Theory Field Examination

August 2023

Instructions: The field exam consists of three questions corresponding to Econ 206, Econ 207A, and PHDBA 269D, respectively. Please choose and answer two out of the three questions. You have a total of two hours. The exam is closed-book. Good luck!

Econ 206 - Field Exam question

August 2023

There are a total of 40 points. Partial credit will be awarded for just giving intuition and/or an incomplete proof.

Consider a monopolist who has a unit mass of a homogeneous good which they can sell to a unit mass of infinitesimal buyers. Each buyer has a type $t \in [\underline{t}, \bar{t}] \subset \mathbb{R}_+$, and the measure of types in the population has CDF F and density f. (Note that types are non-negative.) A (direct) mechanism for the monopolist consists of a pair of allocation and payment rules, (q, p), where $q : [\underline{t}, \bar{t}] \to [0, 1]$ describes the probability of receiving a unit of the good, and $q : [\underline{t}, \bar{t}] \to \mathbb{R}$ describes payments to the monopolist.

We depart from the standard setting by considering buyers whose payoff depends on who else consumes the good: if the allocation rule is q, a buyer with type t who receives the good and makes payment x gets payoff

$$t \cdot \varphi(q) - x \tag{1}$$

where $\varphi(q) = \int_{\underline{t}}^{\bar{t}} zq(z)dF(z)$. (The interpretation is that buyers like the good more the more others are consuming, and they care especially about the good being consumed by high types.) Given these payoffs, a mechanism (q, p) is incentive compatible (IC) iff

$$t \cdot \varphi(q) \cdot q(t) - p(t) \geq t \cdot \varphi(q) \cdot q(t') - p(t') \quad \text{for all } t, t' \in [\underline{t}, \bar{t}] \tag{IC}$$

1. **(5 points)** Prove that any IC mechanism must have an allocation rule, q, that is increasing.

2. **(10 points)** Assume that (q, p) is an IC mechanism and leaves no surplus to the lowest type buyer. The revenue from this mechanism is $\int_{\underline{t}}^{\bar{t}} p(t)dF(t)$. Derive an expression for the revenue from this mechanism as a function of the allocation rule q alone. Denote this function by $Rev(q)$. (*Hint: use the envelope theorem*)

3. **(15 points)** Assume that the type distribution is regular, in the sense of Myerson (1981), i.e. $t \mapsto t - \frac{1-F(t)}{f(t)}$ is increasing. Show that there is a posted price

mechanism that is optimal. (Recall that in a posted price mechanism there is a price p^* at which any buyer can purchase a unit of the good).

4. **(10 points)** How does the optimal posted price mechanism you identified above compare to the optimal mechanism in the standard setting, i.e. in the setting where $\varphi(q) = 1$ for all q? Are there any qualitative differences? (Your answer may include conditions on the parameters, $\underline{t}, \bar{t}, F$, etc.)

References

R. B. Myerson. Optimal auction design. *Mathematics of operations research*, 6(1): 58–73, 1981.

Field Exam Question for Econ 207A

Consider the random assignment model of Bogomolnaia and Moulin (2001).

Definition 1 *A random assignment mechanism* $f : \mathcal{A}^n \to \mathcal{R}$ *is* **envy-free** *if for any preference profile* $\succsim \in \mathcal{A}^n$ *and any two agents* i *and* j*:*

$$f_i(\succsim)\, sd(\succsim_i)\, f_j(\succsim).$$[1]

1. Show that the probabilistic serial mechanism is envy-free.
2. Fix any vector of eating speeds $\omega = (\omega_1, \ldots, \omega_n)$. Let f denote the random assignment mechanism derived from the simultaneous eating algorithm with speeds ω at all preference profiles. Show that if f is envy free then it is the probabilistic serial mechanism.

---

[1] In this footnote, we remind some standard definitions and notation from Bogomolnaia and Moulin (2001). There is an equal number n of agents and indivisible objects. The set of linear orders over objects (representing strict ordinal preferences) is denoted by $\mathcal{A}$ and the set of $n \times n$ bistochastic matrices (representing random assignments) is denoted by $\mathcal{R}$.

$f_i(\succsim)$ is the ith row of the bistochastic matrix $f(\succsim)$ denoting agent i's random allocation under the mechanism f at the preference profile $\succsim$. Analogously $f_j(\succsim)$ is j's random allocation.

$sd(\succsim_i)$ is the first-order stochastic dominance partial order over probability distributions over objects induced by i's ordinal preferences. That is, for any two probability distributions p and q over the set of objects A:

$$p\, sd(\succsim_i)\, q \iff \forall a \in A : \sum_{b \in A : b \succsim_i a} p(b) \geq \sum_{b \in A : b \succsim_i a} q(b)$$

Field Exam Question for PHDBA269D

Construct a knowledge model with a finite state space. Provide an interpretation for each notation/definition you introduce. In that model, define knowledge and common knowledge. State and prove the "agreeing to disagree" theorem of Aumann (1976): "Agreeing to Disagree" *The Annals of Statistics*, 4(6): 1236-1239. If you introduce two definitions of common knowledge that are equivalent to each other, you do not need to prove that they are equivalent.