

MACROECONOMICS FIELD EXAM (Version B)

ANSWER ALL QUESTIONS

You have 3 hours. Each point is intended to correspond to 1 minute.

PART ONE. 70 points.

I. (35 points.) In 2008–9, the advanced economies suffered their worst recession since the Great Depression. Those economies still have not experienced a rapid rebound from the recession. For example, the unemployment rate in the United States remains far above pre-recession estimates of the natural rate; and in many European countries, GDP remains below its pre-recession peak and is falling or barely growing.

Briefly describe two factors that may have contributed to the lack of strong recovery. For each, discuss how that factor could have slowed the recovery and how you might incorporate the factor into a model of the macroeconomy. In addition, for each factor, describe either one piece of empirical evidence bearing on its relevance to the slow recovery or a test that could be performed that would shed light on its relevance.

II. (35 points.)

Let  $Y_t(s^t)$  be the aggregate endowment for this economy, suppose

$$Y_t(s^t) = s_t Y_{t-1}(s^{t-1})$$

where  $s_t$  is the realization for aggregate endowment growth. Suppose that  $(s_t)_t$  follows a two-state Markov process with high and low growth states, i.e.,  $s_t \in \{s_l, s_h\}$ . The Markov chain has a transition matrix  $F$ , where  $F_{ij} = \Pr(s_{t+1} = s_j | s_t = s_i)$ . We let  $s^t = (s_t, s_{t-1}, \dots, s_0)$  denote a history of aggregate growth rates.

This economy is populated with two types of agents  $i = 1, 2$ , with preferences given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \frac{c_{it}(s^t)^{1-\alpha} - 1}{1-\alpha} pr_i(s^t)$$

where  $pr_i(s^t)$  is the agent's  $i$  subjective probability over  $s^t$ . Let  $pr(s^t)$  be the true or correct probability over  $s^t$ .

1. Define and solve the Pareto Problem with weights  $\lambda$  and  $1 - \lambda$  for agents 1 and 2 respectively. **Hint:** Try to cast the solution as  $c_{it}(s^t) = share_i(s^t) \times Y_t(s^t)$  where “ $share_i$ ” is some function.

2. 2.a. Suppose that both agents agree on the law of motion, i.e.,  $pr_1(s^t) = pr_2(s^t)$  for all  $t$ , show that the agents get constant share of the endowment and characterize such shares. **Hint:** In your previous answer  $share_t(s^t) = share$  (i.e. is constant).
- 2.b. Suppose that both agents agree on the law of motion up to time  $T$ , i.e.,  $pr_1(s^t) = pr_2(s^t)$  for all  $t \leq T$ , but at time  $T$  the agent 1 becomes more “optimistic”, i.e.,  $pr_1(s^T, s_h) > pr_2(s^T, s_h)$ , what happens to  $c_{1T+1}(s^T, s_h)$  relative to  $c_{1T+1}(s^T, s_l)$ . Explain the economic intuition.
- 2.c. Suppose that agent 1 is *too* optimistic, i.e.,  $pr_1(s_{t+1} = s_h | s_t = s_h) > pr_2(s_{t+1} = s_h | s_t = s_h) = pr(s_{t+1} = s_h | s_t = s_h)$ , and this remains like this for several periods. What would happen with his realized consumption relative to agent’s 2 realized consumption? **Hint:** An heuristic argument will suffice.
3. Suppose that each agent has equal and constant shares of the aggregate endowment. Solve for the competitive equilibrium in an Arrow-Debreu Economy. Please include all the necessary definitions.
4. Now assume that markets are incomplete. There is only one asset in the economy: a one period non-state contingent bond. Let  $p_t$  denote the price. Take the following convention:  $p_t b_{it} > 0$  means that the agent is saving (and will get  $b_{it} > 0$  of consumption tomorrow).
  - 4.a. Define the competitive equilibrium.
  - 4.b. If  $pr_1(s^t) = pr_2(s^t)$ . Compute the equilibrium consumption allocation.
  - 4.c. If  $pr_1(s^t) = pr_2(s^t) \neq pr(s^t)$  (where  $pr(s^t)$  is the true distribution). Go as far as you can characterizing the equilibrium price of the bond.
  - 4.d (BONUS POINTS) Write down the bellman equation for each agent. Make sure to spell out what is the state of the economy.

**PART TWO. 110 points.**

**III. (55 points.)**

Consider an unemployed worker that is looking for a job. While unemployed, the worker receives a wage offer  $z \sim Q$  with  $\text{support}(z) = [0, B]$ . This offer can be accepted or rejected. If the worker chooses to reject, it receives a payoff of  $c > 0$  (e.g. unemployment insurance) this period and waits until next period for a new draw from  $Q$ . If the worker chooses to accept the offer  $z$ , she receives  $z$  forever.

The worker discounts future using  $\beta \in (0, 1)$ .

1. What is the state of this economy?
2. Consider the “canonical” Bellman equation

$$V(y, z) = \sup_{y' \in \Gamma(y, z)} \{F(y, y', z) + \beta E[V(y', z')|y, z]\}.$$

Spell out what  $y$  (the “endogenous” state variable),  $z$  (the “exogenous” state variable),  $\Gamma$ ,  $F$  and the measure of integration in  $E[\cdot|y, z]$  for this economy.

3. Argue that the Principle of Optimality (PO) holds. **Hint:** It suffices to show that the assumptions to establish the PO hold in this case; you can ignore measurability issues when verifying the assumptions.
4. Show that  $V$  exists in the space of uniformly bounded functions. Is  $V$  continuous in  $z$ ? **Hint:** Feel free to use whatever lemma/theorem/etc you need without proving them (but make sure to verify that their assumptions are met).
5. Show that in this case, the canonical Bellman equation can be simplified to

$$v(z) = \max\left\{c + \beta \int_0^B v(z')Q(dz'), \frac{1}{1 - \beta}z\right\}$$

where  $v(z)$  is the value (in utility terms) of an unemployed worker who receives a wage offer of  $z$  and has the option to accept it.

6. (a) Show that this problem is characterized by a threshold rule, i.e., show that if  $z > \bar{z}$ , then the worker accepts the offer and:

$$v(z) = \frac{z}{1 - \beta}.$$

If  $z < \bar{z}$ , then the worker rejects the offer and:

$$v(z) = c + \beta \int_0^B v(z')Q(dz').$$

Where

$$\frac{\bar{z}}{1-\beta} = c + \beta \int_0^B v(z')Q(dz').$$

(b) Show that  $\bar{z}$  is unique. **Hint:** It could be useful to first show that

$$\bar{z} - c = \frac{\beta}{1-\beta} \int_{\bar{z}}^B (z' - \bar{z})Q(dz'); \quad (1)$$

then show that  $w \mapsto T(w) \equiv \frac{\beta}{1-\beta} \int_w^B (z' - w)Q(dz')$  is decreasing and  $T(0) = \frac{\beta}{1-\beta} E[z]$  and  $T(B) = 0$ .

7. (a) Show that if  $c$  increases, then  $\bar{z}$  increases. (b) What is the economic intuition behind this result.
8. (a) Suppose that  $Q$  is replaced by  $\tilde{Q}$  where  $\int_0^B zQ(dz) = \int_0^B z\tilde{Q}(dz)$  and  $\int_0^z \{\tilde{Q}(y) - Q(y)\}dy \geq 0$ ; i.e.,  $\tilde{Q}$  is a mean preserving spread of  $Q$ . Is  $\bar{z}$  under  $Q$  larger or smaller than  $\bar{z}$  under  $\tilde{Q}$ ? (b) What is the economic intuition behind this result. **Hint for (a):** It may help to use integration by parts in equation 1.

(Exam continues on next page.)

#### IV. (55 points.) Employment and Demand Shocks

Consider an economy made up of  $S$  equally sized counties or “islands” indexed by  $c$ . Each county produces two types of goods, tradable ( $T$ ) and non-tradable ( $N$ ). Counties can freely trade the tradable good among themselves, but must consume the non-tradable good produced in their own county. Labor market is segmented across islands, but can move freely across the tradable and non-tradable sectors within an island.

Each island has  $D_c$  units of total (nominal) consumer demand. Consumers have Cobb Douglas preferences over the two consumption goods, and spend consumption shares  $P_c^N C_c^N = \alpha D_c$  and  $P_c^T C_c^T = (1 - \alpha) D_c$  on non-tradable and tradable good respectively.

All islands face the same tradable good price, while non-tradable good price may be county-specific since each county must consume its own production of non-tradable good. Production is governed by a constant returns technology for tradable and non-tradable goods with labor ( $e$ ) as the only factor input and produces output according to  $y_c^T = b e_c^T$ , and  $y_c^N = a e_c^N$  respectively.

Total employment on each island is normalized to one with  $e_c^T + e_c^N = 1$ . Labor cannot move across islands, but is free to move across tradable and non-tradable sectors on a given island. Goods market equilibrium in non-tradable and tradable sectors implies that  $y_c^N = C_c^N$  on each island and  $\sum_{c=1}^S y_c^S = \sum_{c=1}^S C_c^T$ .

- (a) Assume all islands have the same nominal demand initially at  $D_c = D_0$ . Solve for the symmetric equilibrium output, employment, wages and prices.
- (b) Next suppose counties are hit with differing household expenditure shocks. In particular, normalize initial nominal demand  $D_0 = 1$  and introduce the possibility of negative demand shocks ( $\delta_c$ ) that differ across counties such that  $D_c = 1 - \delta_c$ , and the average of the demand shocks is  $\bar{\delta}$ .

Suppose prices and wages are fully flexible. Solve for the change in prices, wages and employment, i.e.  $\Delta P_c^T$ ,  $\Delta P_c^N$ ,  $\Delta w_c^N$ ,  $\Delta w_c^T$ ,  $\Delta e_c^N$ , and  $\Delta e_c^T$ .

- (c) Explain in words how change in employment in tradable and non-tradable sector may be affected if prices and wages were rigid.
- (d) Intuitively explain if a policy of “nominal GDP targeting” would help in the case of nominal rigidity.