Economic Demography Field Exam  
Department of Economics  
University of California, Berkeley  
August 26, 2013

There are four questions and you may take up to three hours. Answer all parts of all questions. The questions will be weighted equally in the overall grade. You may use a calculator. You may use a special two page list of demographic formulas that will be provided for you. On the methods question 4), please show your work and label your answers clearly. Answers with decimals should be given with six figures beyond the decimal point.

1) 1. (20) Elizabeth, a highly paid professional woman, has two potential marriage partners: Sam who does not earn much money but who is a lot of fun to be with, and George who has a high income but is a bit boring.

   a. Assuming that Elizabeth is constrained by social norms to stay home at least part time to care for any children, discuss how her choice of partner might influence how many children she chooses to have (via the Value of Time model).

   b. Now drop the assumption that the mother is necessarily the one who spends time with the child. Discuss how this would change her choice of number of children with each partner.

   c. Drawing on the theory of marriage, is it more likely that Elizabeth chooses Sam rather than George if she is constrained by social norms to do the child care or is not so constrained?

   d. If the stigma against men staying home to raise children is reduced, how might that affect the relative value of high wage and low wage men and women in the marriage market?

2) Immigration: This question refers to the standard static model of the effect of immigration on returns to labor and to capital assuming immigrants augment the domestic labor supply for which they are perfect substitutes, but immigrants bring no capital and the capital stock remains unchanged. The diagram plots the marginal product of labor on the vertical axis against labor on the horizontal axis.

   a. Draw the standard diagram showing the impact of immigrant labor on wages and profits in the receiving country. Explain its implications. Based on this diagram, does immigration raise or reduce the average income per capita of the original population? Does it raise or reduce national per capital income for the total population? Does it lead to redistribution of income? Discuss.

   b. Discuss at least three important aspects of this static model that are unrealistic.

   c. Consider the diagram you just drew for part a. Now interpret the increase in labor as arising through births to the domestic population rather than immigration, assuming the population is closed to migration. Do these births raise the average income per capita of the original population? Do they raise national per capita income? Do they lead to redistribution of income? In what important ways does this situation differ from the case of immigration?

3) (Actual factual background) The National Institute of Health (NIH) is under pressure from a handful of members of the House, including the chair of the House Appropriation Committee responsible for NIH’s budget, to prohibit NIH funding of economic research programs, projects or activities. Other members of the House are sending a letter to the Director of NIH, stating (in part) the following:

   “It has come to our attention that there have been recent discussions about the relevance of economics research within the scope of the mission of the National Institutes of Health (NIH). We are writing to
affirm our belief that the NIH should sustain its crucial support for behavioral, and social science
research, including economics. Support for these areas of research is consistent with the NIH mission to
‘enhance health, lengthen life, and reduce the burdens of illness and disability’.

Consider the specific topic of socioeconomic disparities in health and mortality, and discuss
systematically what relevant insights and findings economic research brings to this topic. Where
possible, refer to specific studies and articles in your discussion.

4. Men and women in the United States now over the age of 70 were born
before the onset of the Baby Boom, during a period of fairly steady growth
rates, making a roughly stable shape for the top portion of the age pyramid
plausible. Table 1 shows \( \frac{d_n}{d_x} \) and \( \frac{L_n}{L_x} \) values for a female period lifetable for
U.S. women in 2010 with a radix of 1, which (despite limitations) can be used
for stable calculations. Ages over 95 have been combined into an open-ended
age group, after noting that \( e_{95} = 3.543 \) and \( e_{100} = 3.040 \). Assume a value
for Lotka’s \( r \) of 0.007.

a) Find the implied lifetable values for \( \ell_{70}, e_{70}, \) and \( e_{85} \).

b) What is the mean age for those over age 85 in the stable population?

c) Health care costs and associated transfer flows are higher for the “oldest-
old” over 85 than for younger people. Find the ratio of those over 85 to
those 80 to 85 in a stable population based on the estimates you have
been given.

d) About 1.2 million women turned 70 in 2010. In your hypothetical stable
population, how many women would there be who both turn 80 in 2010
and have between 10 and 15 years left to live?

e) Health care costs are seen to depend more closely on time left till death
than on age itself. Estimate the number alive over age 80 in the stable
population in 2010 who have less than 5 years left to live.
Table 1: U.S. Period Lifetable Data for U.S. Women 2010

<table>
<thead>
<tr>
<th>Age</th>
<th>$n_d_x$</th>
<th>$n_L_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.076</td>
<td>3.956</td>
</tr>
<tr>
<td>75</td>
<td>0.110</td>
<td>3.492</td>
</tr>
<tr>
<td>80</td>
<td>0.151</td>
<td>2.839</td>
</tr>
<tr>
<td>85</td>
<td>0.187</td>
<td>1.993</td>
</tr>
<tr>
<td>90</td>
<td>0.178</td>
<td>1.079</td>
</tr>
<tr>
<td>95+</td>
<td>0.127</td>
<td>0.448</td>
</tr>
</tbody>
</table>
HERE ARE THE INSTRUCTIONS AND USEFUL FORMULAS THAT WILL BE SUPPLIED DURING THE FIELD EXAMINATION:

This is a closed-book examination. Please show your work and label your answers clearly. Answers with decimals should be given with six figures beyond the decimal point. Useful formulas are given on a separate handout. Please do not share the questions or your answers with other students.
A Collection of Useful Formulas

Growth Rate: $R = (1/T) \log(K(T)/K(0))$

Exponential Growth: $K(t) = K(0)e^{Rt}$

Survival from hazards: $\ell_{x+n} = l(x)e^{-h_x n}$

Interval Conversions: $(1 - q_x)^n = 1 - nq_x$

Hazard Rate: $h_x = -(1/n) \log(\ell_{x+n}/\ell_x)$

Exponential Slope: $\frac{d}{dx}\alpha e^{\beta x} = \alpha\beta e^{\beta x}$

Exponential Area: $\int_a^b \alpha e^{\beta x} dx = \left(\frac{\alpha}{\beta}\right)(e^{\beta b} - e^{\beta a})$

Gompertz Model: $h_x = \alpha e^{\beta x}$; $\ell_x = \exp\left((-\alpha/\beta)(e^{\beta x} - 1)\right)$

Age Specific Death Rate: $nM_x = nD_x / nK_x$

Period Lifetable: $nq_x = \frac{(n)(nM_x)}{1 + (n - na_x)(nM_x)}$

Age Factors: $1a_0 = 0.07 + 1.7(1M_0)$; $4a_1 = 1.5$; $\infty a_x = 1/(\infty M_x)$

Survivorship: $\ell_{x+n} = \ell_x(1 - nq_x) = \ell_x - nd_x$

Person-Years Lived: $nL_x = (n)(\ell_{x+n}) + (na_x)(nd_x)$
Lifetable death rate: \( n m_x = \frac{n d_x}{n L_x} \)

Remaining Person-Years: \( T_x = n L_x + n L_{x+n} + n L_{x+2n} + \cdots \)

Expectation of Life: \( e_x = \frac{T_x}{\ell_x} \)

In the Presence of Other Causes: \( n q^d_x = (n q_x) s_A \)

In the Absence of Other Causes: \( n q^*_x = 1 - (1 - n q_x) s_A \)

Annuity Price: \[
\frac{B}{\ell_x} \left( \frac{n L_x}{(1+i)^{n/2}} + \frac{n L_{x+n}}{(1+i)^{n+n/2}} + \cdots + \frac{\infty L_{x+max}}{(1+i)^{x+max+x_{max}}} \right)
\]

Brass’s Logit System: \( \ell_x = \frac{1}{1 + \exp(-2\alpha - 2\beta Y_x)} \)

Brass Estimation: \( \alpha + \beta Y_x = (1/2) \log(\ell_x/(1-\ell_x)) \)

Leslie Matrix Top Row: \[
\frac{n L_0}{2\ell_0} \left( n F_x + n F_{x+n} \frac{n L_{x+n}}{n L_x} \right) f_{fab}
\]

Leslie Matrix Subdiagonal: \( \frac{n L_{x+n}}{n L_x} \)

Lotka’s Equation: \( 1 = \sum (1/2) (n F_{x+n} L_x + n F_{x+n+n} L_{x+n}) (f_{fab}/\ell_0) e^{-r(x+n)} \)

Stable Age Pyramid: \( n K_x^{stable} = B(n L_x/\ell_0) e^{-rx} \)

Stable Proportions \( n K_x/\infty K_0 = b \left( n L_x/\ell_0 \right) e^{-rx} \)
Lotka’s Parameter: \( r \approx \log(NRR)/\mu \)

Momentum: \( \frac{K(\infty)}{K(-\epsilon)} = \frac{b(\epsilon)e_0}{\sqrt{NRR}} \)

Coale-Trussell Model: \( n^xF_{xmarital} = Mn(x)exp(-m\nu(x)) \).

Coale-Trussell Estimation: \( \log(n^xF_{xmarital}/n(x)) = \log(M) - m\nu(x) \).

SMAFM = \( \sum (n)(1 - PEM_x/PEM_{ult}) \)

Bourgeois-Pichat’s Formula \( dq_0 = a + b(\log(1 + d))^3 \)

Princeton Indices: \( I_f \approx I_gI_m \)

\[
I_f = \frac{B^{overall}}{\sum (5K_x)(5F_x^{Hutt})} \\
I_g = \frac{B^{marital}}{\sum (5K_x^{married})(5F_x^{Hutt})} \\
I_m = \frac{\sum (5K_x^{married})(5F_x^{Hutt})}{\sum (5K_x)(5F_x^{Hutt})}
\]

Linear Regression slope: \( \frac{\text{mean}(X \ast Y) - \text{mean}(X) \ast \text{mean}(Y)}{\text{mean}(X \ast X) - \text{mean}(X) \ast \text{mean}(X)} \)

Linear Regression intercept: \( \text{mean}(Y) - (\text{slope}) \ast \text{mean}(X) \)

Duncan Dissimilarity Index \( D = \max \left( \frac{\sum_{j=1}^{J} U_{(j)}}{\sum_{j=1}^{J} U_{(j)}} - \frac{\sum_{j=1}^{J} R_{(j)}}{\sum_{j=1}^{J} R_{(j)}} \right) \)