PART ONE. 70 points.

I. (35 points.) In 2008–9, the advanced economies suffered their worst recession since the Great Depression. Those economies still have not experienced a rapid rebound from the recession. For example, the unemployment rate in the United States remains far above pre-recession estimates of the natural rate; and in many European countries, GDP remains below its pre-recession peak and is falling or barely growing.

Briefly describe two factors that may have contributed to the lack of strong recovery. For each, discuss how that factor could have slowed the recovery and how you might incorporate the factor into a model of the macroeconomy. In addition, for each factor, describe either one piece of empirical evidence bearing on its relevance to the slow recovery or a test that could be performed that would shed light on its relevance.

II. (35 points.)

Let \( Y_t(s^t) \) be the aggregate endowment for this economy, suppose

\[
Y_t(s^t) = s_t Y_{t-1}(s^{t-1})
\]

where \( s_t \) is the realization for aggregate endowment growth. Suppose that \( (s_t)_t \) follows a two-state Markov process with high and low growth states, i.e., \( s_t \in \{s_h, s_l\} \). The Markov chain has a transition matrix \( F \), where \( F_{ij} = \Pr(s_{t+1} = s_j | s_t = s_i) \). We let \( s^t = (s_t, s_{t-1}, \ldots, s_0) \) denote a history of aggregate growth rates.

This economy is populated with two types of agents \( i = 1, 2 \), with preferences given by

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t c_{lt}(s^t)^{1-\alpha} - 1 \quad \text{pr}_i(s^t)
\]

where \( pr_i(s^t) \) is the agent’s \( i \) subjective probability over \( s^t \). Let \( pr(s^t) \) be the true or correct probability over \( s^t \).

1. Define and solve the Pareto Problem with weights \( \lambda = 1 \) and \( \lambda = 1 \) for agents 1 and 2 respectively. Hint: Try to cast the solution as \( c_{it}(s^t) = \text{share}_i(s^t) \times Y_t(s^t) \) where “\( \text{share}_i \)” is some function.
2. a. Suppose that both agents agree on the law of motion, i.e., \( pr_1(s^t) = pr_2(s^t) \) for all \( t \), show that the agents get constant share of the endowment and characterize such shares. **Hint:** In your previous answer \( share_t(s^t) = share \) (i.e. is constant).

2.b. Suppose that both agents agree on the law of motion up to time \( T \), i.e., \( pr_1(s^t) = pr_2(s^t) \) for all \( t \leq T \), but at time \( T \) the agent 1 becomes more "optimistic", i.e., \( pr_1(s^T, s_h) > pr_2(s^T, s_h) \), what happens to \( c_{1T+1}(s^T, s_h) \) relative to \( c_{1T+1}(s^T, s_l) \). Explain the economic intuition.

2.c. Suppose that agent 1 is *too* optimistic, i.e., \( pr_1(s_{t+1} = s_h | s_t = s_h) > pr_2(s_{t+1} = s_h | s_t = s_h) = pr(s_{t+1} = s_h | s_t = s_h) \), and this remains like this for several periods. What would happen with his realized consumption relative to agent’s 2 realized consumption? **Hint:** An heuristic argument will suffice.

3. Suppose that each agent has equal and constant shares of the aggregate endowment. Solve for the competitive equilibrium in an Arrow-Debreu Economy. Please include all the necessary definitions.

4. Now assume that markets are incomplete. There is only one asset in the economy: a one period non-state contingent bond. Let \( p_t \) denote the price. Take the following convention: \( p_t b_{it} > 0 \) means that the agent is saving (and will get \( b_{it} > 0 \) of consumption tomorrow).

4.a. Define the competitive equilibrium.

4.b. If \( pr_1(s^t) = pr_2(s^t) \). Compute the equilibrium consumption allocation.

4.c. If \( pr_1(s^t) = pr_2(s^t) \neq pr(s^t) \) (where \( pr(s^t) \) is the true distribution). Go as far as you can characterizing the equilibrium price of the bond.

4.d (BONUS POINTS) Write down the bellman equation for each agent. Make sure to spell out what is the state of the economy.
PART TWO. 110 points.

III. (55 points.)

Consider an unemployed worker that is looking for a job. While unemployed, the worker receives a wage offer \( z \sim Q \) with \( \text{support}(z) = [0, B] \). This offer can be accepted or rejected. If the worker chooses to reject, it receives a payoff of \( c > 0 \) (e.g. unemployment insurance) this period and waits until next period for a new draw from \( Q \). If the worker chooses to accept the offer \( z \), she receives \( z \) forever.

The worker discounts future using \( \beta \in (0, 1) \).

1. What is the state of this economy?

2. Consider the “canonical” Bellman equation

\[
V(y, z) = \sup_{y' \in \Gamma(y, z)} \{ F(y, y', z) + \beta E[V(y', z') | y, z] \}.
\]

Spell out what \( y \) (the “endogenous” state variable), \( z \) (the “exogenous” state variable), \( \Gamma, F \) and the measure of integration in \( E[\cdot | y, z] \) for this economy.

3. Argue that the Principle of Optimality (PO) holds. Hint: It suffices to show that the assumptions to establish the PO hold in this case; you can ignore measurability issues when verifying the assumptions.

4. Show that \( V \) exists in the space of uniformly bounded functions. Is \( V \) continuous in \( z \)? Hint: Feel free to use whatever lemma/theorem/etc you need without proving them (but make sure to verify that their assumptions are met).

5. Show that in this case, the canonical Bellman equation can be simplified to

\[
v(z) = \max \{ c + \beta \int_0^B v(z') Q(dz'), \frac{1}{1 - \beta} z \}
\]

where \( v(z) \) is the value (in utility terms) of an unemployed worker who receives a wage offer of \( z \) and has the option to accept it.

6. (a) Show that this problem is characterized by a threshold rule, i.e., show that if \( z > \bar{z} \), then the worker accepts the offer and:

\[
v(z) = \frac{z}{1 - \beta}.
\]

If \( z < \bar{z} \), then the worker rejects the offer and:

\[
v(z) = c + \beta \int_0^B v(z') Q(dz').
\]
Where

\[
\frac{\bar{z}}{1-\beta} = c + \beta \int_0^R v(z')Q(dz').
\]

(b) Show that \(\bar{z}\) is unique. **Hint:** It could be useful to first show that

\[
\bar{z} - c = \frac{\beta}{1-\beta} \int_{\bar{z}}^B (z' - \bar{z})Q(dz');
\]

then show that \(w \mapsto T(w) \equiv \frac{\beta}{1-\beta} \int_{\bar{z}}^B (z' - w)Q(dz')\) is decreasing and \(T(0) = \frac{\beta}{1-\beta} E[\bar{z}]\) and \(T(B) = 0\).

7. (a) Show that if \(c\) increases, then \(\bar{z}\) increases. (b) What is the economic intuition behind this result.

8. (a) Suppose that \(Q\) is replaced by \(\tilde{Q}\) where \(\int_0^B zQ(dz) = \int_0^B z\tilde{Q}(dz)\) and \(\int_0^\bar{z} \{\tilde{Q}(y) - Q(y)\}dy \geq 0\); i.e., \(\tilde{Q}\) is a mean preserving spread of \(Q\). Is \(\bar{z}\) under \(Q\) larger or smaller than \(\bar{z}\) under \(\tilde{Q}\)? (b) What is the economic intuition behind this result. **Hint for (a):** It may help to use integration by parts in equation 1.

(Exam continues on next page.)
IV. (55 points.)

1. (25 points.) A central question in macroeconomics is determining the effects of monetary policy shocks on the path of real GDP. Define “monetary policy shock” as precisely as possible. (Throughout the problem, assume for simplicity that we are considering a period when monetary policy is not constrained by the zero lower bound and when the Federal Reserve is not using any unconventional tools.) For example, suppose you estimated an equation for the Federal Reserve’s choice of its target for the funds rate as a function of inflation and the output gap. Would departures of the funds rate target from the values predicted by that reaction function necessarily represent monetary policy shocks for purposes of estimating the effects of monetary policy on real GDP? Why or why not? Alternatively, do funds rate “surprises” (specifically, differences between the Federal Reserve’s decisions at FOMC meetings about its target for the federal funds rate and agents’ expectations prior to the meetings) necessarily represent monetary policy shocks for purposes of estimating the effects of monetary policy on real GDP? Why or why not?

2. (25 points.) Suppose you had successfully addressed the issue above, and that you had a series of monetary policy shocks over some sample period. How would you use them to estimate the effects of monetary policy on real GDP? Be as specific as possible. For example, if you propose to estimate a regression, you should specify the functional form of the dependent variable (for example, logs vs. levels, whether it should be differenced, detrended, etc.); whether the regression should include lags (or leads) of any variables you propose to put on the right-hand side; whether the regression should include a constant; whether it should be estimated by OLS or IV (and, if the latter, what the instruments should be); and so on. Likewise, if you propose some other approach (such as narrative analysis or a VAR), be as specific as you can. Explain the reasons for your choices.

3. (5 points.) In addition to estimating how a monetary policy shock affects the path of real GDP, you could estimate how it affects the path of the federal funds rate itself. How, if at all, would findings about the impact of a monetary policy shock on the path of the funds rate affect the interpretation of the findings about the impact on the path of real GDP?