University of California
Field Exam
Labor Economics

Department of Economics
August, 2012

Answer question I and <u>any two</u> of questions II, III, or IV. Be sure to use graphs and/or equations whenever they would be helpful in clarifying your reasoning.

WRITE YOUR ANSWERS FOR EACH QUESTION IN A SEPARATE BOOK

Question I (required of all candidates). Answer all parts of this question

1. Consider an individual who faces an sequence of real wages in the current and future periods, $w_t, w_{t+1}, .. w_T$, and has current (real) assets A_t. (For simplicity assume that the only source of income is labor income). Assume the individual has a per-period utility function $u(c_t, h_t; a_t)$ where $\{a_t\}$ is a sequence of preference shifters over the lifecycle. Assume in addition that the individual discounts the future at a constant rate β, and can earn a real interest rate r_t on savings from period t to $t+1$. Finally, assume that $h_t>0$ in all periods, so we can ignore the extensive margin.

a) Write down the Bellman equation defining the value of an optimal lifecycle plan starting in period t, $V_t(A_t)$.

b) Use the first order conditions for the optimal choices in period t to show that the "Frisch" demand functions can be written in terms of w_t, a_t, r_t and $\lambda_t = V'_t(A_t)$.

c) Show that if $\beta=(1+r_t)^{-1}$ then $\lambda_t = E_t[\lambda_{t+1}]$ where E_t denotes expectations using information available in period t.

d) Show that if $u(c_t, h_t; a_t) = \varphi(c_t, a_{1t}) - \psi(h_t; a_{2t})$ then holding constant λ_t and the preference factors, optimal consumption does not depend on wages. What happens if $u_{ch} > 0$? Suggest a strategy for trying to test whether u_{ch} is positive, 0, or negative.

e) Using your answer in (b), show that the change in the optimal choice of hours from period to $t-1$ to t can be written as:

$$\Delta\log h_t = \alpha + \eta\Delta\log w_t + \varepsilon_t$$

where η is the elasticity of Frisch labor supply with respect to wages. Carefully discuss the "error term" ε_t. In particular, show what terms are included in this "error term". Discuss the likely biases in an OLS approach for estimating η.

f) Altonji suggested that one way to estimate the elasticity η is to include a measure of the change in consumption. Carefully discuss assumptions under which this "works", and the cases in which it leads to a biased estimate of η .

2. Suppose that economy-wide real output (y_t) depends on inputs of capital (K_t) and various types of labor $L_{1t}, L_{2t}, \ldots L_{Jt}$:

$$y_t = f(K_t, L_{1t}, L_{2t}, \ldots L_{Jt}).$$

a) Assume that

$$f(K, L_1, L_2, \ldots L_K) = AK^{1-\alpha} L^{\alpha} ,$$

$$\text{where } L = h(L_1, L_2, \ldots L_K)$$

and that the price of capital, r, is constant. Show in this case that output is linear in the labor aggregate L, and does not depend on the inputs (or wages) of any particular subgroup of labor. Explain how one could test whether this it true (or not).

b) Katz and Murphy assume that there are two types of labor, skilled and unskilled, and that h( ) is CES:

$$h(L_{1t}, L_{2t}) = [\ \theta_{1t} L_{1t}^{-\rho} + \theta_{2t} L_{2t}^{-\rho}\ ]^{1/\rho}$$

where $\rho=(\sigma-1)/\sigma$ and σ is the elasticity of substitution between skill groups. Suppose that there is **skill-biased technical change**, of the form

$$\log \theta_{1t}/\theta_{2t} = \varphi + \lambda t$$

where group 1 are high skilled and $\lambda>0$. Show in this case that the log relative wages of skilled workers will include a trend component and a term that depends on relative employment of skilled to unskilled workers.

c) Suppose that there are 2 types of skilled workers, old ("O") and young ("Y"), and also two types of unskilled workers, old and young. Assume that h is a "nested CES"

$$h(L_{1t}, L_{2t}) = [\ \theta_{1t} L_{1t}^{-\rho} + \theta_{2t} L_{2t}^{-\rho}\ ]^{1/\rho}$$
$$L_{1t} = [\ a_O L_{1Ot}^{-\tau} + a_Y L_{1Yt}^{-\tau}\ ]^{1/\tau} ,$$
$$L_{2t} = [\ b_O L_{2Ot}^{-\tau} + b_Y L_{2Yt}^{-\tau}\ ]^{1/\tau}$$

Show in this case that the relative wages of high skilled older workers to low skilled older workers depends on the overall ratio of high-to-low skilled workers L_{1t}/L_{2t} and on the age-group specific ratio L_{1Ot}/L_{2Ot} .

d) Suppose one is interested in whether young and old workers in the same education group are "perfect substitutes". What does "perfect substitutes" imply about the parameter τ in part (c). How could you test the hypothesis of "perfect substitutes"?

Question II

A1. Becker's classic book, "*The Economics of Discrimination*", discusses a model of taste-based discrimination with two groups of workers: a majority group and a minority group. Assume instead that there are *three* types of workers: B, W and H. Using formulas, develop a model of taste-based discrimination under the assumption that some (but not all) employers have a strong distaste for B and a mild distaste for H. Make sure that you specify all the assumptions that you need and the first order conditions. Make explicit any assumptions on the production technology that combines different type of workers and on the number of biased employers. Derive the equilibrium conditions.

A2. What are the implications of this model for employers and wage differentials in the short run? What are the implications of this model for employers and wage differential in the long run?

A3 Now consider a statistical discrimination model with two type of workers, F and K. Assume that employers have learned from experience that K group members are less productive on average than F group member, and that the dispersion of productivity is the same for both groups. Using formulas, describe this model in detail, making explicit all the assumptions that you need. Derive the wage of F workers and K workers. Show all the steps. Be very specific on the <u>all</u> the steps necessary for the derivation of the expected value of productivity given the signal and group.

A4 In the model described in A3, suppose that a F worker and a K worker are equally productive. Are they paid the same? First, use the formula to answer. Then, in words, explain why.

A5 Now consider the case where employers have learned from experience that the two groups are on average equally productive, but the dispersion of productivity is the higher for group K than F. Explain (i) how the relationship between signal and wage varies for the two groups; and (ii) how the wage of a group K member who has below average productivity compare to the wage of an F group member with the same level of productivity. What is the intuition?

Question III

1. Consider a cross-sectional employer-employee dataset composed of observations on the earnings (Y_i) and educational attainment (X_i) of workers. The dataset also includes information on the assignment of workers to firms. Let the operator $\mathbf{J}(i)$ denote the firm at which worker i works. Define the mean educational attainment and earnings at the worker's firm as $\overline{X}_{\mathbf{J}(i)} \equiv \frac{1}{N_{\mathbf{J}(i)}} \sum_l I[\mathbf{J}(i) = \mathbf{J}(l)] X_l$ and $\overline{Y}_{\mathbf{J}(i)} \equiv \frac{1}{N_{\mathbf{J}(i)}} \sum_l I[\mathbf{J}(i) = \mathbf{J}(l)] Y_l$ respectively, where $N_{\mathbf{J}(i)} \equiv \sum_l I[\mathbf{J}(i) = \mathbf{J}(l)]$ is the number of individuals at worker i's employer.

a) Suppose earnings obey the following relationship:

$$Y_i = \beta X_i + \delta \overline{X}_{\mathbf{J}(i)} + \varepsilon_i \tag{1}$$

where $E\left[\varepsilon_i | X_i, \overline{X}_{\mathbf{J}(i)}\right] = 0$. In this equation one can think of β as the returns to individual schooling and δ as the returns to average schooling at the worker's firm which may reflect a number of different factors including unobserved worker ability, firm wage premia, or spillovers across workers at the same firm.

Suppose a researcher mistakenly neglects the potential influence of firm characteristics when estimating the returns to schooling. Derive an expression for the resulting bivariate regression coefficient $\pi \equiv \frac{Cov(Y_i, X_i)}{Var(X_i)}$ in terms of the parameters β, δ, and $b \equiv \frac{Cov\left(X_i, \overline{X}_{\mathbf{J}(i)}\right)}{Var(X_i)}$.

b) Show that b is bounded between 0 and 1. Provide an interpretation of b. What does it mean if $b = 1$? How about if $b = 0$?

c) Prove that the OLS estimator $\widehat{\beta}_{OLS}$ of β in (1) is numerically equivalent to the "within firm" estimator $\widehat{\beta}_W \equiv \frac{\sum_i \left(X_i - \overline{X}_{\mathbf{J}(i)}\right)\left(Y_i - \overline{Y}_{\mathbf{J}(i)}\right)}{\sum_i \left(X_i - \overline{X}_{\mathbf{J}(i)}\right)^2}$ of the returns to education.

Question IV

Let profit π for a principal be a function of an agent's effort e and compensation w:

$$\pi = g(e) - w.$$

Assume $g'(e) > 0$ and $g''(e) < 0$ exist and are continuous. The agent's utility is $w - e$ and her "participation constraint" is $w - e \geq v$. The principal does not observe the agent's effort level, but instead observes

$$x = e + \epsilon,$$

where ϵ is drawn from a mean-zero, differentiable, symmetric, single-peaked density f (with corresponding cumulative density F). x is observed by both parties; the principal can be trusted to honor commitments in which compensation w is conditioned on x.

(a) There are many compensation policies that would motivate the worker. Let's try a simple one: The principal pays wage w_0 if the observed performance x falls below some cut-off $\bar{x}$ (which we take as given) and pays $w_1 > w_0$ if x is above that cut-off. So the game is:

- The principal announces the policy: w_0, and w_1.
- The agent decides whether to accept the job, and if she does, takes hidden action e.
- Nature plays x.
- Given x, the firm pays the agreed-upon wage.

Characterize the optimal wage policy, (w_0^*, w_1^*). Is it possible that in your contract w_0 will be negative (i.e., the worker pays the firm if x is observed to be low)?

(b) Let's use results from (a) as a building block for an "efficiency wage model of unemployment." The idea is to introduce "limited liability" through an assumption that the only penalty that the firm impose on poorly-performing worker is dismissal. Now assume our game is repeated indefinitely (workers have a discount rate ρ). So we have a repeated game between the firm and a worker with the following order of play in each round:

- The firm offers a wage w.
- If the worker accepts the job, she chooses effort level e.
- Nature plays x using the distribution $f(\ )$.
- The firm pays w.
- The firm retains the worker if x is above the cut-off; otherwise the firm ends the game. If the game ends, the worker becomes unemployed, in which case her utility has present value V^u.

Focus on the perfect Bayesian equilibrium in which the wage is the same in every period; the worker is retained if and only if x exceeds an endogenous threshold $\bar{x}$ (though you do not have to solve for $\bar{x}$ here). Characterize the worker's best response effort level. Show that the job has "value," i.e., the present value of being employed (and providing optimal effort) must exceed V^u.

(c) Sketch out an informal argument about how your result from (b) could lead to *equilibrium* unemployment. Assuming employers are identical, how is the unemployment rate is related to the variance of the density $f(\ )$?