WRITE YOUR ANSWERS FOR EACH QUESTION IN A SEPARATE BOOK

PART I Answer both questions in this part. Each question should take about 30 minutes.

Question 1: LABOR SUPPLY

Consider a static model of labor supply with only two good: consumption goods (x) and leisure (ℓ).

(a) Derive the Slutsky equation for the response of hours of work (h) to a change in the hourly wage (w). Be sure to define all the notation you use.

Hint: carefully define the uncompensated and compensated labor supply functions...

(b) The SNAP program in the US provides a monthly benefit that is reduced by 30 cents for every dollar of family income. Consider the case of a single mother with 2 kids, whose benefit is (for purposes of this question) $600 per month. Assume she earns $15 per hour and works 80 hours per month. You are asked to evaluate the potential effects on such a recipient if the benefit amount were lowered to $450 per month (with no change in the benefit reduction rate).

(i) Explain what term(s) from the Slutsky equation you need to know to predict the effect on her monthly hours.

(ii) Based on your knowledge of the literature about the rough magnitudes of these terms, make a prediction about how much her hours will change, and how much her net family income (earnings plus SNAP benefits) will change.

Suggestion: Ignore all other programs you may be aware of (eg, the EITC). Draw a diagram of the hours choices under the two programs.

(iii) A colleague looks at your prediction from part (ii) and argues that most workers cannot easily change their hours of work unless they change jobs. How does this comment affect your answer?
Question 2: SEARCH

(a) Assume that a job searcher is infinitely lived, risk neutral, and has a discount rate $\beta=1/(1+r)$ where $r>0$. Assume further that each period there is one job offer that can be accepted or rejected; that a job pays a fixed wage $w$ forever (no job destruction), where $w$ is a positive random variable with density $f(w)$; and that unemployed job searchers receive a benefit of $b$. (Assume there is no cost of search).

(i) Set up a Bellman equation for the value of unemployment, $V$, assuming the individual follows an optimal strategy of accepting a job or continuing to search in all periods.

(ii) Define the reservation wage $w^*$ as the lowest wage the job searcher will accept. Restate the Bellman equation from (i) in terms of $w^*$. Show that $w^*$ is an increasing function of $b$.

Hint: What is the relationship between $V$ and $w^*$ if jobs last forever and the individual has a discount rate $\beta$?

(iii) Define the "job finding rate" as the probability that a job searcher will find an acceptable job in a given period. How is this related to the reservation wage? Using your answer from (ii) show that the job finding rate is decreasing in $b$.

(iv) Now suppose there is a cost of search, $c$. An unemployed person has two options: search (receiving a net utility of $b-c$ in each period of search with some option value of finding a job) or not search (receiving a net utility of $b$ in each period with no option value of search). Discuss the determinants of the decision whether to search or not.

(v) Briefly discuss how your answer to (iv) would be changed if the model is expanded to allow a "job offer rate" of $\lambda < 1$ in each period of unemployment.
Part II

Consider the following two-period model. In period 1, individuals decide to go to school or work. Potential earnings for individual \( i \) in this period are

\[
y_{1i} = \alpha_1 + a_i,
\]

where \( a_i \) is ability and \( E[a_i] = 0 \). Earnings are zero if an individual goes to school, and school costs \( c \). Everyone works in period 2, and earnings are

\[
y_{2i} = \alpha_2 + \gamma a_i + \beta s_i + \delta a_i s_i,
\]

where \( s_i \) equals one individual \( i \) attends school in period 1 and zero otherwise, \( \gamma > 0, \beta > 0, \) and \( \delta \geq 0 \). Individuals seek to maximize earnings net of schooling costs and there is no discounting between periods.

1. Characterize the decision rule that determines who attends school in period 1.

2. Under what conditions will higher-ability people attend school? Under what conditions will lower-ability people attend school? Give some intuition linking your answer to the theory of human capital investments.

3. A researcher has data \( \{s_i, y_{2i}\}_{i=1}^N \) for a random sample of individuals that behave according to this model. The researcher is interested in estimating the average causal effect of schooling on the earnings of students who go to school (the “effect of treatment on the treated”, TOT). She runs the OLS regression

\[
y_{2i} = \lambda + \theta s_i + e_i
\]

Write an expression relating the OLS coefficient \( \theta \) to the causal effect of interest. When is the OLS coefficient upward biased, and when is it downward biased? How is this related to your answer in part (2)?

4. Suppose the researcher learns of a policy that shifts the cost of attending schooling. The cost of attending school for individual \( i \) is

\[
c_i = c_0 + c_1 p_i,
\]

where \( c_1 > 0 \) and \( p_i \in \{0, 1\} \) is a policy-related variable that is independent of \( a_i \). Describe a strategy the researcher could use to estimate a causal effect of schooling on earnings with data on \( p_i \).

5. What parameter is recovered by the estimation strategy you described in part (4)? How is this parameter related to the OLS parameter \( \theta \) and the TOT?

6. Briefly describe the relationship between IV and OLS estimates in the literature on the returns to schooling, and relate this relationship to your answer in part (5).
A large literature reviewed in Solon (1992) considers estimation of intergenerational earnings elasticities (IGEs). Recently, a debate has emerged regarding “which” elasticity to estimate. Suppose we have a dataset \( \{X_i, Y_i\}_{i=1}^N \) giving the lifetime earnings of \( N \) father-son pairs, with \( X_i \) being the father’s lifetime earnings and \( Y_i \) the son’s lifetime earnings. While the traditional approach has been to estimate an OLS regression of the form:

\[
\ln Y_i = \alpha + \beta \ln X_i + \varepsilon_i,
\]

Mitnik et al. (2014) have instead suggested fitting a pseudo-maximum likelihood Poisson regression model that imposes the conditional mean restriction:

\[
\ln E[Y_i|X_i = x] = \alpha + \beta x
\]

To think about the difference between these approaches, suppose that:

\[
Y_i|X_i \overset{iid}{\sim} F_{Y|X}(.).
\]

Assume the earnings distributions of fathers and sons have no mass points and that everyone works at some point in their lifetime (i.e., earnings are strictly positive). We can define the \( \tau \)’th conditional quantile of earnings among sons whose fathers earn \( x \) as:

\[
q(x, \tau) \equiv F_{Y|X=x}^{-1}(\tau)
\]

1. Prove that we can therefore write:

\[
Y_i = q(X_i, U_i),
\]

where \( U_i|X_i \sim \text{Uniform}(0, 1) \).

2. Assuming that the conditional quantile function \( q(x, \tau) \) is differentiable in both its arguments, we can define the quantile-specific intergenerational elasticity function:

\[
\sigma(x, \tau) \equiv \frac{dq(x, \tau)}{dx} \frac{x}{q(x, \tau)},
\]

which summarizes how each quantile of son’s earnings depends (locally) upon father’s earnings when father’s earnings are \( x \). Evaluate the following three derivatives in terms of \( \sigma(x, \tau) \):

a) \( \frac{d}{d \log x} E[Y_i|X_i = x] \)

b) \( \frac{d}{d \log x} \log E[Y_i|X_i = x] \)

c) \( \frac{d}{d \log x} E[\log Y_i|X_i = x] \)

3. Describe a situation where you would expect \( \frac{d}{d \log x} E[Y_i|X_i = x] > \frac{d}{d \log x} E[\log Y_i|X_i = x] \).

4. Describe a possible drawback of using \( \int \frac{d}{d \log x} \log E[Y_i|X_i = x] dF_X(x) \) as the preferred IGE concept (\( F_X(.) \) is the CDF of father’s earnings).

5. Describe a possible drawback of using \( \int \frac{d}{d \log x} E[\log Y_i|X_i = x] dF_X(x) \) as the preferred IGE concept.

6. Chetty et al. (2014) examine a “rank-rank” IGE specification of the form:

\[
E[F_Y(Y_i)|F_X(X_i) = p] = \alpha + \beta p
\]

where \( F_Y(.) \) is the CDF of son’s earnings. Derive an expression for \( \frac{d}{dp} E[F_Y(Y_i)|F_X(X_i) = p] \) in terms of the derivative \( \Delta(p, \tau) \equiv \frac{dq(F_Y^{-1}(p), \tau)}{d F_X^{-1}(p)} \). Interpret your answer.