Field Exam: Advanced Theory

There are two questions on this exam, one for Econ 219A and another for Economics 206. Answer all parts for both questions.

Exercise 1: Consider a $n$-player all-pay auction auction with a single object where the highest bid wins and every agent $i$ pays her bid $t_i$. Let $\theta_i$ be the value of agent $i$ for the object, and assume that the values are i.i.d. distributed according to $F : [\theta, \bar{\theta}] \to [0, 1]$. The utility of the agent equals $\theta_i - t_i$ if she gets the object and $-t_i$ otherwise.

(a) Find a symmetric equilibrium of this auction. Is this an equilibrium in dominant strategies?

(b) What is the expected revenue in this auction.

(c) Is this all-pay auction revenue maximizing?

(d) Derive the bid in the corresponding first and second price auction.

(e) Compare the bid agent $i$ makes in this auction to the bid she makes in a first or second price auction when she has the valuation $\theta_i$. Explain intuitively, why the bids are ordered between the different auction formats.

(f) Now derive an equilibrium if there are $1 < k < n$ objects.
(g) Suppose now that the agent’s utility is given by $\theta_i - c(t_i)$ if she gets the object and $-c(t_i)$ otherwise, where $c$ is a strictly increasing function. Characterize a symmetric equilibrium bidding strategy.
Exercise 2: Consider a simple model of gym attendance (following DellaVigna and Malmendier 2004), where in period 0 individuals choose whether or not to sign a contract that requires them to pay a lump-sum membership fee $L$ in period 1 and then an additional attendance price $p$ if they attend the gym in period 2. They receive a health benefit $b$ of attending the gym, which is a delayed benefit realized only later. They also incur a hassle cost $c$ in period 2, which they feel immediately in period 2. In period 0 they only know that $c$ will be drawn from a uniform distribution on the unit interval $[0,1]$, with $c$ realized only at the beginning of period 2.

Individuals are present-biased, with a common present bias factor $\beta \leq 1$. They thus attend the gym in period 2 if and only if $\beta b - p - c \geq 0$. The present-biased individuals are sophisticated, and in period 0 they choose to sign the contract if $\beta \int_{c=0}^{1} b - p - c dc - L \geq 0$.

Part a. Let $V(p,L)$ denote an individual’s expected utility from signing the contract, from the period 0 perspective. Show that for $p < \beta b$,

$$V(p,L) = \beta (\beta b - p) \left( b - \frac{\beta b + p}{2} \right) - \beta L$$

Part b. Show that $\frac{dV}{dL} = -\beta$ and $\frac{dV}{dp} = -\beta (b - p)$ for $p < \beta b$.

Part c. Suppose that the gym incurs a cost $\psi$ whenever an individual attends the gym. And suppose also that $p$ and $L$ must satisfy the zero profit condition $L + Pr(\text{attend}) \cdot p = Pr(\text{attend}) \cdot \psi$. This zero profit condition allows us to write $L$ as a function of $p$. What is $L(p)$?

Part d. If $L(p)$ is determined from the zero profit condition above, show that the value of $p$ that maximizes $V(p,L(p))$ is given by $p^* = \psi - (1 - \beta)b$.

Part e. Now let’s generalized everything from parts a through f. If $L(p)$ is determined from the zero profit condition, prove that $p^* = \psi - (1 - \beta)b$ without assuming that $c$ is distributed uniformly.
on [0,1]; assume only that the distribution of c has a continuous density function with full support on the unit interval.

**Part f.** Please provide intuition for the \( p^* \) formula above. In particular, explain why \( p^* = \psi \) when \( \beta = 1 \) and why \( p^* < \psi \) when \( \beta < 1 \).

**Part g.** Suppose that the “gym economy” consists of many identical gyms, each of which incurs a cost \( \psi \) per attendance. Show that in a competitive equilibrium of this economy, gyms will set \( p = \psi - (1 - \beta) b \) and set \( L \) to satisfy the zero-profit condition.

**Part h.** Keep assuming the competitive equilibrium from part (g). Suppose that Calvin Voltt, a renowned researcher applying behavioral economics to health decisions, decides that it is a good idea to provide incentives for gym attendance to counteract the fact that most people seem to go to the gym less than they wanted to due to self-control problems. Calvin runs a large scale field experiment with a particular gym branch and finds that financial incentives do indeed increase gym attendance. Assuming that the gym branch maintains its standard pricing \( p^* = \psi - (1 - \beta) b \) during the experiment, explain why this field experiment actually created socially inefficient gym attendance while it was being run.

**Part i.** Keep assuming the competitive equilibrium from part (g). Suppose that Calvin cleverly measures people’s present bias \( \beta \) and attendance health benefits \( b \), and convinces the government to provide financial incentives of \( r = (1 - \beta) b \) per gym attendance. To maintain a balanced budget, these incentives must be funded through a lump-sum tax equal to \( T = r \cdot Pr(\text{attend}) \) per individual. Assume that the attendance incentives are obtained by individuals instantaneously in period 2, while the lump-sum tax is paid in period 1, alongside the membership fee \( L \).

In the long-run, the competitive gym economy will adjust its contract terms \((L, p)\) in response to this government incentive policy. Show that when the equilibrium adjusts, gyms will set \( p = \psi \),
and the net effect of the government intervention on individuals’ (long-run) welfare will be zero.

**Part j.** Please comment on the broad lesson that parts (h) and (i) are conveying about “behaviorally-informed” interventions.

**Part k.** How would things change if consumers were (partially) naive and over-estimated their future self-control? In particular, would the interventions in parts (h) and (i) be more helpful, less helpful, or equally helpful in this case? Feel free to solve for the equilibrium \( p^* \) as a function of the actual \( \beta \) as well as what consumers in period 0 think their \( \beta \) will be in period 2, denoted \( \hat{\beta} \).