3562	0	0	-23	19
alt_resid	3562	0.00000	-0.00009	-0.27504	0.18256
alt_risk_index	3562	0.000	-0.009	-27.504	18.256
alt_resid_2	3563	0.00000	-0.00023	-0.30910	0.22514
alt_risk_index_2	3563	0.000	-0.023	-30.910	22.514

Table 3.

Risk Index as Dependent Variable

	OLS	FE	OLS	FE
Age	-0.062***	-0.082***	-0.063***	-0.085**
	(0.01)	(0.02)	(0.01)	(0.03)
Events	0.002	0.017	0.004	0.020
	(0.01)	(0.02)	(0.01)	(0.02)
Owgr<=50	0.021	-0.125	0.025	-0.131
	(0.16)	(0.21)	(0.16)	(0.21)
Rookie	-0.047	-0.397	0.115	-0.249
	(0.22)	(0.24)	(0.22)	(0.25)
Superstar			0.064	-0.043
			(0.14)	(0.26)
Loss domain			0.345*	0.318*
			(0.14)	(0.14)
Constant	2.150***	2.596*	1.964***	2.511
	(0.58)	(1.01)	(0.58)	(1.34)
R-squared	0.013	0.008	0.015	0.009
Deg. freedom	3557	663	3555	663

Notes. For the OLS regression, this table reports estimates and robust standard errors. For the player fixed effects regression, this table reports estimates and heteroskedasticity and autocorrelation consistent standard errors, clustered by playername.
* p<0.05, ** p<0.01, *** p<0.001

Table 4.

Par 5 Scoring Relative to Skill as Dependent Variable
OLS Estimation

	Linear	Quadratic
Risk_Index	0.0025***	0.0026***
	(0.0003)	(0.0003)
Risk_Index^2		0.00011*
		(0.00005)
Constant	0.00055	-0.0012
	(0.001)	(0.001)
R-squared	0.029	0.032
Deg. freedom	3560	3559

Notes. This table reports estimates and robust standard errors.
* p<0.05, ** p<0.01, *** p<0.001

Table 5.

Par 4 Scoring Relative to Skill as Dependent Variable

OLS Estimation

Risk_Index	-0.0002
	(0.0001)
Constant	0.0004
	(0.0005)
R-squared	0.0011
Deg. Freedom	3560

Notes. This table reports estimates and robust standard errors.

* p<0.05, ** p<0.01, *** p<0.001

Table 6.

Par 3 Scoring Relative to Skill as Dependent Variable

OLS Estimation

Risk_Index	-0.0001
	(0.0002)
Constant	0.0011
	(0.0007)
R-squared	0.0001
Deg. Freedom	3560

Notes. This table reports estimates and robust standard errors.

* p<0.05, ** p<0.01, *** p<0.001

Table 7.

Log of Season Earnings (in 2018$) as Dependent Variable

OLS Estimation

SG total	0.560***
	(0.01)
Constant	5.774***
	(0.01)
R-squared	0.564
Deg. freedom	3799

Notes. This table reports estimates and robust standard errors.
* p<0.05, ** p<0.01, *** p<0.001

Table 8.

Risk Index as Dependent Variable

	OLS	FE	OLS	FE
Age	-0.060***	-0.076***	-0.061***	-0.076*
	(0.01)	(0.02)	(0.01)	(0.03)
Events	0.001	0.014	0.002	0.016
	(0.01)	(0.01)	(0.01)	(0.01)
Owgr <= 50	0.024	-0.076	0.026	-0.081
	(0.16)	(0.21)	(0.16)	(0.21)
Rookie	-0.068	-0.354	0.050	-0.240
	(0.21)	(0.24)	(0.22)	(0.24)
Superstar			0.062	-0.003
			(0.13)	(0.25)
Loss domain			0.257*	0.246
			(0.13)	(0.14)
Constant	2.098***	2.424**	1.967***	2.257
	(0.51)	(0.85)	(0.52)	(1.28)
R-squared	0.013	0.007	0.014	0.008
Deg. freedom	3525	661	3523	661

Notes. ~1% of observations have been dropped, corresponding to the most extreme (sixteen highest and lowest) risk index values.
For the OLS regression, this table reports robust errors. For the player fixed effects regression, this table reports estimates and HAC standard errors, clustered by playername.
* p<0.05, ** p<0.01, *** p<0.001

Table 9.

Alternate Risk Index as Dependent Variable

	OLS	FE	OLS	FE
Age	-0.063***	-0.080***	-0.064***	-0.084*
	(0.01)	(0.02)	(0.01)	(0.03)
Events	0.005	0.020	0.007	0.023
	(0.01)	(0.02)	(0.01)	(0.02)
Owgr<=50	0.010	-0.109	0.013	-0.114
	(0.16)	(0.21)	(0.16)	(0.21)
Rookie	-0.033	-0.391	0.128	-0.249
	(0.22)	(0.24)	(0.23)	(0.25)
Superstar			0.064	-0.043
			(0.14)	(0.27)
Loss domain			0.341*	0.306*
			(0.14)	(0.15)
Constant	2.104***	2.463*	1.921**	2.388
	(0.58)	(1.01)	(0.59)	(1.34)
R-squared	0.013	0.008	0.015	0.010
Deg. freedom	3557	663	3555	663

Notes. For the OLS regression, this table reports estimates and robust standard errors. For the player fixed effects regression, this table reports estimates and heteroskedasticity and autocorrelation consistent standard errors, clustered by playername.
* p<0.05, ** p<0.01, *** p<0.001

Table 10.

Second Alternate Risk Index as Dependent Variable

	OLS	FE	OLS	FE
Age	-0.058***	-0.075**	-0.060***	-0.076*
	(0.01)	(0.02)	(0.01)	(0.03)
Events	-0.006	0.009	-0.004	0.013
	(0.01)	(0.02)	(0.01)	(0.02)
Owgr<=50	-0.315	-0.210	-0.310	-0.217
	(0.16)	(0.22)	(0.16)	(0.22)
Rookie	0.205	-0.254	0.412	-0.054
	(0.22)	(0.24)	(0.23)	(0.25)
Superstar			0.080	-0.018
			(0.14)	(0.27)
Loss domain			0.440**	0.427**
			(0.14)	(0.15)
Constant	2.232***	2.507*	1.996***	2.249
	(0.59)	(1.04)	(0.60)	(1.37)
R-squared	0.012	0.005	0.014	0.008
Deg. freedom	3558	663	3556	663

Notes. For the OLS regression, this table reports estimates and robust standard errors. For the player fixed effects regression, this table reports estimates and heteroskedasticity and autocorrelation consistent standard errors, clustered by playername.
* p<0.05, ** p<0.01, *** p<0.001

Figure 1. Example Utility as Function of Score for Irrational Risk-Averse Golfer.

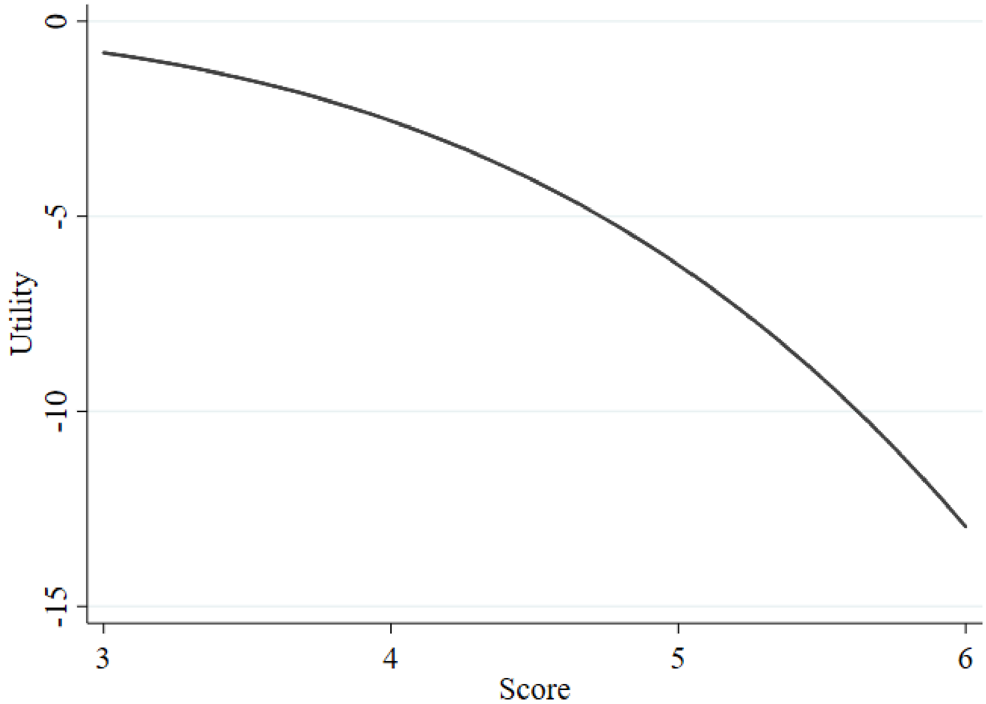


Figure 2. Example Utility as Function of Score for Irrational Risk-Seeking Golfer.

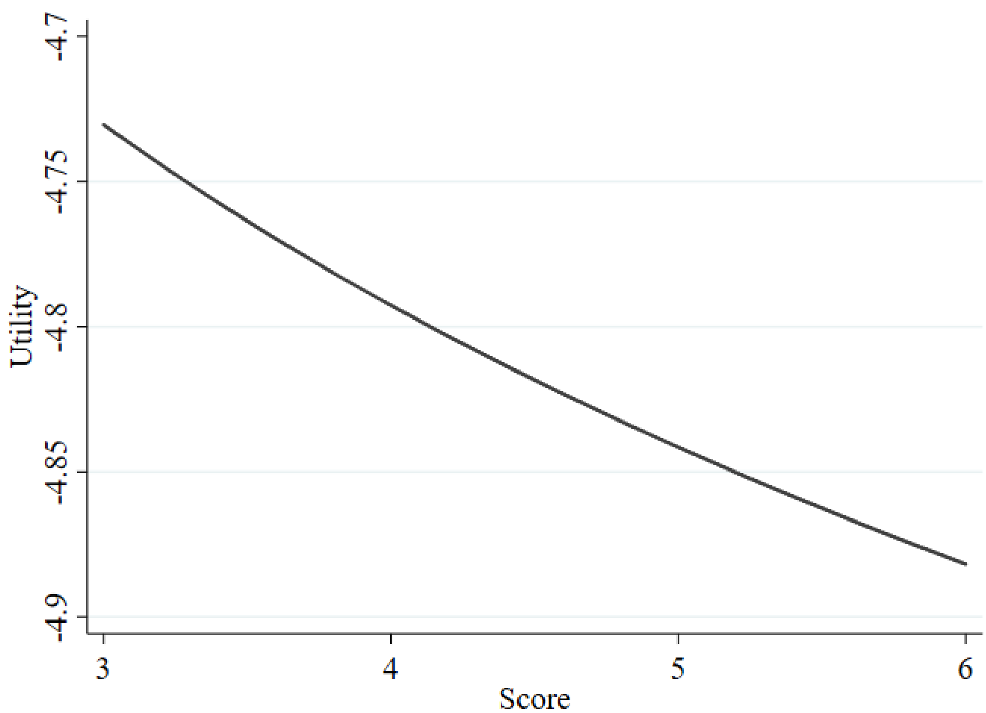


Figure 3. Example Utility as Function of Score for Rational Risk-Neutral Golfer.

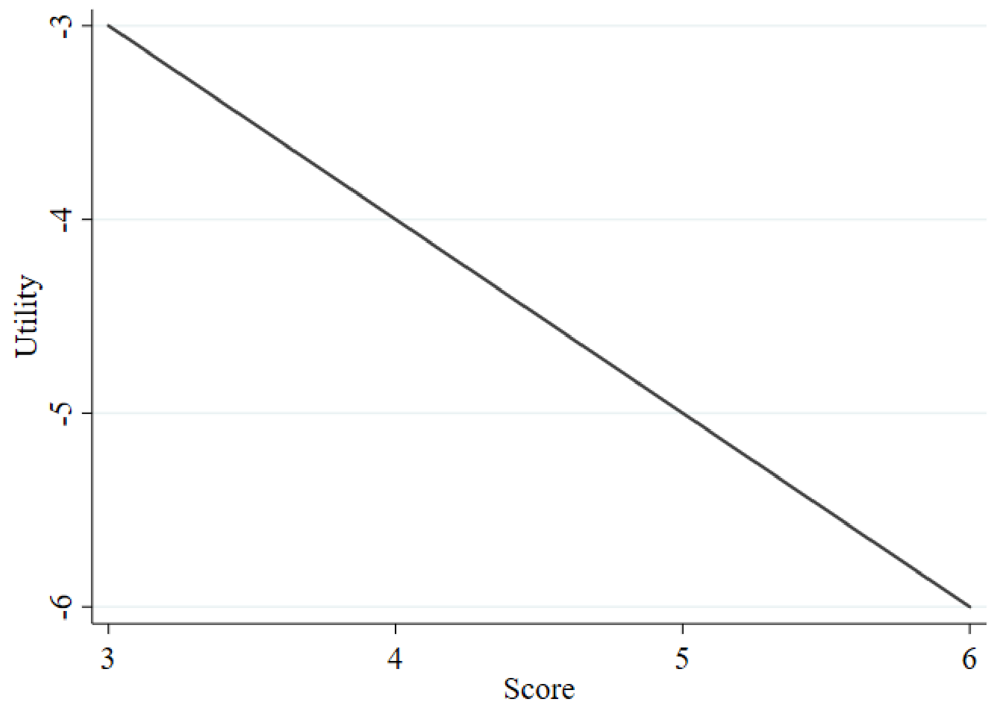


Figure 4. Scatter Plot and Best Fit Lines for Risk Index against Age. Note since age is a discrete variable, a horizontal jitter has been added to avoid overplotting.

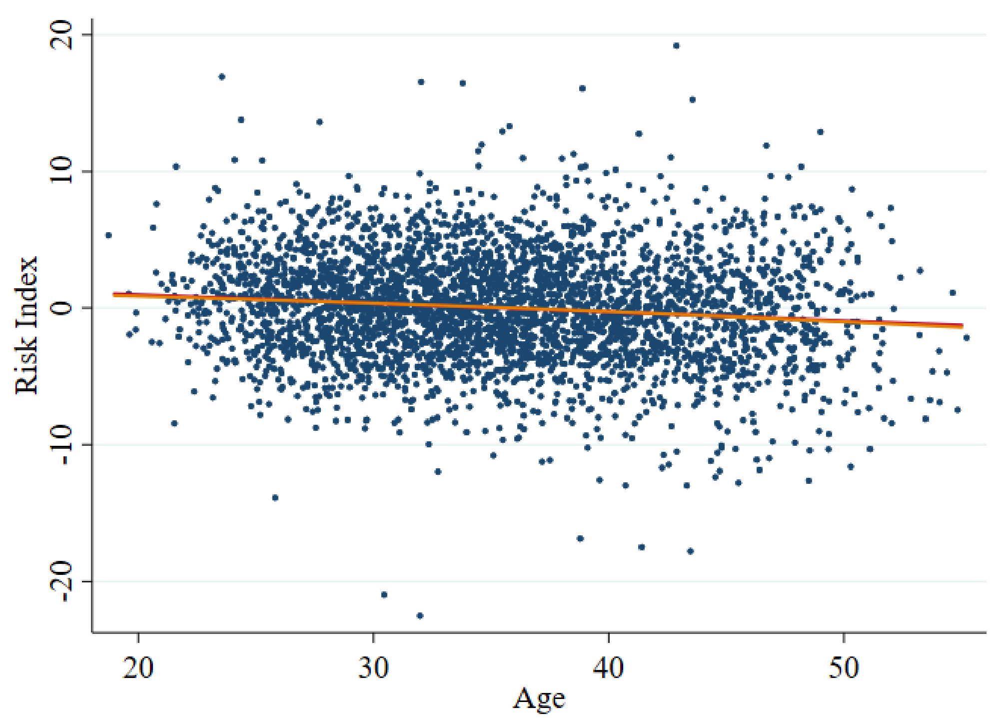


Figure 5. Scatter Plot and Best Fit Lines for Risk Index against Events.

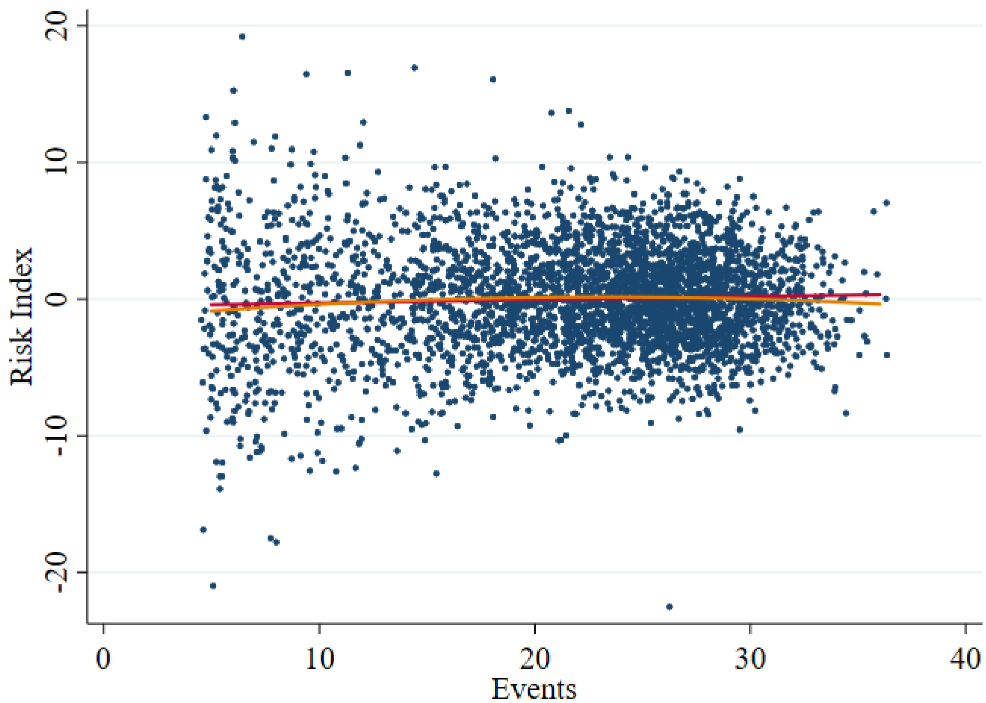


Figure 6. Prospect Theory Golfer Utility over Money Rank.

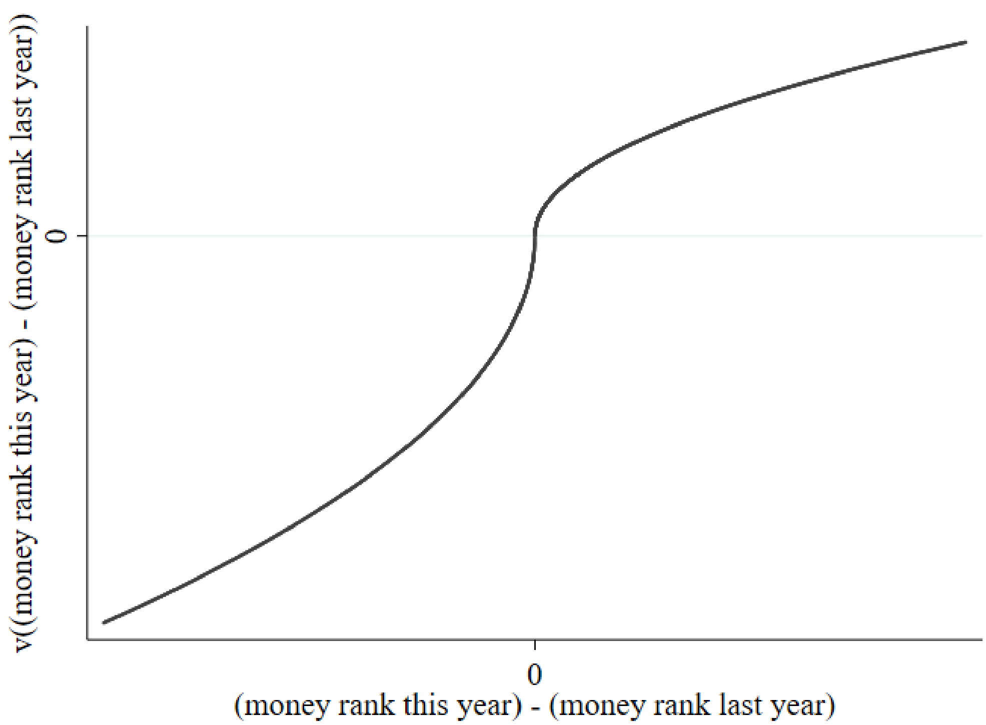


Figure 7. Scatter Plot and Best Fit Lines for Par 5 Scoring Relative to Skill against Risk Index.

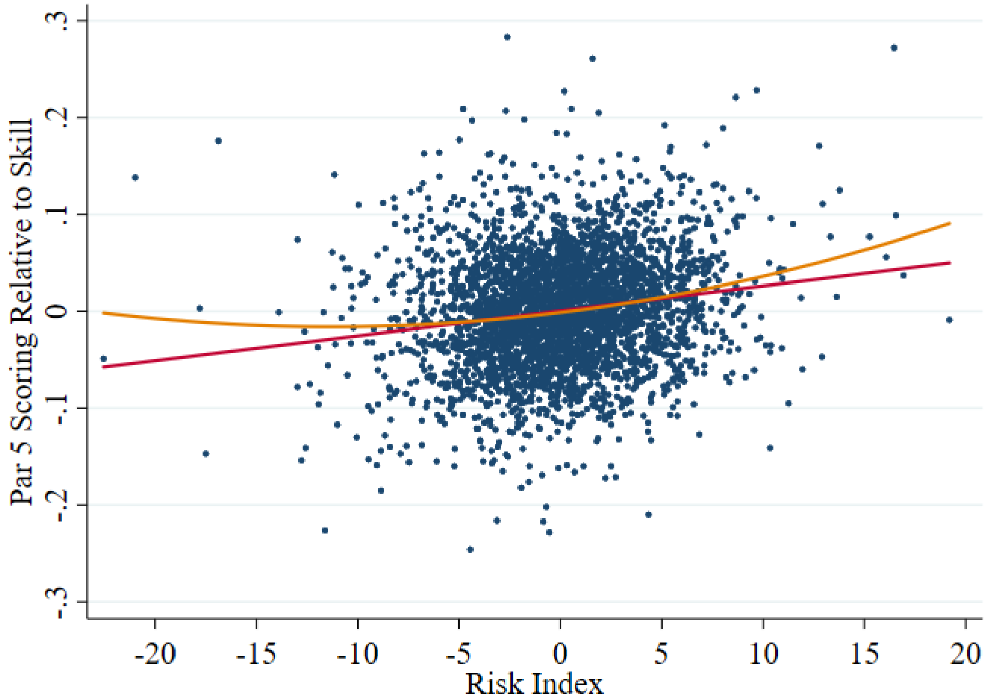


Figure 8. Hypothetical Par 5 Scoring Distribution against Risk Index for Average Tour Player.

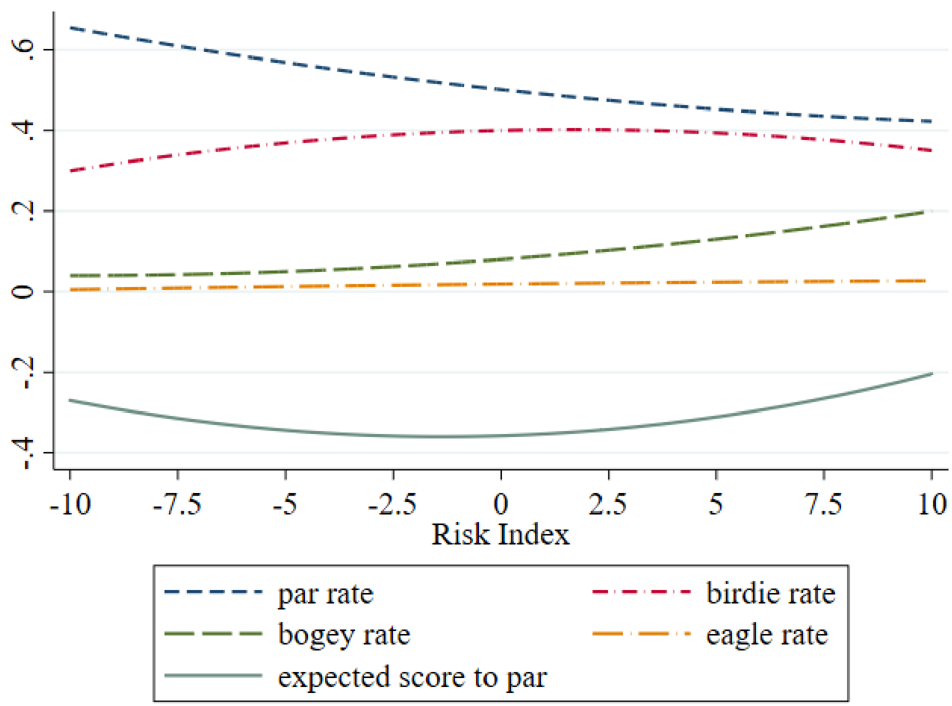


Figure 9. Scatter Plot and Best Fits for Log of Season Earnings against Strokes Gained Total.

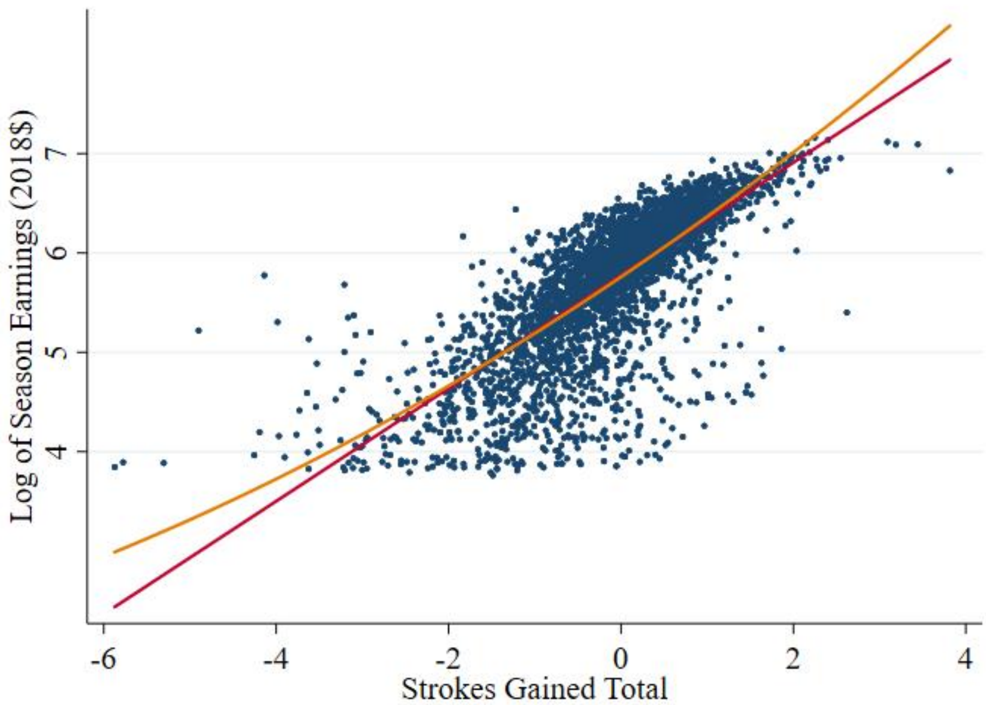


Figure 10. Scatter Plot and Best Fit Lines for Par 4 Scoring Relative to Skill against Risk Index.

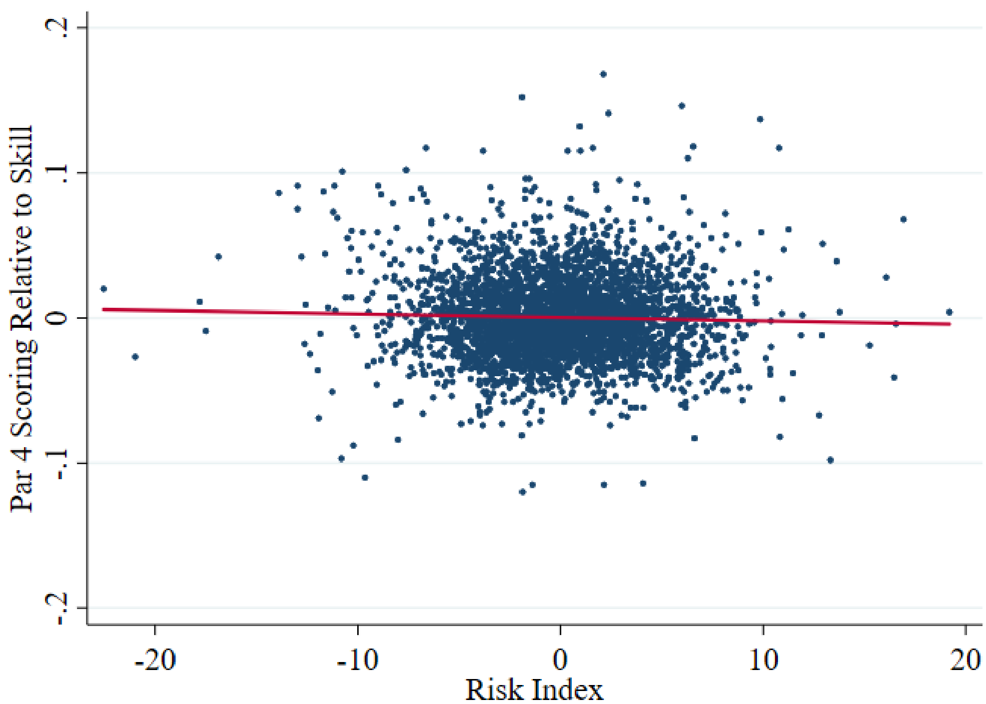
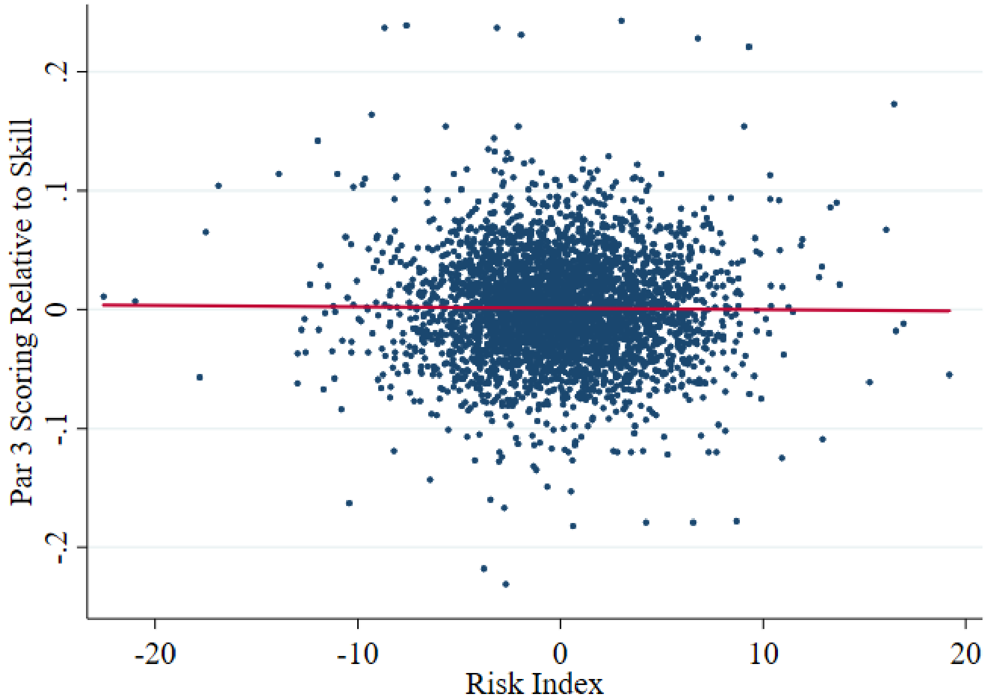


Figure 11. Scatter Plot and Best Fit Lines for Par 3 Scoring Relative to Skill against Risk Index.



Appendices

A

We shall analyze risk behavior in golf through the lens of expected utility theory. First, let's consider a risk-averse golfer on a hypothetical par 5 hole. Our golfer can either play it safe and hit the green in three, which guarantees a par, or play a risky two-shot strategy which gives a 51% chance of making birdie and a 49% chance of bogey. Our risk-averse golfer, having a concave utility function in score (see figure 1), wants to avoid bogeys more than he wants to shoot birdies. So his expected utility is higher with the safe play than the risky play, and he chooses the safe play. Now, let's think about the mirror situation. A different hypothetical par 5 hole yields a guaranteed par to safe play, and a 49% chance of birdie and 51% chance of bogey to risky play. A risk-seeking golfer, with a convex utility function in score (see figure 2), will take on the two-shot gamble since this maximizes his expected utility. Note both these golfers have behaved rationally given their utility function. However, note also that both chose strategies that did not minimize expected score. This seems odd, given that the aim of the game in golf is to shoot the lowest score. This leads us to objective rational choice in golf.

Prize money in professional golf follows a convex distribution, where the winner takes a large percentage of the total purse. However, professional golf is a marathon as opposed to a sprint – tournaments are played over seventy-two holes in four days. To quote McHale and Forrest in their paper *The Importance of Recent Scores in a Forecasting Model for Professional Golf Tournaments* (2005): "Score in a golf tournament reflects performance over a full 4 days of competition and therefore there is less scope than in other sports for variation in scores to be attributable to random noise" (p. 3). Continuing in this vein, let's consider every golfer as having a population distribution for per-hole score, determined by their skill and their strategy. Now, if a golf tournament were played over just one hole, the best strategy for winning would be one which gives the best chance of a birdie (or eagle), even if this strategy also gives a high probability of bogey. In other words, a high volatility, risky strategy, which doesn't necessarily minimize expected score. But tournament golf is played over seventy-two holes, where the score which matters is the total score, the summation of all per-hole scores. Seventy-two is a large sample size. The law of large numbers tells us that over this many holes, a strategy which produces the lowest expected score on every hole is going to beat the more volatile strategy almost every time. The one possible exception to this in practice is the case of players in contention to win, playing the final few holes of a tournament. This, however, accounts for only a tiny fraction of the total number of holes played by all competitors in a tournament. A thorough exploration of how mean and standard deviation in tournament golf scores translate to win probabilities is beyond the scope of this paper, but if interested see Grober's *PGA Tour Scores as a Gaussian Random Variable* (2008). Suffice it to say that, virtually 100% of the time, the unique best strategy in golf is the one which minimizes expected score. This sentiment was voiced long ago by hall of fame golfer Sam Snead – "Forget your opponents; always play against par."

Now we'll move on to constructing a utility function for a rational golfer. We've covered why on almost any hole in a tournament, a player should be trying to minimize his expected score. That is to say, for a rational golfer, minimizing expected score is both a necessary and sufficient condition for maximizing expected utility. It turns out we've already disproved by counterexample the possibilities of risk-averse play and risk-seeking play being rational. Both entail choosing a strategy to maximize expected utility which gives a strictly higher expected score than the minimum expected score available on the hypothetical par 5 holes above. By definition, an individual is risk-neutral if she is indifferent between a gamble and a sure outcome equal to the expected value of the gamble. Such behavior can only be represented by a linear utility function, a result economics undergraduates hear countless times in micro theory classes. Ergo, let's write the rational golfer's utility function as u(hole score) = –(hole score), or any linear and non-negative transformation. With this family of functions, it is always true that, given a list of score distribution choices for the hole, the option which maximizes expected utility also minimizes expected score. We have a winner: the unique objectively rational strategy in golf, at least for the vast majority of situations, is risk-neutrality. See the associated utility function in figure 3.

B

At first glance, the new risk index measure we'll derive in our paper appears quite similar to McFall and Roffhoff's pseudo-risk measure. McFall and Rotthoff use a linear probability model, with both golfer (entity) and year (time) fixed effects, to determine intended two-shot strategy rate. Independent variable is the go-for-green rate, in other words the fraction of total par 5 holes for which a given player attempts to hit the green in two shots. They take this rate as a proxy for risk appetite; in this way they link riskiness of player strategies with decision-making on par 5 holes, which is what we do. The dependent variable of interest to them is a binary indicating the presence of Tiger Woods. Hence the crux of their analysis lies in comparing the risk appetites of all pro golfers when Tiger Woods is in a tournament, versus when he's not.

They deal with the important confounding variable of driving distance, although their controls are traditional golf statistics and not strokes gained statistics. They use Jim Furyk as an example of a conservative player. To say that Jim Furyk is conservative simply because he has a low go-for-green ratio is incorrect, because Furyk is a short hitter and physically can't reach as many par 5 holes in two as other players. However, including driving distance as a control, the authors claim Furyk has a lower go-for-green ratio than expected for a player with his driving distance, and so his strategy must be more conservative than average. This is where McFall and Rotthoff stop their argument, yet the claim is still not entirely correct: approach shot dispersion is an endogenous confounder in their regressions. Here's why. McFall and Rotthoff use PGA Tour data for years 2004-2013. The PGA Tour adopted strokes gained statistics in 2011, which were then retroactively applied as far back as 2004.[7] However, the superiority of strokes gained over traditional golfing statistics was only elucidated in 2014, with Broadie's work *Every Shot Counts*. McFall and Roffhoff published *Risk-Taking Dynamics in Tournaments* in 2016, which, given the

[7] The PGA Tour had been keeping the requisite data for strokes gained statistics since 2004.

average timespan from first draft to publication in the field of economics, means they likely wrote the paper in ignorance of the power of strokes gained. In any case, they don't use strokes gained controls, and the fact they don't control for strokes gained approach means approach shot dispersion is an endogenous regressor. They do include fixed effects, however, these do not absorb the kind of variation in skill we observe, where a player's approach shot performance can yo-yo significantly and unpredictably from year to year. Tiger Woods in his prime, they claim, played a high risk strategy. However, from strokes gained statistics we know Tiger Woods' approach shot accuracy was exceptionally good, even by his standards, in the 2006 and 2008 seasons (but not 2007). So second shots to par fives which would have been risky for most players, and even for Woods in most years, were not all that risky for him in those seasons! Thus to some degree McFall and Rotthoff mix up risk-taking, a psychological trait, with approach shot skill, a physical attribute. In our paper we actually find Tiger Woods has been more conservative than the average tour pro throughout his career.

Notwithstanding the above, McFall and Rotthoff's punchline is pro golfers are less willing to take risks when Tiger Woods is in a tournament, which causes them to play worse. This is equivalent to saying most pro golfers perform worse if they employ a less risk-seeking strategy. If true, it must follow that becoming more risk-averse takes most golfers further away from the optimal strategy of risk-neutrality. Which in turn implies most pro golfers are playing risk-averse strategies. In our paper, we'll reach a result at loggerheads with this – we find virtually all pro golfers are excessively risk-seeking.

C

To find evidence for loss aversion, Pope and Schweitzer first carry out a logit regression, where the dependent variable takes value 1 if a putt is holed and 0 else. Independent variables are binaries for par and birdie; putt length, a good proxy for difficulty, is included as one of the controls. They find that, holding all else constant, pro golfers are better at par putts than birdie putts. The conclusion drawn here is consistent with loss aversion: golfers try harder when putting for par because a miss here would put them in the loss domain.

Pope and Schweitzer consider several alternative mechanisms for their findings. Easily the most important of these is the learning effect, which works as follows. Whenever a golfer misses a putt, they are faced with another putt to the same hole. But now they've already seen how the ball moves around the hole, so they have a better handle on the exact slope and speed of the green near the hole. Now, since a missed birdie putt is followed by a par putt, we have a big confounder at play. Birdie putts tend to be first putts, where a golfer hasn't had opportunity to learn from a previous putt. Par putts, on the other hand, tend to be second putts, where the golfer benefits from having seen what the first putt did. Thus, the observation that golfers are more successful with par putts than birdie putts could, in theory, by driven entirely by this mechanism, not loss aversion. Pope and Schweitzer control for this learning effect by including dummy variables for number of putts already hit. They find high statistical significance. Including these dummies for learning effects in the specification reduces the loss aversion estimates by around 25%.

However, this isn't where learning stops. Players also gain insight into the nuances of the green near the hole through short game shots. A high level golfer learns almost as much from hitting a chip, pitch or bunker shot as they do from hitting a putt. And since the vast majority of short game shots arise from a missed green in regulation, most short game shots are followed by a par putt. Hitting a short game shot gives a player an advantage for the upcoming par putt. Additionally, on average a PGA Tour player misses almost as many greens in regulation as they hit per round. All in all, this means that by not including dummies for number of short game shots already hit, Pope and Schweitzer underestimate the total learning effect. It is likely the total learning effect accounts for close to 50% of what was at initially thought to be loss aversion. Note this opinion is not exclusive to me. Mark Broadie, in his book "Every Shot Counts" (2014), writes:

> We already know that second putts are easier than first putts because of the learning effect. After controlling for first-putt-second-putt differences, and controlling for uphill, downhill, and sidehill differences, the par-birdie effect is reduced by more than half… But even this computation overestimates the birdie-par effect. A short first putt for par can happen after chipping from off the green, so the golfer gets to see the path of the chip before hitting his putt. (p. 160)

On a personal side note, we've seen in this and the previous appendices how academic research using the setting of golf can be aberrated by the authors not possessing elite-level-golf-specific knowledge. Frustratingly, the overwhelming majority of professional golfers and their coaches are not at all in touch with this research, such that oversights like these are left undetected and uncorrected.

D

Firstly, golf is a sport in which both skill and strategy are tested. Further, golf courses, which consist of eighteen "holes", are divided up between par 3, par 4, and par 5 holes. Par 5 holes are the longest holes, where professional golfers can reach the "green" in two or three shots. Par 5 holes on PGA Tour courses are designed to elicit excitement: the golfer must choose to either "go for the green" in two, which is risky, or "lay up" conservatively in a three shot strategy.

The strategic decision to be made on par 5 holes revolves around how much risk to take on, and so in this paper the words risk and strategy are synonymous.

E

Par 5 mean score is very well explained by the four strokes gained statistics, since together these statistics are a complete measure of a golfer's skill. However, we are interested in scoring dispersion about a player's mean score. Therefore we separate driving skill into its orthogonal constituents of power and accuracy, measured through club head speed, and fraction of fairways hit, respectively. This allows us to tell apart a powerful but inaccurate player from an accurate but short-hitting player. The two could be the same in terms of strokes gained driving, however, the former has larger spread because he sets up more birdie opportunities with long drives, but also makes more bogeys through errant drives.

The regression is run four times, once for each year, because like strokes gained and scoring average, the risk index statistic describes a given player in a given year. Before running the regressions, though, all observations of golfers who played fewer than five events in a year are dropped. Five events corresponds to playing at least forty par 5 holes. With a smaller sample size than about forty, a player could have a spuriously idiosyncratic scoring distribution and hence a spuriously extreme risk index value, which is why we drop such observations.

F

Suppose we take two hypothetical golfers with exactly the same physical skill level in every area of the game, but who have different spreads in their par 5 scoring distribution. This difference could arise due to statistical noise, a difference in their risk attitudes, or some combination. The average PGA Tour golfer plays around three-hundred and twenty par 5 holes in one season, and with such a large sample size the effect noise has on scoring distribution is minimal. We are left with risk attitude as the only candidate for explaining the difference. Holding physical skill constant, a player with a riskier strategy will make more eagles and more birdies on par 5 holes, but also more bogeys. Hence a risk-taking player will have greater spread in their scoring distribution, which translates into a positive residual in the above regression of standard deviation on physical skill factors. Conversely, a safe-playing golfer's residual will be negative.

G

To begin, we note the assumptions under which an OLS regression produces unbiased and consistent estimates, and consistent standard errors, are the same as the assumptions under which an entity fixed effects regression produces these things. The first OLS/FE assumption is conditional mean error of zero, i.e. $E(u_i \mid age_i, events_i, \ldots, lossdomain_i) = 0$. Expressed otherwise, it must be true there are no excluded variables which help explain risk_index, and which are correlated with any of our explanatory variables. We do not think there are any more observable factors which help explain risk taking in golf: we've covered all bases with respect to the related literature,[8] and we've conjectured there may also be a rookie learning mechanism. There is the possibility of unobserved cohort effects, but we save this discussion for the section on robustness checks. Relatedly, a low R-squared should not worry us, because risk attitude differs widely from person to person on account of individual psychology, which, with the exception of age, we cannot observe. We can take the component of this psychology not explained by age as being randomly assigned by nature, and thus independent of events, owgr50, and so on.

[8] To the best of our knowledge, that is.

The second OLS/FE assumption is that of random draws, requiring (age_i, $events_i$, $owgr50_i$, $risk\_index_i$) for i = 1,…, n, be independent and identically distributed draws from their joint distribution. Our population of interest is all PGA Tour golfers who have played during the "modern" era of golf, which began in around 2000. The dataset we have covers all players from the 2004 to 2018 seasons inclusive; we maintain our sample is representative of the population.

The third OLS/FE assumption is that age_i, $events_i$, $owgr50_i$, and $risk\_index_i$ have finite fourth moments. Age by its nature takes on finite values with no large outliers: all PGA Tour golfers are between about 20 and 55 years old. Events also takes on finite values with no large outliers: we omitted players with fewer than five events when we calculated risk index values, and the maximum events played in our sample is 36. Owgr50 is a binary variable, so there are no outliers. The risk index distribution is fairly Gaussian, with most values falling between -10 and +10. There are a small number of extreme values, but we deal with this threat to the third OLS/FE assumption in a robustness check. The fourth and final OLS/FE assumption of no perfect multicollinearity is clearly satisfied.

The first threat to internal validity is omitted variable bias. We've already discussed the first OLS/FE assumption, and as long as it holds we know omitted variable bias is absent. The next threat to internal validity is measurement error. The nature of golf statistics means they are objective and recorded with no uncertainty. Hence there are no errors in variables. Another threat is that of sample selectivity bias. We've already stated how the population of interest is PGA Tour golfers who've played during the modern era. The dataset we have covers all players from 2015 to 2018 – there is no bias in our sample selection. Simultaneous causality is also a threat. The effect we care about is that of age on risk, but age is evidently not caused to vary by any other variable, risk included. Ergo there is no reverse causality. The last threat to internal validity is specification error. For reasons given in the results section, we believe the linear model we arrive at is a suitable specification.

H

To reiterate, zero is the average risk index. This comes about from how risk index is defined. Zero does not mean risk-neutral. Where risk-neutral actually lies is what we're trying to find.

Let's take a tour player of average skill. Looking at our summary statistics in table 2, we see the mean rates of bogey, birdie and eagle made on par 5 holes. Further, through an understanding of how golf is played at the professional level, we are able to reason out, approximately, how the probabilities of the different scores vary with risk. We'll begin with the straightforward: bogeys and eagles. Within common risk levels, bogey probability obviously increases with risk index, and we conjecture this occurs at an increasing rate. Eagle probability also increases with risk index, but this time we suspect a decreasing rate. For birdies, we conjecture that birdie probability is increasing up to some (positive) risk index and then falls back down as play becomes reckless. It would make sense that par probability is highest for very safe play and falls with risk at a decreasing rate. Since we are working under the assumption that PGA Tour players only ever make eagle, birdie, par or bogey on par 5 holes (this assumption is not 100% accurate but is extremely close), we can actually back out par probability as one minus the sum of the other probabilities. This is how we trace out par rate in Figure 8. The fact the curve is shaped as we expect from our above intuition is reassuring.

It is easy enough to calculate average score to par from a known scoring distribution. This is what the lowest curve, "expected score to par", shows. To recap our thought experiment:

we've taken a PGA Tour player of average skill, and we've then varied his risk index to cover all values in the -10 to +10 interval, with the intention of seeing how his scoring is affected. And what we see is very interesting. The expected score curve is a U shaped parabola. Note this is robust to tweaking the bogey, birdie, and eagle curves – we plotted an expected score curve for 100 plausible variations on the bogey-birdie-eagle frequencies with risk, and found a U shaped parabola every time. What does change is the minimum point of the parabola. The minimum point is important because it marks the lowest possible expected score a player of a given skill can attain. By definition, this makes the risk index at the minimum point the unique optimal strategy. Since we know the optimal strategy in golf is risk-neutral, we can now mark where risk-neutral is on our risk index scale.

Figure 8 depicts a hypothetical world in which PGA Tour players are in general too risk-seeking. We know this because the minimum point of the expected score parabola in this figure occurs at a risk index of -1.25. Thus risk-neutral, which is optimal, occurs at -1.25, yet the average risk index is 0. Our intention all along was to find where our actual PGA Tour players lie relative to risk-neutral, and we now have all the tools to do this.

I

First, we'll consider the economic magnitude of our findings from an ageing perspective. Table 3 shows us four slightly different regressions for risk index on age. Since it turns out lossdomain is significant (a point we'll return to later), the two right-hand side regressions are better specifications than those on the left. Of these, the OLS regression answers the question of association between risk and age, but without controlling for cohort effects. The FE regression tackles the time series question of how a player's risk evolves as they age. This latter question pinpoints our primary research goal in this paper, hence we look to the fixed effects regression.

What it tells us is, on average and holding all else constant, a one year increase in age causes (we're confident it's a causal relation for reasons given in appendix G) a 0.085 point decrease in risk index.

Moving across to table 4, we see that, on average, a one point increase in risk index is associated with a 0.0025 increase in par 5 score relative to skill. Note up until now we've argued for a quadratic relation between par 5 score relative to skill and risk index. However, figure 7 shows the quadratic and linear fits are very similar for risk index values between about -5 and +5, which is most players. Since it's easier to make general statements with a linear relation, as change doesn't depend on level, we'll work here with the linear regression from table 4. Putting these two results together, if we take a player whose skill doesn't change as he ages, this player, when he's ten years older, is expected to have a 0.85 point lower risk index and thus a ~0.85*0.0025≈0.0021 lower scoring average on par 5 holes.

There are four par 5 holes in a round of golf, so the older version of the player is expected to be ~0.0085 shots better per round, which is equivalent to saying the older player would have a ~0.0085 better strokes gained total. Strokes gained total is a very good predictor of season earnings, as we see in figure 9 and table 7. All else equal, a 0.0085 better strokes gained total is associated with a 0.0085*0.560~0.0048 increase in log of season earnings, i.e. a 0.48% increase in season earnings. The average PGA Tour player earns $1.24 million per year (in prize money alone, not including endorsements or appearance fees), and 0.56% of this is ~$6000.

Now, we shall consider things from an optimal strategies perspective. The average risk index of zero is associated with an average par-5-score-relative-to-skill of 0.00. The risk index of -10, however, which we tentatively take as the risk-neutral mark, is associated with an average of

-0.02. On the other hand, a risk index of +5, which is in the 90th percentile for risk, is associated with an average of +0.02. There are four par 5 holes in a round of golf, so, all else equal, a player having risk index -10 has a 0.08 better strokes gained total than an average player, and a 0.16 better strokes gained than a +5 risk index player. But what do these translate to economically? We look to table 7 again, which tells us how strokes gained total relates to season earnings. Our risk-neutral (-10 risk index) player is expected to be 0.08*0.560*100%~4.5% better off than our average (0 risk index) tour pro in terms of season earnings. Assuming both are of average skill, this means a ~$60000 difference. And the earnings gap between the risk-neutral player and the 90^{th} percentile risk player is double this at ~$110000! Note the accuracy of these numbers hinges on the accuracy of the quadratic fit curve, especially the minimum point, in figure 7, hence there is some uncertainty.

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The linear fits in figures 10 and 11 have very slight negative slopes. Although not statistically significant at 5% (shown by the coefficients on risk index in tables 5 and 6), the negative slopes are not random noise. So what's going on? We've already found how players with higher risk index values perform worse, relative to their skill level, on par 5 holes. But skill, for our purposes, is defined as the strokes gained total statistic, which is very strongly correlated with scoring average. Say we have two equally skilled players, i.e. two players with the same strokes gained total measure. Player X has risk index +10, and player Y has risk index 0. On average, player X performs worse on par 5 holes, but to be at the same overall skill level as player Y he must be doing better on the par 4 and par 3 holes. Thus we have reverse-causality at play. This is the mechanism which drives the negative slopes in those figures. However, since there are three and a half par 4 and par 3 holes for every one par 5 hole, this reverse-causality effect is diluted down compared to the risk effect we see on par 5 holes, and so it does not show up as statistically significant.