GENERAL INSTRUCTIONS:

This is a 3-hour (180 min) field exam. 
There are 4 questions in total, but you only need to answer 3 questions. 
Questions 1 is from course 280A. Question 2 is from course 280B. Question 3 is from course 280C. Question 4 is from course 280D.  
Each question has equal weight, so you have 1 hour for each of the 3 questions that you answer.

**Question 1 (280A)**

This question has two parts (A and B). Both parts have equal weight (30 min each). Answer all questions.

**Part A:** In the Armington and Krugman models, the trade elasticity is \((1 - \sigma)\), and yet \(\sigma\) does not affect the trade elasticity in the Eaton-Kortum and Melitz-Pareto models.  

Explain why.

**Part B:** This question is about Hottman, Redding and Weinstein (2016) “Quantifying the Sources of Firm Heterogeneity”.

i) Describe the components into which the paper decomposes cross-sectional differences in firm sizes.

ii) Explain how taking into account differences in the number of products offered by firms affects estimates of TFP differences across firms.

iii) Explain how the authors estimate differences in markups across firms, and how this matters for the decomposition.
**Question 2 (280B)**

**The International Monetary System**

This question is loosely based on Farhi and Maggiori (2016).

There are two periods ($t = 0, 1$) and two classes of agents: the Hegemon country and the Rest of the World (RoW), composed of a competitive fringe of international investors. There is a single endowment good at $t = 0$, split equally between the Hegemon and the RoW: $w = w^*$. Starred variables denote RoW variables. There are two assets, a *risky bond* in perfectly elastic supply and a *nominal bond* issued exclusively by the Hegemon and denominated in its currency. The risky asset exogenous returns between time $t = 0$ and $t = 1$ are $\{R^r_H, R^r_L\}$ with $R^r_H > 1$ and $0 < R^r_L < 1$. The low realization of the risky asset at $t = 1$ occurs with probability $\lambda \in (0, 1)$ and we shall refer to this low realization as a disaster.

The RoW representative agent has mean-variance preferences over consumption at time $t = 1$ and does not consume at $t = 0$:

$$U^*(C^*_1) \equiv \mathbb{E}[C^*_1] - \gamma \text{Var}[C^*_1].$$

The Hegemon representative agent is risk neutral over consumption in both periods:

$$U(C_0, C_1) \equiv C_0 + \delta \mathbb{E}[C_1],$$

where we assume $\delta^{-1} = \mathbb{E}[R^r]$.

At time $t = 1$, after uncertainty about the risky asset is resolved, the Hegemon decides whether to adjust its exchange rate vis-a-vis the RoW, denoted by $e$, with the convention that an increase in $e$ represents a Hegemon currency *appreciation*. For simplicity, we normalize the exchange rate at time zero to be $e_0 = 1$ and assume that the Hegemon can only choose two values of $e = \{e_H, e_L\}$, with $e_H = 1$ and $e_L < 1$. It follows that the Hegemon bonds’ ex-post return in units of the foreign currency is $Re$, where $R$ is the nominal yield determined at $t = 0$. We assume throughout this exercise that $e_L = \frac{K^r_L}{K^r_H}$. We assume also that the Hegemon can only decide to depreciate after a disaster, and if it chooses to do so it pays a utility cost proportional to the depreciation rate: $\tau (1 - e_L)$, with $\tau > 0$.

The RoW budget constraints are:

$$w^* = s^* + b,$$

$$s^* R^r + b Re = C^*_1,$$

where $s^*$ is the real value invested in the world risky asset, $b$ is the real value of Hegemon nominal debt.

The Hegemon budget constraints similarly are:

$$w - C_0 = s - b,$$  \hspace{1cm} (1)

$$s R^r - b Re = C_1.$$  \hspace{1cm} (2)
1. The RoW optimization problem is given by:

\[
\max_b \ E[C^*_1] - \gamma \ Var(C^*_1),
\]

s.t. \( w^* = s^* + b \), \( s^* \geq 0 \), \( b \geq 0 \),

s.t. \( s^* R_r^* + b R e = C^*_1 \).

Assume that the Hegemon debt is expected to be safe (i.e. no depreciation), then derive the optimality condition for the portfolio choice of the RoW. Provide a simple economic intuition behind this optimality condition. Show that this defines a mapping \( R = R^s(b) \) between the nominal interest rate \( R \) and the demand for safe debt \( b \).

For future reference, simply assume that if the Hegemon’s debt is expected to be risky, then \( R = R^r_H \), i.e. the risky nominal debt and risky asset are perfect substitutes.

2. In what follows we assume that the Hegemon has full commitment and can therefore promise not to devalue the currency.

a) Show formally (or argue intuitively) that the Hegemon value function solves

\[
\max_{b \geq 0} V^{FC}(b) \equiv b(E[R^r] - R^s(b)),
\]

where \( R^s(b) \) is the mapping derived in the previous question. (Hint: start from the Hegemon consumption/investment problem, then argue that in full commitment the Hegemon does not depreciate, and finally use \( \delta^{-1} = E[R^r] \).)

b) Recall that the Hegemon is a Monopolist, derive its optimal issuance of debt under full commitment. Discuss intuitively this result.

c) Derive the equilibrium issuance \( b^{FC} \) and show that it satisfies:

\[
b^{FC} = \frac{1}{2} w^*
\]

Is \( R^s(b^{FC}) < E[R^r] \)? If so, why? What empirical phenomenon is this trying to capture?

d) Show that in the previous point, if the Hegemon were competitive (i.e. took \( R \) as given) then it would provide full insurance to the RoW.

3. In what follows we assume that the Hegemon has limited commitment. We do not study the optimal issuance problem, but actually focus on the possibility of multiple equilibria for a given \( b \).

a) At \( t = 1 \), if a disaster has occurred, the Hegemon decides whether to appreciate or depreciate by solving:

\[
\max_{e \in \{1,e_L\}} C_1 - \tau (1 - e),
\]

s.t. \( s R^r_L - b R e = C_1 \).

Derive a threshold property for the decision to depreciate in terms of the amount to be repaid \( b R \).
b) For a given level of issuance $b$ at $t = 0^-$, the structure of continuation equilibria for $t = 0^+$ onwards is as follows:

i) For $b > \bar{b}$ there is an equilibrium, the collapse equilibrium, in which Reserve currency debt has no safety premium ($R = R_H^r$) and the Reserve currency depreciates conditional on a disaster. Prove this statement and provide an expression for $\bar{b}$.

ii) For $0 < b < \bar{b}$ there is an equilibrium, the safe equilibrium, in which the Hegemon does not depreciate in the disaster state at $t = 1$. The yield on Reserve currency debt is given by:

$$R^s(b) = \mathbb{E}[R^r] - 2\gamma(w^* - b)\sigma^2$$

and is increasing in $b$. Prove this statement (You do not have to provide an expression for $\bar{b}$).

iii) Show formally or argue informally that $\bar{b} < b$ and that therefore there is a range of debt values $b$ for which multiple equilibria exist. Explain why this depends on the particular timing of decisions in Calvo also adopted in this paper. What economics is this trying to capture?

In what follows we denote the Safety region the interval $[0, \bar{b}]$, the Instability region the interval $(\bar{b}, \bar{b})$, and the Collapse region the interval $(\bar{b}, w^*)$.

4. Discuss whether, in your view, this is a good model of the ‘Triffin dilemma’.

5. (Bonus question) Suppose that a sunspot determines which equilibrium occurs in the Instability region and that the probability of collapse is $0 < \alpha < 1$. Discuss informally how the optimal bond issuance of the Hegemon varies with $b^{FC} = w^*/2$ under limited commitment.
Question 3 (280C)

International Risk Sharing

Assume that household preferences are represented by the following utility function

\[ U_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[ \log c_t - \frac{1}{1+\phi} n_t^{1+\phi} \right] \]

and that technology is given by

\[ y_t = A_t k_t^{1-\alpha} n_t. \]

Denote initial capital by \( k_0 \) and assume that capital depreciates fully each period so that the resource constraint in a closed economy is

\[ c_t + k_{t+1} \leq y_t. \]

Assume that productivity \( A_t \) follows some stochastic process. Maintain these assumptions throughout the entire question.

A) Closed Economy (financial autarky)

Show that in the closed economy, equilibrium consumption follows

\[ c_t = (1 - \alpha \beta) y_t \]

Discuss the properties of hours \( n_t \) and investment \( k_{t+1} \) in this model. How do the predicted series compare to those observed in developed countries such as the US?

B) Complete Markets

B1) Small open economy

Assume now that the country can borrow and lend one-period state-contingent bonds. The flow budget constraint becomes

\[ c_t(s^t) + k_{t+1}(s^t) + \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) q_t(s^{t+1}|s^t) b_{t+1}(s^{t+1}) \leq b_t(s^t) + A_t(s^t) k_t(s^{t-1}) n_t(s^t)^{1-\alpha}, \]

where \( \pi(s^{t+1}|s^t) \) is the probability of state \( s^{t+1} \) occurring conditional on state \( s^t \). Explain the remaining terms in this flow budget constraint.

Discuss the following statement: Assuming that the small open economy can borrow from risk neutral foreign lenders implies that \( q_t(s^{t+1}|s^t) \equiv \frac{1}{1+r^*}. \)

Assume that \( 1 + r^* = \frac{1}{\beta} \).

- Solve for \( c_t, k_{t+1}, n_t, y_t \). Compare these allocations to the allocations you obtained in the closed economy. Discuss.
• Assume that $A_t$ follows an AR(1) process in logs. How do the business cycle properties of $y_t$ differ between the closed and the small open economy.

• Discuss the Feldstein-Horioka puzzle within the context of this model.

B2) Closing the economy: two large countries

Assume now that there are two large identical countries that can perfectly share risk—i.e. can trade state contingent bonds. Characterize the equilibrium allocation and discuss its properties. You can either solve the social planner’s problem or work with the decentralized economy in this case.

• What is the equilibrium asset portfolio for each country?

• Are risks shared across the countries?

• Discuss the consumption correlation puzzle within the context of this model.
Question 4 (280D)

An Economic Geography Model with Migrations

In this problem, we build on the model of Artuc, Chaudhuri and Mac Laren (2010) to write an economic geography model with migrations, in the spirit of Desmet, Nagy and Rossi-Hansberg (2016) or Dvorkin, Caliendo and Parro for example.

Consider an economy with $N$ cities indexed by $n$. Time is discrete. Workers maximize lifetime utility $U$ defined by $\log U = \sum_{t=0}^{\infty} \beta^t \log(U(t))$ where $\beta < 1$ is the workers’ discount factor. Equivalently, $U = \prod_{t=0}^{\infty} U(t)^{\beta^t}$.

Workers choose the city where they live. The timing is as follows. At the beginning of each period, each worker draws a vector of preference shocks $a_n(t)$, iid across workers, cities and time. Shocks $a_n$ are distributed Frechet, with mean $A_n$ in city $n$ and shape parameter $\kappa$. Upon observing their draws, workers decide whether (and where) to migrate. If they move, they incur a migration cost that takes the form of a disutility cost $T_{in} > 1$, where $i$ is the current city of the worker and $n$ is the new one (in contrast, $T_{ii} = 1$). The flow utility in period $t$ of a worker who moves from city $i$ to city $n$ at the beginning of period $t$ is:

$$U(t) = \frac{a_n(t)w_n(t)}{P_n(t)T_{in}},$$

where $w_n(t)$ is the local wage for period $t$ and $P_n(t)$ the local price index in period $t$. The equations that pin down wages and prices are left unspecified for now.

1. Write the utility maximization problem of a worker who starts the period in city $i$, observes his draws $a_n(t)$ and chooses where to move to. To that end, it will be useful to use the function $V_n(t)$ defined recursively as:

$$V_n(t) = \mathbb{E}\left[\max_{j} a_j(t)w_j(t)P_j(t)T_{nj}V_j(t+1)^{\beta}\right].$$

What does $V_n(t)$ represent?

2. What is the fraction $\lambda_{in}(t)$ of city $i$ workers who migrate from $i$ to $n$ in period $t$ (still using the function $V_n(.)$)? Write down the law of motion for city sizes, i.e. the equation that defines city size $L_n(t+1)$ as a function of $\lambda_{ij}(t)$ and $L_n(t)$.

3. From now on, we focus on the steady state of this economy. All quantities and prices are constant, so we drop the $t$ subscript.

Write down the average welfare of workers who start a given period in city $i$, as a function of the vector $\{A_m, w_m, P_m, V_m\}_m$ and the matrix $\{T_{im}\}_{i,m}$. Is (average) welfare equalized across all cities? Why or why not?

4. Show that in the steady state, city sizes can be written as:

$$L_n = A_n \left(\frac{w_n}{P_n}V_n^{\beta} \Phi_n\right)^{\kappa},$$

where $\Phi_n$ is a function of migration frictions (use the results from (2)). What is the role played by migration frictions in shaping city sizes?
5. Assume that we close the model with a production side as in Eaton and Kortum. In particular, the real wage can be expressed as a function of the trade share of a city with itself: \( \pi_{ii}^{-\frac{1}{\theta}} = \frac{w_i}{P_i} \). Express welfare in city \( i \) as a function of \( \pi_{ii} \) and of \( \lambda_{ii} \), the share of workers who do not migrate away from city \( i \).

6. Using exact hat algebra, write the change in welfare \( \hat{V}_i = \frac{V_i'}{V_i} \) in city \( i \) between two equilibria with different frictions (trade or migration frictions), as a function of the change in endogenous variables of the model. Is there a “sufficient statistics formula” for the gains from trade between cities that only requires observing data from the current equilibrium and key elasticities? Why or why not? Same question for (a) the gains from migration between cities, and (b) the gains from both trade and migration.

Note: Gains from trade (resp. migration) between cities = gains of going from a world with no trade (resp. no migration) between cities, to the equilibrium we observe in the data.

7. Discuss the pros and cons of this model.
For instance: what does the steady state of this model bring to the table compared to a static model as in Allen and Arkolakis (2014) or Redding (2016)? What are the limitations of this dynamic model of migration? What assumptions would you try to relax to make it more realistic?