Labor Field Exam January 2012

Answer three of the following four questions. You have three hours. Try to explain your reasoning wherever possible.

1. Consider a simple search model applied to an experiment on the labor supply of undergraduates. You take a random sample of 100 undergraduates. For each such undergraduate, you explain that they will receive a series of wage offers that will vary from day to day. Each offer can be accepted or not and acceptance or rejection of a day’s offer does not affect subsequent wage offers. Each such task takes 10 minutes to accomplish and can be done over the internet. Wage offers will be given daily at 6am over the internet by e-mail; the task can be accomplished any time prior to 5:59am the next day. You do not specify a final period to the experiment and describe it as being an open-ended arrangement. Students are indexed by $s$, and let their wage offers be distributed exponential with parameter $\lambda_s$. You randomly assign the parameters from a list of possible parameters. Let $B_{st}$ denote the random variable associated with the wage offer student $s$ on day $t$. You inform students of the range of offers they are likely to get by giving them 100 examples of draws from their wage distribution. Assume that the students are able to infer the parameters from the list. Assume that undergraduates have utility of zero associated with not working for you on the menial task. Assume linear utility.

   a. Write down the Bellman equation for this problem.

   b. Give a formula for the expected value of the value function associated with a random draw $B_{st}$. Your formula may depend on the reservation wage.

   c. Give a formula for the reservation wage. Your formula may depend on the expected value of the value function associated with a random wage offer.

   d. Derive the log-likelihood function for the set of binary labor supply decisions recorded by the experiment, as a function of the discount factor.

   e. Would the model be identified if you only gave one wage offer to the undergraduates?
2. Imbens, Rubin and Sacerote (2001) use Social Security earnings data for participants in the Massachusetts lottery to estimate models of the form:

\[ y_{it} = \alpha_i + \beta L_{it} + d_t + \varepsilon_{it} \]

where \( y_{it} \) represents annual labor earnings of person \( i \) in year \( t \), \( \alpha_i \) represents a person effect, \( L_{it} \) represents the annuitized amount of lottery winnings received by person \( i \) in year \( t \), and \( d_t \) represents a year effect for calendar year \( t \). For each winner they have several years of data from before the person won, and several years after: in the pre-winning years they set \( L_{it} = 0 \). Their sample includes observations on people who won very small amounts (e.g. $10) for whom \( L_{it} = 0 \) for all years, as well as “big winners” who receive as much as $30,000 per year for each of the 20 years following their victory.

a) Suppose that individuals choose hours of work (\( h \)) and consumption (\( c \)) each period, given a wage \( w \) and non-labor income \( y \) (i.e., no borrowing or lending), and that the per-period utility function has the Stone-Geary form:

\[ U(c, h) = a \log (T-h) + b \log (c-\gamma) \]

where \( a, b \) and \( \gamma \) are parameters and \( T \) is maximum time available. Use this model to derive an interpretation of the coefficient \( \beta \) in Imbens et al’s estimating equation. Be sure to carefully discuss what you are assuming about wages and other forms of income (which are not observed in the data set they use).

b) Suppose that individuals can borrow and/or lend over time, and that they have Stone Geary preferences within each period. Explain how this changes (or does not change) your answer to part (a).
3. A researcher in the distant future is interested in studying whether earnings inequality has increased on the planet Mars. Data from two cross-sectional surveys are available – one from the year 2210, the other from the year 2220. Unfortunately, the Martian Census Bureau, like our present day one, is quite concerned about confidentiality issues and censors the earnings data from above in order to protect the identities of high earners. Fortunately, having learned something over the past two hundred years, the Martians topcode earnings at the 95th percentile each year in order to facilitate comparisons across time.

The data available to the researcher is given in the table below:

| Martian Earnings Survey (Summary Stats) |
|-----------------------------|--------|
| **Year** | **2210** | **2220** |
| \( \bar{y}_{y < c} \) (mean of uncensored obs) | 10 | 8 |
| \( P(y \geq c) \) (% topcoded) | 5 | 5 |
| \( c \) (topcode value) | 15 | 20 |

Notes: the symbol \( y \) denotes deflated log earnings in 2210 dollars. \( c \) is the topcode value.

a) Suppose that log earnings \( (y) \) are distributed according to a normal distribution in each year with mean \( \mu \) and variance \( \sigma^2 \). Write a pair of expressions linking the moments \( \bar{y}_{y < c} \) and \( P(y \geq c) \) to the parameters \((\mu, \sigma)\) of the normal distribution and the censoring point \( c \).

b) Solve one of these expressions for \( \mu \) in terms of \( \sigma \) and the observed moments.

c) Use your answer from b) to solve for \( \sigma \) in terms of the observed moments.

d) According to your calculations, did inequality increase between 2210 and 2220?

e) Suppose you had access to the censored microdata from both years. Is there a measure of inequality you could compute in each year that corrects for the censoring yet doesn’t rely on normality?
Consider the following stylized model of “job search.” The set up is a dynamic game that has two firms, each of which has a job opening, and two workers, each of whom hopes to find a job. The game proceeds as follows:

- In Stage 1, Firm 1 posts a wage $w_1$ and Firm 2 posts a wage $w_2$.
- In Stage 2, each of two workers applies to one firm only.
- Given choices in the two stages, agents get payoffs. The payoff to a worker is the posted wage if she is the only worker applying to a firm, but $1/2$ the wage if both workers apply:

<table>
<thead>
<tr>
<th>Worker 2</th>
<th>Apply to Firm 1</th>
<th>Apply to Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker 1</td>
<td>Apply to Firm 1</td>
<td>$\frac{1}{2}w_1$, $\frac{1}{2}w_1$</td>
</tr>
<tr>
<td>Apply to Firm 2</td>
<td>$w_2$, $w_1$</td>
<td>$\frac{1}{2}w_2$, $\frac{1}{2}w_2$</td>
</tr>
</tbody>
</table>

As for the firms, Firm $i$ has a payoff of

$$(v_i - w_i)\Pr(\text{Firm } i \text{ receives at least one applicant}|w_1, w_2),$$

where $v_i$ is the value created by the worker.

(a) Show that each firm’s optimal wage depends not only the value created at the firm by the worker, but also on the wage posted by the other firm. Specifically, find the “best response functions,”

$$w_1^* = f_1(v_1, w_2)$$

and

$$w_2^* = f_2(v_2, w_1).$$

(b) Given your answer in (a), show that:

- The firm with the higher $v$ will offer the higher wage. (The high-wage firm might plausibly be the more capital-intensive firm.)
- “Queue length” will vary across firms, with the high-wage firm having a longer queue.