Psychology and Economics Field Exam

August 2018

There are 3 questions on the exam. Please answer the 3 questions to the best of your ability. Do not spend too much time on any one part of any problem (especially if it is not crucial to answering the rest of that problem), and don’t stress too much if you do not get all parts of all problems.
Problem 1

Consider experimental subjects who are asked to work through some tedious experimental tasks, such as pressing A-B keys for 10 minutes, as some cruel experimenter is paying them to do. Let \( e \) be the number of units of effort (say, number of A-B presses). There is a piece rate \( p \) for each unit of effort. Let \( c(e) \) be the cost of effort, with \( c' > 0 \) and \( c'' > 0 \). To simplify, we will assume a power cost of effort \( c(e) = k (e^{1+\gamma}) / (1 + \gamma) \).

**Part a.** Write down the utility maximization problem as maximizing \( pe - c(e) \). Plug in for a power cost of effort function and solve for the optimal effort \( e^* \). Solve for the elasticity of effort to the piece rate, and comment on the property of the solution.

**Part b.** Economist A runs a real-effort experiment with two groups, \( p_{HI} \) and \( p_{LO} \), with \( p_{HI} > p_{LO} > 0 \) with, say, 100 participants in each group. The results yield average effort \( e_{HI} \) and \( e_{LO} \) in the two groups, with \( e_{HI} > e_{LO} > 0 \). The economist wants to structurally estimate the model above and reasons “I have two treatments and two parameters to estimate, this will work!” Can you outline at least one and (preferably two) estimation strategies to estimate the model above on the data? Be as detailed and precise as you can.

**Part c.** Economist B hears about the model above, the results by economist A, and the structural estimation. Economist B is skeptical about the model. The economist re-runs the experiment with groups \( p_{HI} \) and \( p_{LO} \), but also includes a third group with no incentive, that is, \( p = 0 \). We will call effort in this group \( e_0 \). The economist finds \( e_{HI} > e_{LO} > e_0 > 0 \), that is, the effort in the no-incentive group is significantly larger than zero. Is this result consistent with the model above of economist A? Discuss and suggest an extension of the model above that would accommodate the observed finding that \( e_0 > 0 \).

**Part d.** Economist C independently had run a real effort experiment with the same three treatments, \( p_{HI}, p_{LO}, \) and \( p = 0 \). This experimenter is of reduced-form training though and simply runs an OLS regression

\[
\log(e_i) = a + \beta_{LO}T_{LO} + \beta_{HI}T_{HI} + \epsilon_i, \quad (1)
\]

where \( T_i \) is a treatment indicator for treatment \( i = HI, LO \), with the no-piece-rate treatment as omitted category. Economist B writes a report on the paper of Economist C and comments “The OLS model (1) admits a structural interpretation; that is, one can interpret the estimates of \( a, \beta_{LO}, \beta_{HI} \) in light of a model where the workers optimize utility in setting the level of effort. Here is how one can do that”. Can you continue this sentence and, to the extent that you can, derive the model that would yield equation (1)? Continue assuming a power cost of utility as above. In this model what are the parameters from the model underlying \( \beta_{LO} \) and \( \beta_{HI} \)?
Part e. Economist D re-analyzes the data of economist C but uses the OLS equation

\[ e_i = a + \beta_{LO}T_{LO} + \beta_{HI}T_{HI} + \epsilon_i, \]

(2)

that is, with effort as dependent variable, instead of log effort. Along the lines of what you did in Part d, can you outline a model that would yield estimating equation (2)? Discuss.
Problem 2

Consider a reference-dependent tax-filer who is deciding how much effort to expend on finding charitable receipts to lower the tax bill, as in the Alex Rees-Jones paper. This agent has a marginal utility of money \( \phi \) and thus gets a utility benefit \( \phi e \) for every dollar \( e \) that he saves from tax filing. Searching for receipts has a cost \( c(e) \) which for simplicity we assume satisfies \( c(e) = (e^2)/2 \).

The agent also has gain-loss utility with a reference point \( r \), which we assume to be the amount of taxes due pre-tax-elusion. So \( e < r \) will indicate owing taxes, \( e = r \) implies no tax due and \( e > r \) indicates a tax refund. The agent thus has overall utility function

\[
\max e \phi e + \eta \phi [e - r] - e^2/2 \quad \text{for } e \geq r
\]

and

\[
\max e \phi e + \eta \lambda \phi [e - r] - e^2/2 \quad \text{for } e < r
\]

Part a. Discuss briefly the various components in the model above and identify the gain and loss component.

Part b. Derive the first-order condition with respect to effort and plot it as a function of effort. Assume \( \eta > 0 \) and \( \lambda > 1 \). Distinguish three cases. Comment on the shape of the marginal utility of effort.

Part c. Derive the solution for \( e_{RD}^* \) for this reference-dependent filer. Comment on the qualitative features.

Part d. Plot the solution for \( e_{RD}^* \) as a function of \( \phi \) (the marginal utility of money). Comment on the qualitative features.

Part e. Solve now for the non-loss averse standard case (\( \lambda = 1 \)), solve for \( e_{St}^* \) and plot it as a function of \( \phi \). How does this differ from \( e_{RD}^* \)?

Part f. Now assume that we live in a world in which \( \phi_j \) is uniformly distributed across tax-payers between 0 and 10: \( \phi \sim U[0, 10] \). Also assume \( \eta = 1 \). We are interested in what the distribution of tax-filing would look like to an econometrician who cannot observe the \( \phi_j \), but can observe the ultimate distribution of \( e \) (since it determines the tax payment). Assume first no loss aversion (\( \lambda = 1 \)) and plot the distribution of \( e_{St}^* \) observed by the econometrician.

Part g. Now, we turn to the reference-dependent case. Assume \( \lambda = 2 \), keeping \( \eta = 1 \) and \( \phi \sim U[0, 10] \). Also assume that the reference point \( r \) equals 10 (\( r = 10 \)). Plot the distribution of \( e_{RD}^* \) which the econometrician would observe. If you do not fully solve this through, use your
intuition to go as far as you can. In doing this, remember that $e < r$ is the case in which the person owes taxes and $e > r$ is the case of getting a refund. Provide intuition.

**Part h.** Relate what you found to the “bunching” test and “shifting” tests that Alex describes in his paper.

**Part i.** In what sense is it true that the “bunching” and “shifting” have to be of the same magnitude? Discuss.

**Part j.** In light of this discussion, comment on the key finding in Alex Rees-Jones paper of the distribution of tax returns relative to the amount withheld (see Figure). Does it looks like the “bunching” and “shifting” have the same size?
Consider individuals choosing to take an action with delayed benefits and immediate costs, such as an investment in education, health, or other types of human capital. In period 1 agents choose the action \( a \in \{0, 1\} \). The direct cost of taking the action, realized in period 1, is \( c \sim F \), and the distribution \( F \) has a continuous density function \( f \). The benefits of taking the action, which are realized in period 2, are \( b - p \), where \( b \) is the intrinsic benefit (e.g., the impact on one’s health) and \( p \) is the price of taking the action (e.g., the price of attending the gym). Suppose that the distribution has positive density on \([0, \bar{c}]\), where \( \bar{c} \geq b \).

Individuals are present biased. So in period 1, individual \( i \) takes the action if \( \beta_i (b - p) > c \). His long-run utility (period 0 perspective) when choosing \( a = 1 \) and incurring cost \( c \) is \( V_i = b - c + x \), where \( x \) is the remaining money that he spends on a numeraire good. Otherwise, when \( a = 0 \), his long-run utility is simply \( V_i = x \).

The policymaker sets a subsidy \( s \) for choosing \( a = 1 \), which she funds through a lump-sum tax \( T \). In the absence of the subsidy \( s \), the price of taking \( a = 1 \) is zero, while in the presence of the subsidy \( s \) the price is \( p = -s \). An individual with an initial endowment of \( z \) thus receives long-run utility \( V_i = z - T + (b + s - c) \) when choosing \( a = 1 \) and \( V_i = z - T \) otherwise. Each individual’s expected utility under this tax scheme is given by

\[
W_i(s) = \int_{c=0}^{\beta_i(b+s)} (z - T + b + s - c) f(c) dc + \int_{c=\beta_i(b+s)}^{\bar{c}} (z - T) f(c) dc
\]

where \( T = \sum_i F(\beta_i(b+s)) \). The government maximizes the expected long-run utility of the individuals: \( \mathcal{W} = \sum_i W_i \). We call \( \mathcal{W} \) social welfare. We call first-best welfare the welfare that obtains in the counterfactual case in which no individuals are present biased, and the government sets \( s = T = 0 \).

Part a)

Using the identity \( T = \sum_i F(\beta_i(b+s)) \), show that

\[
\mathcal{W} = \sum_i \int_{c=0}^{\beta_i(b+s)} (z + b - c) f(c) dc + \sum_i \int_{c=\beta_i(b+s)}^{\bar{c}} z f(c) dc
\]

Part b)

Suppose that all individuals have a common present bias \( \beta \). Show that in this case the policymaker will set a subsidy equal to \( s = \frac{1-\beta}{\beta} b \). Explain the intuition, drawing a connection to standard Pigovian taxation in the presence of externalities.

Part c)

Explain why the solution above leads to the same outcomes and the same social welfare as if individuals were time consistent (first-best welfare).
Part d)
Suppose that individuals either have $\beta_i = 0$ or $\beta_i = 1$. Show that in this case the optimal subsidy is $s = 0$. What is the economic intuition for this result?

Part e)
Suppose again that there are two levels of present bias in the population, but now that all $\beta_i > 0$. Specifically, there are some individuals with $\beta_i = 1$ and some individuals with $\beta_i = \beta^\dagger$, where $0 < \beta^\dagger < 1$. Now show that the optimal subsidy is $s > 0$. What is the intuition? In particular, comment on why no matter how many individuals are time-consistent and how few are present-biased (as long as there are some), the optimal subsidy will never be zero.

Part f)
Explain why, when $\beta_i$ is heterogeneous as in parts (d) and (e), social welfare will always be lower than first-best welfare. Contrast this to part (c) and explain the intuition for the difference.

Part g)
Consider the setup of part (e), but suppose that the policymaker can also offer individuals a commitment contract. A commitment contract is a contract that individuals can freely sign up for in period 0, and which requires them to pay the government $t$ unless they choose $a = 1$ in period 1. Assume that individuals are sophisticated, and thus choose whether or not to sign up for a commitment contract based on the expected value of $\beta_i V_i$. Assume also that the cost $c$ is realized only in period 1; in period 0 the individual only knows that it will be drawn from the distribution $F$. For this part, assume that $\bar{c} = b$. Show that now first-best welfare can be obtained by setting $s = 0$ and offering a commitment contract with stakes $t > \frac{1-\beta^\dagger}{\beta^\dagger}$.

Part h)
Suppose now that the present biased individuals are only partially sophisticated. That is, in period 0 they think that in period 1 they will behave according to a present bias parameter equal to $\hat{\beta}^\dagger > \beta^\dagger$. Show that as long as $\hat{\beta}^\dagger < 1$, the first best welfare can still be obtained by setting $s = 0$ and offering a commitment contract with stakes $t > \frac{1-\hat{\beta}^\dagger}{\beta^\dagger}$.

Part i)
Explain the intuition for the above results. In particular, comment on the sense in which the commitment contract is a better targeted policy tool than the subsidy.

Part j)
Suppose now that the costs $c$ are uniformly distributed on $[0, \bar{c}]$, where $\bar{c} > b + (1 - \beta^\dagger)b$. Show that when $s = 0$, there is no value of $t$ such that individuals will want take up a commitment contract that forces them to pay $t$ in the event that they don’t choose $a = 1$. Explain the intuition for this result.
Part k)
Suppose now that the costs $c$ are uniformly distributed on $[0, \bar{c}]$, where $\bar{c} > b + (1 - \beta^\dagger)(b + k)$. Show that when $s \leq k$, there is no value of $t$ such that individuals will want take up a commitment contract that forces them to pay $t$ in the event that they don’t choose $a = 1$.

Part l)
Suppose now that the costs $c$ are uniformly distributed on $[0, \bar{c}]$, where $\bar{c} > b + (1 - \beta^\dagger)2b$. Using part (k), explain why the policymaker will not bother offering commitment contracts, and instead will simply set an optimal subsidy as in part (e).