Consumption Commitments: A Foundation for Reference-Dependent Preferences and Habit Formation*

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Abstract

We build a theory of reference-dependent preferences based on adjustment costs in consumption. The endogenous evolution of the reference point in our model captures several features of existing theories in which the evolution of the reference point is specified exogenously. In particular, our model predicts that reference points (1) depend on past consumption levels, (2) reflect recent expectations, and (3) diminish in importance when agents experience large shocks. When the ratio of idiosyncratic to aggregate risk is large, the model is isomorphic to standard habit formation specifications, in which the reference point is a weighted average of past consumption. To illustrate the implications of endogenizing the reference point, we apply the model to analyze aggregate consumption dynamics, changes in policy parameters, and the welfare cost of shocks. In each application, the model confirms certain intuitions from existing theories but yields some starkly different predictions. For example, we show that insuring idiosyncratic shocks increases welfare as in standard models, but can also raise the welfare cost of aggregate shocks by making reference points more persistent.

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1 Introduction

Models of reference dependent preferences – in which utility depends upon the difference between outcomes and a pre-determined benchmark – are now widely used in economics. A central question in these models is how the reference point is determined, to which the existing literature has proposed two general answers. In habit formation and status-quo models, the reference point is modeled as an average of recent consumption. In these models, the agent evaluates outcomes relative to recent experience, to which he has habituated himself.[1] In contrast, a more recent literature has argued that the reference point is determined by recent expectations, i.e. the agent compares outcomes to what he expects to obtain.[2] The difference between these two competing approaches is analogous to the debate between adaptive- and rational-expectations theories in macroeconomics.

Existing theories of reference-dependent behavior specify the evolution of the reference point exogenously. Such models require that researchers make ad hoc assumptions about habit weights or the time horizon that determines recent expectations. In this paper, we propose a theory of endogenous reference points based on fixed (mental or physical) adjustment costs. In the spirit of Stigler and Becker (1977) and Postlewaite (1998), we show that variants of existing reference-dependent preference specifications can be endogenously generated as a function of prices and income.

The central premise of our model is that changing the consumption of certain goods – which we term "consumption commitments" – is costly. These costs could reflect either transaction costs or mental costs such as the effort required in changing plans (Grossman and Laroque 1990, Chetty and Szeidl, 2007). Consumption commitments act as a reference point by altering the agent's indirect utility over total consumption. Because commitments are themselves chosen to maximize expected utility, we obtain an explicitly dynamic theory for the evolution of the reference point. The only additional primitive that must be specified exogenously to calculate these dynamics is the adjustment cost, a parameter that can at least in principle be measured empirically.

We show that the commitment-based reference point unifies the intuitions in many existing models of reference dependence. As in forward-looking models, the reference point evolves based

[1] Ryder and Heal (1973) specify habit as an exponential average of past consumption. Sundaresan (1989), Constantinides (1990) and Campbell and Cochrane (1999) use variants of this model in finanace. In macroeconomics, Carroll, Overland and Weil (2000), Fuhrer (2000), Christiano, Eichenbaum, and Evans (2003) and a large literature in monetary policy building on this work use models where habit is a function of lagged consumption.

[2] Koszegi and Rabin (2006) develop such a forward looking model, which is then used by Koszegi and Rabin (2007, 2009), Koszegi and Heidhues (2008), Yogo (2008) and Crawford and Meng (2009) in various applications.

on expectations because the choice of commitment consumption is based upon expectations about future income. But due to infrequent adjustment, current commitments reflect *recent* expectations, which in turn are also reflected in recent consumption choices by the permanent income hypothesis. As a result, the current reference point is also related to recent consumption, as in backward-looking models. In addition, models of reference dependence sometimes posit that the utility has two components – one that features reference dependence and another that has a neoclassical form – so that the importance of the reference point diminishes for large fluctuations. Our model endogenously generates this property because agents abandon commitments when they face large shocks and thus no longer have reference-dependent preferences. The degree to which these various properties are manifested depends upon the primitives of the environment and the realization of shocks, creating rich dynamics for the reference point.

To characterize these dynamics, we study aggregate behavior in a model economy populated by many agents. Individual discrete adjustments in the reference point are smoothed out in the large population, allowing us to focus on the central trends. This environment allows us to establish a tight connection between our model and one of the most popular models of reference-dependent preferences, habit formation, which is typically used to study macroeconomic aggregates. The main theoretical result of the paper is that when the ratio of idiosyncratic to aggregate consumption risk is large, aggregate commitments are well approximated by a weighted average of past consumption with fixed weights. As a result, aggregate dynamics are close to a representative agent habit-formation model where current habit is an average of past consumption. Hence, backward-looking habit models can be viewed as a "reduced form" representation of forward-looking reference point models based on adjustment costs. From an applied perspective, this equivalence result provides conditions under which the reduced-form habit formation specifications widely used in applied work can be motivated by simple adjustment costs.[3] Intuitively, the impulse response to aggregate shocks in our model depends upon the distribution of agents in the inaction region for commitment consumption. When idiosyncratic risk is large, this distribution remains close to its steady state, because the covariance across agents choices is low. As a result, the reference point follows dynamics that are well approximated by a fixed, state-independent impulse response, which can be generated with a habit model that has fixed weights.

[3] Our characterization result is analytical. Previous studies of aggregate dynamics with adjustment costs use numerical techniques (Marshall and Parekh, 1999), time-dependent adjustment (Lynch, 1996, Gabaix and Laibson, 2001, Reis, 2006) or approximate solutions (Caballero and Engel, 1993). These studies focus on a model with a single illiquid good, as in Grossman and Laroque (1990).

We illustrate the implications of endogenizing the reference point using three applications. Each application confirms certain intuitions from existing "reduced form" reference-dependent models but yields some starkly different predictions because of endogenous changes in the reference point. We first consider the model's implications for aggregate consumption behavior. Two well-documented empirical regularities are that consumption does not respond fully to contemporaneous shocks (excess smoothness, Deaton 1987) and that anticipated changes affect current consumption (excess sensitivity, Flavin 1981). Fuhrer (2000) argues that both of these features of the data can be explained by a habit formation model, essentially because habit responds sluggishly to shocks. The commitment theory of reference dependence also generates the same sluggish response and therefore also explains excess sensitivity and smoothness. However, the commitment model predicts that sluggish responses will only arise in normal times with moderate shocks; during extreme events such as large recessions and crises, many households will choose to pay the fixed cost of adjustment, and therefore aggregate consumption responds quickly as the reference point becomes less important. As a result, vector autoregression (VAR) estimates linking macro aggregagates and their lags are likely to be unstable during extreme events, when correlations break down due to rapid adjustment.[4]

Our second application considers the welfare cost of shocks. As in habit models, the reference point created by commitments amplifies the welfare cost of shocks. However, the endogenous evolution of the reference point again modifies the models predictions for large shocks. Agents can abandon commitments in extreme events, while they are prohibited from doing so in reduced-form models of reference dependence. This distinction is strongest when the habit stock is slow-moving, in which case the high habit set by past consumption reduces the welfare of the agent for an extended period during an negative shock. For example, in a commitment economy with low idiosyncratic risk, consumption is quite sluggish due to infrequent adjustment, implying that the habit model that matches these data would be quite persistent. In this economy, a researcher estimating a standard habit model would incorrectly predict high aversion to extreme events, whereas in the true model agents would simply abandon commitments in response to a large shock.

These two applications show that our model generates different predictions from existing theories because the reference point responds endogenously to the shocks that agents face. In our third application, we show that the endogenous response of the reference point to primitives also

[4] The inattention model of Reis (2006) also generates excess sensitivity and smoothness of consumption, but does not predict correlations breraking down in extreme events because it features time-dependent adjustment.

leads to different predictions. Because commitments are chosen by the consumer, they respond endogenously to policy changes. Hence, the parameters of a reduced-form habit model chosen to match data generated by the commitment economy will change when policy or environmental parameters change. This observation is a variant of the Lucas (1976) critique, and calls for caution in using reduced form habit models in policy analysis, where these endogenous responses are not taken into account. Reductions in risk or in expected growth increase sluggishness in the commitment economy, because people find it optimal to update less frequently. Insuring idiosyncratic shocks – e.g. by expanding social insurance programs – substantially increases welfare, as in models of habit. However, it can also raise the welfare cost of aggregate shocks by making habit more persistent, as individuals change their plans and commitments less frequently. One potential implication is that recessions will last longer in European welfare states, which have large social safety nets, than in economies like the U.S. where idiosyncratic risk is higher.[5] Another implication is that consumption will respond more rapidly to shocks – and recessions may be shorter lived – in rapidly growing economies, where agents will change reference points quickly.

In addition to the literature on reference-dependence and habit that motivates this paper, our analysis relates to and builds on several other strands of research. The qualitative similarity between the commitment and reference-dependent models has been pointed out in several recent papers. Dybvig (1995) examines ratcheting consumption demand in a model with extreme habit persistence, and motivates these preferences by pre-commitment in consumption. Flavin and Nakagawa (2008), analyze asset pricing in a two-good adjustment cost model, and note the similarity to habit. Postlewaite, Samuelson and Silverman (2008), Fratantoni (2001), and Li (2003) also study two-good models and note this similarity in different contexts. We contribute to this literature by analyzing aggregate dynamics, characterizing conditions under which commitments, reference-dependence, and habit formation are similar, and contrasting the implications of exogenous vs. endogenous reference points.

Our analysis is also closely related to a growing literature that examines the implications of commitments in different contexts. Postlewaite, Samuelson and Silverman (2008) show that commitments generate incentives to bunch uninsured risks together, potentially explaining real wage rigidities. Chetty and Szeidl (2007) show that consumption commitments amplify moderate-stake risk aversion. Shore and Sinai (2009) show that households who are subject to exogenously higher

[5] In a general equilibrium model, these effects are likely to be amplified further, when non-adjustment by some agents affects the choices of others.

income risk optimally undertake larger housing commitments. Olney (1999) gives historical evidence that exposure to installment finance commitments forced households to cut back on other consumption, exacerbating welfare loss in the great Depression. The present paper shows the close connection between this body of work and models of reference dependence and habit formation.

The remainder of the paper is organized as follows. Section 2 shows how adjustment costs generate reference-dependent preferences in a general model. Section 3 introduces the model with heterogenous agents. In section 4, we establish the main result showing the equivalence between the aggregated commitment model and models of habit formation. Section 5 presents the three applications. Section 6 concludes.

2 Adjustment Costs Generate Reference-Dependent Preferences

In this section, we show that adjustment costs in consumption lead to indirect utility that exhibits reference dependence. We also establish some basic properties of the commitment-based reference point.

Consider a consumer who maximizes the present discounted value of expected utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(a_t, x_t\right)$$

where u is bounded from above, goes to $-\infty$ if $a \to 0$, concave, twice continuously differentiable, and $u_{12} \geq 0$. There are two components of consumption: commitment (x) and adjustable (a). The goods x and a may be viewed as different components of the same consumption good c – e.g., adjustable margins of housing such as the the quality of a kitchen vs. fixed components such as the amount of land – or as two distinct goods. We normalize units so that the relative price of the two goods is one. Adjustment of commitment consumption from x_{t-1} to x_t generates a monetary cost of $\lambda\left(x_{t-1}, x_t\right) = \lambda_1 x_{t-1} + \lambda_2 x_t$ where $\lambda_1, \lambda_2 \geq 0$ and at least one of them is positive. This cost could reflect the transaction cost inherent in changing consumption of illiquid durables such as houses, cars, or appliances or the cost of renegotiating service contracts (Grossman and Laroque 1990, Bertola and Caballero 1990).[6] The switching cost may also arise from consumption plans and mental accounts (Ameriks, Caplin and Leahy, 2003, Thaler, 1999).[7] In this case, the adjustment

[6] More recently, several studies have examined state-dependent models with two consumption components, one freely adjustable and one that is costly to adjust (Flavin and Nakagawa 2003, Fratantoni 2003, Li 2003).

[7] Fudenberg and Levine (2006) introduce an explicit planning phase into a dual-self model of impulse control.

costs can be interpreted as the psychological cost of changing plans or the contemplation costs required to make new choices (Ergin 2003).

The agent has two types of investment opportunities: a risk free asset with constant return $R_f > 0$, and a pool of risky assets with a stochastic net return vector R_{t+1}. The agent also earns labor income $Y_t \geq 0$ in period t. Let $h_t = \{Y_0, R_0, ..., Y_t, R_t\}$ describe the history of income and return realizations up to time t. We assume that there exists a state variable $\omega_t \in R^n$ that summarizes expectations at time t about the future distribution of shocks. Formally, the distribution of $(Y_{t+u}, R_{t+u})_{u=1}^{\infty}$ conditional on time t is given by a joint density $f(...y_u, r_u, ...|\omega_t)$ which is a smooth function of ω_t. We assume that this joint density has full support on $[0,\infty) \times ... \times [0,\infty)$, so that earnings and net returns arbitrarily close to zero cannot be ruled out with certainty. Finally, assume that ω_t is a smooth function of the history h_t.

The agent's dynamic budget constraint is

$$W_{t+1} = (1 + R_{p,t+1}) \cdot [W_t - x_t - a_t - 1\{x_{t-1} \neq x_t\} \cdot \lambda(x_{t-1}, x_t)] + Y_{t+1}$$

where W_t is wealth in period t and R_p is the portfolio return given the agent's investment choices. When x is a durable good, this budget constraint assumes that rental payments are made period by period; this assumption is equivalent to having a fixed lump-sum payment in the absence of borrowing constraints.

Definition 1 *Given a set of choice objects C, a utility function U exhibits reference dependence if for each choice object $c \in C$ and reference point $x \in C$ it associates a utility. The reference point at time t is pre-determined if it is determined at time $t-1$.*

Let $v(c,x) = u(c-x,x)$ be the period indirect utility over total consumption $c = x + a$ given commitments x. We now show that this indirect utility over consumption expenditure c exhibits reference-dependence for a range of realizations, where dollar commitments x_{t-1} act as the pre-determined reference point.

Theorem 1 *[Adjustment costs generate reference-dependent preferences] If $\lambda_1 < 1 + 1/R_f$, then*

(i) $\partial^2 v(c,x)/\partial c \partial x > 0$: holding fixed total consumption c, higher commitments x increase marginal utility.

(ii) Reference point is pre-determined for small wealth shocks but abandoned for sufficiently large wealth shocks: in the optimum, given x_{t-1}, there is an open set of (W_t, ω_t) where $x_t = x_{t-1}$;

but for any fixed ω_t, if W_t is close enough to zero or infinity, $x_t \neq x_{t-1}$.

(iii) Reference point is determined by recent expectations: If the consumer adjusts commitments at time t, then there is a function F such that $x_t = F(W_t - \lambda_1 x_{t-1}, \omega_t)$.

(iv) Reference point and recent consumption are determined by the same parameters: if the consumer last adjusted at date $s < t$, then $x_t = F(W_s - \lambda_1 x_{s-1}, \omega_s)$, while there exists a G function such that $c_s = G(W_s - \lambda_1 x_{s-1}, \omega_s)$.

Part (i) of this theorem establishes that preferences are reference dependent in the sense that marginal utility (and hence also total utility) is evaluated relative to the current level of commitment consumption x_t. Intuitively, when x_t is high, a given drop in total consumption c_t results in a larger proportional reduction in a_t, driving up marginal utility more quickly. As a result, in periods when the consumer does not adjust x_t, it acts as a pre-determined reference point.

Parts (ii)-(iv) of the theorem characterize the determinants and evolution of the commitment-based reference point. Part (ii) shows that commitments are adjusted for big, but not for small shocks, consistent with the prior literature on (S,s) models. In the context of reference dependence, this result implies that x_{t-1} acts as a pre-determined reference point when shocks are small, but becomes irrelevant for sufficiently large shocks. The model thus endogenously predicts that the importance of the reference point diminishes for large fluctuations, a feature which Koszegi and Rabin (2006) assume by postulating that utility has two parts, one reference dependent and the other neoclassical.

Part (iii) shows that when the consumer updates, the choice of commitments is determined by current wealth and expectations ω_t. In periods where x_t is not changed, the current reference point is determined by expectations in the period where the reference point was last adjusted. Thus, the commitment-based reference point is set in a forward-looking manner, as postulated by Koszegi and Rabin (2006, 2008, 2009). Because agents are generally more likely to have adjusted commitment consumption in the recent past than the distant past in an environment with shocks, this result suggests that the current reference point is largely determined by recent expectations.

Finally, part (iv) shows that the commitment based reference point is also connected to past consumption levels, as in backward-looking models of reference dependence. Intuitively, the permanent income hypothesis implies that expectations and permanent income were also the determinants of total consumption in the recent past. Because both the reference point and recent consumption levels are determined by permanent income in the recent past, the reference point is connected to past consumption, in a manner similar to models of habit formation (Constantides 1990, Campbell and Cochrane 1999).

This theorem establishes that adjustment costs endogenously generate a reference point that captures many of common intuitions about how the reference point. These features of the reference point are exogenously assumed in existing theories of reference dependence. The key benefit of the adjustment cost microfoundation for the reference point is that one does not need to take an ad hoc stance on how the reference point is determined and can instead characterize its evolution simply by specifying the adjustment cost.

In the rest of the paper, we provide an explicit characterization of the dynamics of the reference point under specific functional form assumptions about utility and the stochastic processes. We use this characterization to establish a tight connection between our model and habit formation preferences and to contrast the implications of the two models in various applications.

3 Aggregation in a Heterogenous Agent Economy

To facilitate the characterization of dynamics, we consider an economy populated by many consumers. In a heterogenous agent economy, individual lumpy adjustments are smoothed out, allowing us to focus on the "average" dynamics of the reference point. In addition, this approach enables us to contrast the results with habit formation, which is generally used to study the behavior of macroeconomic aggregates.

3.1 Setup

To study aggregate dynamics, we specialize the model in two ways: we switch to a continuous time setting and assume specific functional forms for utility and the stochastic processes governing asset prices. Consider an economy inhabited by a continuum of consumers, each of whom maximizes expected lifetime utility given by

$$\mathrm{E}\int_0^\infty e^{-\rho t}\left(\kappa\frac{{a_t}^{1-\gamma}}{1-\gamma}+\frac{{x_t}^{1-\gamma}}{1-\gamma}\right)\mathrm{d}t \tag{1}$$

where ρ is the discount rate and κ measures the relative preference for adjustables. When $\kappa \to \infty$ this framework converges to the standard setting with no adjustment costs, and when $\kappa = 0$ we obtain a model with only commitment consumption, as in Grossman and Laroque (1990). Because utility is time-separable, γ measures the elasticity of intertemporal substitution as well as relative risk aversion for an individual who is free to adjust both x and a. We use this functional form for tractability; we believe that the intuition behind our main results extend to more general specifications.[8]

As above, we assume a constant riskfree return and let $r = \log(1 + R_f)$ denote the instantaneous interest rate. We allow two types of risky investments, both with i.i.d. returns. The instantaneous return of the stock market is

$$\frac{dS_t}{S_t} = (r + \pi)dt + \sigma_t dz_t \tag{2}$$

where z_t is a standard Brownian motion that generates a filtration $\{\mathcal{F}_t, 0 \leq t < \infty\}$, π is the expected excess return, and σ is the standard deviation of asset returns. Households also face idiosyncratic risk in the form of a household-specific risky investment opportunity. This background risk can be thought of as entrepreneurial investment or labor income risk (where "investment" is investment in human capital). The return of household i's entrepreneurial investment is given by

$$\frac{dS_t^E}{S_t^E} = (r + \pi_E)dt + \sigma_E dz_t^i$$

where the z^is are standard Brownian motions uncorrelated across households. Each household is free to invest or disinvest an arbitrary amount into her private asset at any time.

As above, consumers can change consumption of x_t at any time, but must pay a transaction cost $\lambda_1 x_{t-} + \lambda_2 x_t$ to do so. We let $\overline{\lambda}_i = \lambda_i r$ denote the transaction cost as a share of the present value of the commitment good x/r.

3.2 Household Behavior

The optimal adjustment policy of each household in this class of models can be written in terms of an (S,s) band over a state variable x/w, where w stands for household wealth (Flavin and Nakagawa 2008). For our purposes here, it proves useful to describe the household's policy for x in terms of an (S,s) band over x/a instead of x/w. Intuitively, a_t is a more convenient scaling variable because it adjusts immediately and fully to shocks, while the dynamics of w_t also takes into account expenditures on commitments which respond sluggishly. For this reason, a_t is usefully thought of as a measure of the permanent income of the agent. To obtain an (S,s) band for x/a, note that adjustable consumption a_t is a strictly increasing function of current wealth w_t, and hence the consumption function $a_t(w_t, x)$ can be used to map the (S,s) band over wealth into an (S,s)

[8] For example, we have shown that Cobb-Douglas preferences also permit a habit representation result. In that case, the representative consumer has proportional habit utility (as in Abel, 1990) over adjustable consumption.

band over adjustable consumption for any given x. Since utility is homogenous of degree $1-\gamma$, the (S,s) band can be written in terms of x/a.

Define $y = \log(x/a)$. Then each household's optimal policy for the committed portion of consumption can be described by three numbers, $\{L, U, M\}$. For $y \in (L, U)$, the household does not adjust x from its prior level; as soon as y reaches L or U, the household resets x so that $y = M$. Since households have identical preferences and the model is scalable in wealth, the numbers $\{L, U, M\}$ do not vary across households in the economy. However, because of the idiosyncratic noise, households have different values of y in general – that is, they will be in different locations within their (S,s) bands. The non-degenerate cross-sectional distribution of y at each time t yields a smooth path for aggregate consumption because only a small fraction of households adjust x in response to a shock.

Given $\{L, U, M\}$, we can completely characterize the behavior of a household by the dynamics of a_t. The appendix shows that $\log a_t$ is a random walk with drift that satisfies

$$d \log a_t^i = \mu_a \cdot dt + \frac{\pi}{\gamma\sigma} \cdot dz_t + \frac{\pi_I}{\gamma\sigma_I} \cdot dz_t^i$$

where μ_a is the constant mean growth rate. The second and third terms measure how permanent income a_t responds to aggregate shocks dz_t and idiosyncratic shocks dz_t^i. Motivated by this expression, we define aggregate and idiosyncratic consumption risk by $\sigma_A = \pi/(\gamma\sigma)$ and $\sigma_I = \pi_E/(\gamma\sigma_E)$. These variables measure the standard deviation of adjustable consumption due to aggregate and idiosyncratic risk, respectively. Let $\sigma_T^2 = \sigma_A^2 + \sigma_I^2$ measure total consumption risk.

3.3 Preferences of the Representative Consumer

We now show that aggregate dynamics in the adjustment cost model coincide with those of a single-agent economy where aggregate commitments act as a habit-like reference point for the representative consumer. Let capital variables denote unconditional aggregates, so that X_t, A_t, and C_t denote aggregate commitment, adjustable, and total consumption at time t.

Proposition 1 *The aggregate dynamics of consumption are the optimal policy of a representative consumer with external habit formation utility*

$$E \int_0^\infty e^{-\delta t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} dt \tag{3}$$

as long as the discount rate $\delta = \rho - \frac{\pi_I^2}{2\sigma_I^2}\left(1 + \frac{1}{\gamma}\right) > 0$*, where* X_t *follow the dynamics of aggregate commitments.*

This result is an analogue of part (i) of Theorem 1 for the aggregate economy, establishing that marginal utility of the representative consumer is increasing in the aggregate commitment stock. The intuition for the existence of a representative consumer is that idiosyncratic shocks cancel out in the aggregation, as in Grossman and Shiller (1982). However, the presence of idiosyncratic risk increases both the mean and the variance of household consumption growth. To compensate for the increase in mean consumption growth in the aggregate, the representative consumer must be more patient than the individual households.

An interesting implication of Proposition 1 is that the functional form for the utility of the representative consumer obtained here is identical to the commonly used "additive habit" specification in the literature (Constantinides, 1990, Campbell and Cochrane, 1999). Hence, the only observational difference in the aggregate between the commitment model and habit formation models comes from the dynamics of the reference point itself. In the next section, we characterize the dynamics of the aggregate reference point X_t in the representative agent economy to evaluate whether the assumptions made in the existing literature are consistent with an adjustment-cost microfoundation.

4 Dynamics of the Reference Point

The goal of this section is to characterize the aggregate dynamics of the commitment-based reference point. We start out by connecting aggregate dynamics to the evolution of the cross-sectional distribution of households, and use this to write aggregate commitments as a moving average of past shocks, with state-dependent weights. We then show that models where the reference point is an average of past consumption, such as habit formation, admit a similar moving average representation with state-independent weights. This leads us to our main result, which is to identify conditions under which the commitment model produces state-independent weights, establishing a partial equivalence with habit formation models. Finally, we present an illustrative calibration to show how this result can be used to guide specification of habit weights if one believes that fundamental source of habit is adjustment costs.

4.1 Evolution of Aggregate Commitments

Characterizing the dynamics of X_t requires keeping track of where households are in their (S,s) bands. Using the interpretation that a_t measures the permanent income of an agent, define $f(y,t) = a_t(y)/A_t$ to be the share of aggregate adjustable consumption at time t accounted for by households i for whom $y_t^i = y$. The density $f(y,t)$ can be interpreted as the cross-sectional distribution of permanent income in the (S,s) band, measured relative to aggregate permanent income.

Our first aim is to characterize how $f(y,t)$, which summarizes the state of the economy on date t, evolves over time. Let μ_A denote the instantaneous drift of A_t.

Proposition 2 *$f(y,t)$ satisfies the stochastic partial differential equation*

$$df(y,t) = \left[\left(\mu_A + \frac{\sigma_I^2}{2}\right)\frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2}\frac{\partial^2 f(y,t)}{\partial y^2}\right]dt + \sigma_A \frac{\partial f(y,t)}{\partial y}dz \tag{4}$$

together with the following boundary conditions:

$$\frac{\partial f(M,t)}{\partial y}^+ - \frac{\partial f(M,t)}{\partial y}^- = \frac{\partial f(U,t)}{\partial y}^- - \frac{\partial f(L,t)}{\partial y}^+$$
$$f(U,t) = f(L,t) = 0 \text{ and } f(M,t)^+ = f(M,t)^-.$$

Aggregate commitments follow the dynamics

$$dX_t = A_t \frac{\sigma_T^2}{2} \cdot \left(f_y(L,t)(e^M - e^L) + f_y(U,t)(e^U - e^M)\right) dt. \tag{5}$$

This result is based on Propositions 1 and 2 in Caballero (1993). The first statement follows from the Kolmogorov forward equation, which characterizes the evolution of marginal distributions in the presence of aggregate shocks. In our situation, the Kolmogorov equation cannot be applied directly, because $f(y,t)$ is not a marginal distribution, since it is weighted by A_t. To deal with this, we introduce a new probability measure Q that weights the sample path of y_t for each agent by his consumption share a_t/A_t. Under Q, we obtain $f(y,t) = \Pr_Q\left[y_t^i = y | A_{[0,t]}\right]$, i.e., f can be written as the marginal distribution of y under this new measure. We can now apply the Kolmogorov equation; we just need to note that by the Girsanov theorem, the evolution of y under Q is equivalent to changing the drift of the underlying Brownian motion z_t^i. Intuitively, Q means that we should assign higher weight to households with high a/A: this turns out to be equivalent to having a higher mean growth rate. With the new drift, the Kolmogorov forward equation immediately implies (4). The boundary conditions follow as in Caballero (1993).

Equation (5) shows that the evolution of commitments is smooth in the aggregate in the sense that it is a bounded variation process (has no dz term). Intuitively, the cross-sectional densities go to zero near the boundary of the (S,s) band. The total mass agents who adjust in response to an aggregate shock of size dz is proportional to the area under the density at the boundaries, which is of order $(dz)^2 = dt$.

It is well-known that the distribution of y under Q converges to a unique steady-state distribution f^*, which is also the time-invariant solution of (4) in the case where there is no aggregate noise ($dz = 0$). It follows that this f^* is also the long term expected cross-sectional distribution of the commitments model. In the presence of aggregate shocks, the actual cross-sectional distribution f is constantly perturbed relative to f^*, but in the long term the system returns to f^* in expectation. Figure 1 illustrates this result by plotting the steady-state distribution f^* in two environments: one with high aggregate risk and low idiosyncratic risk and the other with low aggregate and high idiosyncratic risk. The bottom panels show the actual cross-sectional distribution sampled twenty times from simulating the two environments. The actual distributions are generally quite similar in shape to the steady state distribution. The similarity is particularly strong when idiosyncratic risk is high relative to aggregate risk, a result that we revisit and explain below.

4.2 Impulse Responses and a Moving-Average Representation

To connect the dynamics of X_t to the dynamics of reference points in existing exogenous specifications of the reference point, we introduce a moving average (MA) representation for X_t. Define the de-trended processes $\overline{A}_t = e^{-\mu_A t} A_t$, which is a martingale, and $\overline{X}_t = e^{-\mu_A t} X_t$.

Definition 2 *The impulse response function of the commitments model in state f is the function*

$$\xi\left(t|f\right) = \frac{\partial E_0\left[\overline{X}_t|f\right]}{\partial \overline{A}_0}.$$

The function $\xi\left(t|f\right)$ measures the effect of a shock to aggregate permanent income on consumption commitments t periods later, given that the initial cross-sectional distribution is given by f. The Appendix shows that $\xi\left(t|f\right)$ is well-defined. Impulse responses depend on the initial distribution f: when many households are on the verge of downsizing, a negative aggregate shock will reduce commitments at a faster rate. Figure 2 plots impulse-responses in our model in four environments (assuming $f = f^*$). As $t \to \infty$, these impulse responses converge to a limit which is normalized to one in the figures, which corresponds to full adjustment to the initial aggregate shock. For finite t, the figures indicate partial adjustment. Higher risk leads to more rapid convergence, as commitments are updated more quickly.

The impulse response function allows us to write X_t as an explicit function of past aggregate shocks.

Proposition 3 *De-trended aggregate commitments admit the moving average representation*

$$\overline{X}_t = \int_0^t \xi\left(t-s, f\left(s\right)\right) d\overline{A}_s + E_0 X_t. \tag{6}$$

This result follows from Ito's lemma combined with the observation that the impulse response function is smooth. Equation (6) is a MA representation of commitments, where the MA coefficients $\xi\left(t-s, f\left(s\right)\right)$ depend on the state of the economy through f.

4.3 State-Independent MA Representation of Reduced-Form Habit

A leading special case of the moving-average representation in (6) is where the weights ξ are state-independent. This is an environment where the impulse response to a shock does not depend on history. We now show that this special case coincides with reduced-form habit models in which X_t is specified as an average of past consumption with fixed weights. To see why, suppose that in the representative agent economy where external habit preferences are given by (3), the habit stock is exogenously determined as

$$X_t^h = o^h(t) X_0^h + \int_0^t \zeta^h(t-s) C_s ds \tag{7}$$

with fixed weights ζ^h and o^h. In the optimum, this representative agent chooses consumption so that his surplus consumption $A_t = C_t - X_t$ follows the same path as aggregate adjustables in the commitments model – this observation forms the basis of the proof of Proposition 1. Thus we can continue to think of A_t as capturing aggregate shocks to permanent income.

Lemma 6 in the appendix shows that the habit representation (7) can be written as

$$X_t^h = \int_0^t \theta(t-s) A_s ds + \theta_0(t) X_0^h \tag{8}$$

where habit is now the weighted average of A_t. This is because C, X and A are linked by an accounting identity, and hence any linear representation of X in terms of C can also be written as a linear representation in terms of A. Detrending the habit variables by the mean growth rate μ_A, i.e., letting $\overline{X}_t^h = e^{-\mu_A t} X_t^h$ and then integrating by parts, (8) implies that

$$\overline{X}_t^h = \int_0^t \xi^h (t-s) \cdot d\overline{A}_s + E_0 \overline{X}_t^h \tag{9}$$

where $\xi^h(u) = \int_0^u e^{-\mu_A v} \theta(v)\, dv$. Equation (9) establishes a MA representation for the detrended habit stock with state-independent weights in the reduced-form habit model.

4.4 Equivalence Result: A Fixed-Weight Representation in the Commitments Model

The results above imply that the only difference between reduced-form habit models and the commitment-based model comes from the state-dependent nature of impulse-responses in the latter case. Hence, the commitment model of reference dependence is identical to existing habit models when the evolution of aggregate commitments is determined by a fixed-weight average of past consumption. We establish the main equivalence result of the paper by showing that aggregate commitments evolve approximately according to a fixed weight specification when the ratio of idiosyncratic to aggregate risk is high. To establish this result, we first introduce a fixed-weight model that generates reference point dynamics that match the evolution of commitments on average.

Definition 3 *A habit representation X_t^h as given by (8) matches the steady state impulse response of commitments if $\partial E_0 \overline{X}_t^h / \partial A_0 = \xi(t|f^*)$ for all t.*

The impulse-response in the commitments model, while state-dependent, generates on average – that is, in steady state f^* – the same impulse response as the habit model defined to match it.[9] Because matching the impulse response pins down all MA coefficients $\xi^h(u) = \xi(u, f^*)$ in the representation (9), there exists a unique fixed weight habit model that satisfies the definition. We denote the corresponding impulse response weights by $\xi^* = \xi(u, f^*)$ and the habit model generated by them as X_t^{h*}.

Since both idiosyncratic (σ_I) and aggregate (σ_A) consumption risk are endogenous in our model, to state the result, we need to look at sequences of exogenous parameters where the implied ratio $\sigma_I/\sigma_A \to \infty$. We consider a sequence of models Θ_n with the property that as $n \to \infty$, 1) $\sigma_I/\sigma_A \to$

[9] An alternative definition would be to choose fixed weights that generate dynamics which minimize a mean-squared error. All of our formal results hold for that specification as well (proofs available upon request), and in simulations the differences between the two models are small. Here we develop the model that matches the average impulse-response because it is easier to analyze.

∞; 2) γ, κ and $\overline{\lambda}_i$ remain fixed; 3) r stays bounded away from zero; 4) μ_A remains bounded; and 5) r/ρ is bounded away from zero and infinity.

Theorem 2 *For any sequence of models Θ_n specified above and any $p \geq 1$,*

$$\limsup_t \left\| \frac{X_t - X_t^{h*}}{A_t} \right\|_p = o\left(\frac{\sigma_A}{\sigma_I}\right).$$

The small order $o(.)$ on the right hand side means that commitments converge rapidly to the fixed-weight specification: the distance between the two models becomes an arbitrarily small share of σ_A/σ_I when this ratio goes to zero. For example, on a sequence where $\sigma_A \to 0$, the theorem says that the difference between commitments and a fixed-weight representation goes to zero even *relative to* σ_A. When the size of aggregate shocks shrinks, the difference between the two models becomes small even compared to the these shocks. Similarly, when the magnitude of idiosyncratic risk grows, the distance between the two models goes to zero at a rate that is faster than the growth in σ_I.

Simulations presented in Figure 3 illustrate the theorem.[10] The figures plot the evolution of X_t, the aggregate habit stock, in the commitments model and the habit representation that is the best match for the commitments model. The figures have σ_I and σ_A equal to either 5% or 10%, resulting in four different environments. We choose the other parameters to keep μ_A fixed, and use the same sequence of shocks in all cases. Figure 3a plots commitments (red) and habit (green) paths in these four environments, while the blue curve represents the evolution of permanent income (A_t). The main lesson from this figure is that the difference between the two models is small in all four scenarios. The fixed-weight reduced-form habit model approximates the commitment model very well even when σ_I/σ_A is not very high. The two models are closer when idiosyncratic risk is higher (right panels) and when aggregate risk is lower (bottom panels). This point is illustrated more saliently in Figure 3b, which plots the ratio X_t^{h*}/X_t and shows that it is closer to 1 when σ_I/σ_A is larger.

The general intuition underlying Theorem 2 is that when consumers face a high degree of idiosyncratic risk, the cross-sectional distribution is usually close to it's steady state. Hence aggregate shocks generate the same pattern of adjustment on most dates, like in a habit formation model. The proof of the theorem involves several technical steps that are presented in the appendix. The main concept is to analyze both models using their MA representations. Differencing (6) and (9)

[10] The parameters used for these figures are given in Appendix B.

yields

$$\overline{X}_t - \overline{X}_t^{h*} - E_0\left[\overline{X}_t - \overline{X}_t^{h*}\right] = \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot d\overline{A}_s$$
$$= \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot \sigma_A \cdot \overline{A}_s dz_s$$

where we use $d\overline{A}_s = \overline{A}_s \sigma_A dz_s$. Focusing on the final integral, consider a sequence of models where the level of aggregate risk $\sigma_A \to 0$. Since the integrand involves σ_A, its value goes to zero for each s – as aggregate shocks become smaller, both models will stay close to their unconditional expectation. But the equation also reveals an important additional effect. As σ_A becomes small relative to σ_I, much of the shock each household experiences is idiosyncratic. This pushes the cross-sectional distribution f close to its steady state f^*. This is because the force pushing for convergence toward f^*, which is proportional to σ_I for reasons explained above, becomes stronger relative to the force of divergence, which is determined by σ_A. This mechanism is labeled the "attractor effect" by Caballero (1993). As result, f and f^* are close in most periods, as illustrated above in Figure 1. This in turn implies that $\xi^*(t-s) - \xi(t-s, f(s))$ is typically small: when the system is close to the steady state, the impulse response is also close to its steady state shape. Thus $X - X^h$ is on average small even relative to σ_A.

A similar argument applies when $\sigma_I \to \infty$. Higher idiosyncratic risk implies that adjustment takes place more frequently for each household. As a result, aggregate shocks are absorbed by commitments at a faster rate, so that the amount of aggregate risk that accumulates during a typical non-adjustment period is small. This mechanism reduces the distance between the two models at a rate proportional to $1/\sigma_I$. At the same time, a high σ_I/σ_A term also implies that the cross-sectional distribution is usually close to its steady state, so that absorption of these aggregate shocks follows the same pattern at all times, explaining why convergence is faster.

The mechanism driving the rapid convergence of the commitment model to fixed-weight habit is illustrated in the bottom panel of Figure 1. As noted above, there is much more "variance" in the evolution of the cross-sectional distribution in the left panel (low σ_I/σ_A), because the forces of divergence are stronger. This creates fluctuations in the impulse-response across periods, and hence behavior that diverges from a fixed-weight habit model. In contrast, the cross-sectional density varies much less in right panel. As a result, the impulse-responses are approximately constant, creating aggregate consumption dynamics that are very close to those produced by a fixed-weight habit specification.

We believe that the special case where σ_I/σ_A is large is not just of theoretical interest but may also be the most empirically relevant scenario. Empirical evidence indicates that idiosyncratic labor income and entrepreneurial risk is generally much greater in magnitude than economy-wide shocks. Coupled with the illustrative calibrations in Figure 3, this suggests that in practice, the dynamics of the reference point the commitment model are most of the time well approximated by state-independent weights. Hence, existing models of habit formation are likely to be a good reduced-form method of describing aggregate behavior if reference dependent behavior arises from adjustment costs.

4.5 Illustrative Calibration of Habit Weights

One benefit of deriving the habit formation model from a microeconomic model of adjustment costs is that one can calibrate the weights used to define the evolution of the habit stock by specifying the adjustment cost.

As an illustration, Figure 4 compares the habit weights implied by the commitments model with the exponential habit specification of Ryder and Heal (1973) and Constantinides (1990). The green solid line shows the log of these exponential consumption weights using the calibration of Table 1, column 5 in Constantinides (1990). The blue dashed line shows the equivalent log habit weights from the low aggregate, high idiosyncratic risk environment of Figure 3. The relative preference for commitments κ was calibrated so that the sum of weights is the same across the two models; this implies that the reference point is in the range of 80% of total consumption in both environments.

The figure reveals two differences between these habit sepcifications. First, the commitment-based model places larger weight on the recent past (last two quarters). The intuitive reason is that in the commitment environment, there are always some households at the boundary of the inaction region; in response to a shock these agents update immediately, and when they do, they change their consumption discontinuously, resulting in quick initial adjustment in the aggregate. Second, the commitment model has lower consumption weights for intermediate horizon of up to about 3 years. This is because the mass of households in the middle of the inaction region need a longer time to update.

These features suggest that the habit dynamics of the commitments model forms a middle ground between reduced-form specifications where habit equals last period's consumption, such as the macroeconomic models of Fuhrer (2000) or Christiano, Eichenbaum, Evans (2005), and the exponential weight models used in finance, like Constantinides (1990).

5 Applications

Thus far, we have established that adjustment costs provide foundations for the reference-dependent models currently used in the literature. As shown above, these results facilitate calibration of existing reduced-form models of reference-dependent preferences. But perhaps a more important lesson is that the endogenous reference points generated by our model lead to different predictions in applications. We demonstrate this lesson by comparing the predictions of our micro-founded model of habit and reduced-form habit models in some common applications.

5.1 Consumption Dynamics

Two well-documented features of aggregate consumption behavior are excess sensitivity and excess smoothness to shocks (Flavin 1981, Deaton 1987). Excess smoothness means that consumption fails to respond to contemporaneous shocks to the extent predicted by the permanent income model. Excess sensitivity is the idea that current consumption responds to past shocks to permanent income. A major reason for using habit preferences in applied macroeconomic models is that they generate such delayed consumption responses (Fuhrer 2000). In this section, we show that adjustment-cost based foundations for reference points also create these patterns, but with some important caveats that challenge existing results.

Consider the following linear regression specification for consumption growth:

$$\log\left(C_{t+\Delta t}\right) - \log\left(C_t\right) = \alpha + \beta_1 \cdot \left[\log A_t - \log A_{t-\Delta t}\right] + \varepsilon. \tag{10}$$

Definition 4 *Consumption is excessively smooth if* $\beta_1 < 1$ *for some* $\Delta t > 0$*.*

Next, let $s_1 < s_2$ and consider the regression

$$\log\left(C_{t+s_2}\right) - \log\left(C_{t+s_1}\right) = \alpha + \beta_2 \cdot \left[\log A_t - \log A_{t-\Delta t}\right] + \varepsilon. \tag{11}$$

Definition 5 *Consumption is excessively sensitive if there exist* $0 < s_1 < s_2$ *and* $\Delta t > 0$ *such that* $\beta_2 > 0$*.*

Proposition 4 *In the commitments model, consumption is both excessively sensitive and excessively smooth:* $\xi^*(0) = 0$*,* $\lim_{t\to\infty} \xi^*(t) = \overline{x}$ *and* ξ^* *is continuous.*

In our model, excess smoothness is captured by the result that the impulse response $\xi^*(0) = 0$: commitments do not respond at all to instantaneous shocks. In contrast, in a model with instantaneous response to aggregate shocks (i.e., if X were immediately adjustable), $\xi^*(0)$ would be equal to $\overline{x}$. Excess sensitivity follows because for any s, by continuity $\xi^*(s+t) - \xi^*(s) > 0$ for all t sufficiently large. Since $\xi^*(s+t) - \xi^*(s)$ is the effect of an initial shock on the growth in X between s and $s+t$, a positive value indicates delayed effects. These delayed responses are illustrated in Figure 2 above, which plots the impulse response ξ^* normalized by $\overline{x}$.

Large shocks and changing correlations. While both commitments and habit generate sluggish dynamics on average, this similarity breaks down in extreme times, when households abandon commitments and no longer reference-dependent preferences. This can also be seen in Figure 3, which shows that on dates with large permanent income shocks, there are differences of several percentage points between the two models. To illustrate, fix $\Delta A > 0$ and $\Delta t > 0$, and consider the event $B(\Delta A)$ in which $\left|\log \overline{A}_{t+\Delta t} - \log \overline{A}_t\right| \geq \Delta A$. This event represents realizations where aggregate permanent income changes by more than ΔA during a Δt time period. We focus on the case when Δt approaches zero, holding fixed ΔA; since A is driven by a Brownian motion with infinitesimal shocks, this implies an unusually large shock per unit of time. In the extreme when $\Delta t = 0$, shocks of this sort would correspond to discontinuous jumps in permanent income.

Proposition 5 *(i) Greater adjustment of reference point in commitment model than habit model:* $\lim_{\Delta t \to 0} E\left[\left|X_{t+\Delta t} - X_t\right| \mid B(\Delta A)\right] > \lim_{\Delta t \to 0} E\left[\left|X^h_{t+\Delta t} - X^h_t\right| \mid B(\Delta A)\right] = 0$.

(2) If (10) is estimated around an extreme event $B(\Delta)$, as $\Delta t \to 0$, the coefficient estimate $\beta_{\Delta t} > 0$ in the commitment model whereas $\beta_{\Delta t} \to 0$ in the habit model.

Part (i) of this proposition compares the expected change in the reference point in the commitment and habit models conditional on a big shock. The short-term response of reduced-form habit is zero: in habit models the impulse-response is completely state-independent and hence does not differ depending on the size of the shock. In contrast, in the commitment model, the response is positive, because many households choose to adjust during a short period. As (ii) shows, this implies that with commitments, the correlation between consumption and permanent income changes in extreme events. As shown above, $\beta_{\Delta t}$ estimated during a representative sample of data is zero. But in an extreme event $B(\Delta A)$, to coefficient turns positive with commitments as a positive measure of people adjust. This does not happen in the habit model, where impulse responses are identical in normal and extreme times.

These changing correlations have implications for both theoretical and empirical studies of stabilization policy. Building on Fuhrer (2000) and Christiano, Eichenbaum and Evans (2005), habit specifications are a now a standard piece of equilibrium models used to study monetary policy. If adjustment costs are responsible to the sluggish dynamics that are typically modeled using reduced-form habits, our results indicate that the predictions of these models will be invalid in extreme periods. Since periods of crisis are of greatest interest from a policy perspective, models based on reduced-form habits may not be useful tools to evaluate monetary policy. As a counterpart to these models, empirical work often relies on linear vector autoregressions (VARs) estimated using historical time series to evaluate policies. The proposition shows that the coefficients of these linear models can break down in crises, when people make costly adjustments to abandon their reference points.

5.2 Welfare Costs of Shocks

Thus far, we have focused exclusively on positive differences between the adjustment-cost based and other models of reference points. We now turn to explore differences in normative implications of reduced-form habit models and our adjustment-based model of reference dependence. One benefit of the adjustment-cost based reference point model is that it offers a natural welfare measure. In contrast, in habit and prospect theory models, the appropriate measure of welfare is open to debate – for instance, should habit consumption be included in welfare calculations?

The differential response to large shocks of the commitment and reduced-form habit models also implies different risk preferences and, by extension, differences in the welfare cost of shocks. To demonstrate this, consider the effect of a wealth shock that hits with probability p and reduces total wealth by a share b. Suppose that in the commitment economy, this shock affects all agents, while in the habit economy it affects the representative agent. Define the risk premium to be dollar amount that agents in either economy are willing to give up in excess of the expected value of the gamble to avoid this risk. The proportional risk premium is the risk premium normalized by total wealth in the economy.

Proposition 6 *If $\lambda_1 = 0$ but $\lambda_2 > 0$, as $b \to 1$,*

(i) the proportional risk premium in the reduced-form habit economy exceeds that in the commitment economy

(ii) the portfolio share of stocks in the commitment economy following the negative shock exceeds

that in the habit economy.

Part (i) of this result implies that habit agents are more averse to large shocks than are commitment agents. Intuitively, in response to a big shock, the reference point in the commitment model adjusts immediately, mitigating the impact of the shock. In contrast, reduced-form habits adjust sluggishly for shocks of all sizes by assumption, and thus agents suffer greatly when hit by a large shock in such models.[11] These differences in the welfare cost of shocks imply differences in the risk appetites of the commitment and the habit economies following a negative shock. Intuitively, the reason why habit agents are hurt so much by a large shock is that marginal utility goes up very quickly as the size of the shock increases. This also implies that following a large negative shock their marginal utility is highly curved, and hence they are very averse to any additional risk, which means that they reduce their stockholdings. Conversely, in the commitment economy after a big negative shock everybody readjusts, and the stock share of total wealth is the same as before the shock.

Figure 5 illustrates these results by plotting the ratio of the proportional risk premium in the habit and the commitment model as a function of shock size b, when $p = 1\%$. The blue line is for an environment with low idiosyncratic risk. As shown before, here consumption responds sluggishly to shocks, and hence the matching habit model is highly persistent. As a result, the habit agent will be much more averse to big shocks. In contrast, the green line computes the ratio of risk premia in an economy with high idiosyncratic risk. Here habit is less persistent, and hence risk premia are similar for a wide range of shocks in the two economy.

The lesson from this application is that a reduced-form habit model that matches observed dynamics of consumption well may nevertheless yield misleading conclusions about the welfare costs of shocks, particularly large shocks. Even if consumption is highly persistent for typical shocks, agents may not be extremely averse to big fluctuations, due to the ability to adjust at a fixed cost. This result can have important policy implications: for instance, the optimal size of social insurance programs that insure large, long-term shocks such as disability or job displacement may be smaller than predicted by analyses using habit models such as Ljungqvist and Uhlig (2000).

[11] The assumption that $\lambda_1 = 0$ guarantees that when moving, the commitment agents can get rid of all pre-commitments; otherwise, even when moving they would still have promised expenditures of $\lambda_1 X_{t-}$, which behave like sluggish habits. In simulations, we find that unless λ_1 is very high, the conclusion of the proposition is unaffected. Intuitively, the promised moving costs are much smaller than the habit expenditures.

5.3 Comparative Dynamics and Policy Analysis

The two applications above have shown that reduced-form habit models and a model of habit or reference dependence built from microfoundations of adjustment costs can have very different predictions for certain realizations of shocks. We now show that even when the shock processes do not feature extreme events, the two models make different predictions about the effects of changes in policy or other exogenous parameters of the system. This is because in a reduced form habit model, changes in the economy do not affect the degree of sluggishness, which is pinned down purely by the exogenous consumption weights. In contrast, with commitments, the degree of sluggishness is determined by households' optimizing behavior, and therefore responds endogenously to policy or environmental changes.

To illustrate the implications of this observation, we define a measure of the responsiveness of the reference point to shocks – $T(p,f) = \inf_t \{\xi(t|f) \geq p \cdot \overline{x}\}$ – which is the time required for the expected reference point to adjust a share p in response to a unit shock when the economy is started from a distribution f. By definition, in a reduced form habit model $T(p)$ is pinned down by the exogenous habit weights and hence does not change when other parameters are varied. Table 1 reports $T(p|f^*)$ for the commitments model for $p = .25$, $.5$ and $.75$ for different parameters. In the top panel, the adjustment cost equals one year's consumption value of the commitment good, which corresponds to 1% of its' capitalized value when the riskfree rate is 1%. The first row shows that when both $\sigma_A = \sigma_I = 10\%$ and $r_f = 1\%$, it takes on average 1.7 years for a 50% adjustment of the reference point to a shock. The next three rows illustrate the effect of reducing σ_A or σ_I, changing r_f to make sure that individual consumption growth remains unchanged in these comparisons. The table shows that reducing either idiosyncratic or aggregate risk results in more sluggish response to shocks. The intuition for this result is that higher risk forces consumers to update their commitments more frequently, resulting in faster response to aggregate shocks. Comparing the first and last rows in the top panel shows the effect of increasing consumption growth by setting a higher interest rate. This again reduces sluggishness, because higher growth results in more frequent adjustments. The bottom panel shows that when the adjustment cost is increased to five times annual consumption, the reference point responds in a more sluggish fashion, but the basic effects of changing risk and growth are preserved.

Sluggish adjustment and the welfare cost of shocks. The preceding comparative dynamics imply that aggregate shocks have longer-lasting effects in an economy with lower idiosyncratic risk, because agents have fewer opportunities to adjust for other reasons. To illustrate this point, let $V(W,x)$ denote the value function of an agent, and define the coefficient of relative risk aversion at (W,x) to be $CRRA(W,x) = -V_{ww}w/V_w$. We will use the CRRA as a measure of the consumer's aversion to shocks. Consider a sequence of economies $\Theta_n \to \Theta^*$ such that along the sequence r_f and λ are constant, all parameters bounded, and $\sigma_T = \mu_a = 0$ in the limit economy Θ^*. Suppose that at (W_0, x_0) in economy Θ^* the agent does not want to move.

Proposition 7 *(i) For any $p > 0$, $T_n(p|x_0) \to \infty$.*

(ii) If $\lambda_1 = 0$ but $\lambda_2 > 0$, then for all n, $CRRA_n(W_0, x_0) < CRRA_*(W_0, x_0)$.

Part (i) of this result shows that as risk and growth rates approach zero, the economy becomes infinitely sluggish: the expected time until adjustment takes place converges to infinity. Part (ii) shows that risk aversion is highest in the limit economy with no growth and risk. Intuitively, when σ and μ become small, agents expect to adjust infrequently, and hence the effect of all shocks is concentrated on the adjustables margin: the absence of adjustment implies that the welfare cost of shocks is high. In contrast, shocks are less costly to agents who adjust frequently for other reasons.[12] This result is related to Koszegi and Rabin's (2007) finding that the presence of background risk can reduce risk-aversion where agents have forward-looking reference points. The logic for their result is that background risk has a smoothing effect on the kink of their gain-loss utility. In contrast, our mechanism is that background risk allows for more frequent updating of commitments, reducing the welfare cost of additional risk.

A practical lesson of Proposition 7 is that recessions will be longer and possibly more costly in welfare states with large social insurance against idiosyncratic risk, because people have weaker incentives to change their plans and commitments. In contrast, an economy with more idiosyncratic risk, such as the US, responds faster to aggregate shocks, because agents update frequently for other reasons. This result suggests that expanding social insurance programs in order to reduce households' exposure to idiosyncratic risk may increase the welfare cost of aggregate shocks by making the adjustment to a given shock longer lasting. One would not obtain this result in the conventional reduced-form habit model, because the degree of social insurance would have no bearing on the weights that determine the evolution of the habit stock.

Following similar reasoning, one can show that growth-promoting policies make aggregate consumption more responsive to shocks due to more frequent changes in commitments. The general

[12] As before, $\lambda_1 = 0$ guarantees that when moving, the agent can get rid of all commitments.

lesson is that the dynamics of the commitment-based reference point respond to policy changes in systematic ways that are not captured in a reduced-form habit specification. This point is analogous to the Lucas (1976) critique, and calls for caution in using reduced form models of reference dependent preferences in policy analysis because the endogenous response of the evolution of the reference point are not taken into account.

6 Conclusion

The results in this paper provide both a critique and synthesis of the literature on reference dependent preferences and habit formation. We have built a theory of reference dependent preferences based on adjustment costs in consumption that synthesize the intuitions of existing models of reference dependence. In particular, our theory predicts that reference points are determined by recent expectations, recent consumption patterns, and become less relevant when agents face large shocks. In the special case where idiosyncratic risk is prevalent, the model provides foundations for precisely the fixed-weight habit specifications that are most widely used in the existing literature and can be used to guide the specification of weights in such models.

Although our results indicate that the preference specifications used in the existing literature can be viewed as convenient reduced forms, they also reveal that the predictions of these reduced-form models may be inaccurate in some cases. In applications such as the analysis of aggregate consumption dynamics and the welfare cost of shocks, the reduced-form habit model and the model we develop here deliver starkly different predictions in some domains of the parameter space. This is because the evolution of the reference point is endogenous to policies and the types of shocks that agents face. Hence, in some applications, it may be necessary to build a model of reference dependence from the foundations of adjustment costs rather than directly employing a convenient reduced-form. More generally, these findings underscore the importance of modeling the foundations of non-standard preferences, and serve as a call for further theoretical and empirical research on the sources of reference-dependent behavior.

Appendix A: Proofs

Proofs for Section 2

Proof of Theorem 1. (1) This follows from $u_{12} \geq 0$.

(2) At date $t-1$, x_{t-1} was weakly preferred to adjusting, given W_{t-1} and ω_{t-1}. By continuity,

x_{t-1} is strictly preferred at ω_{t-1} and a slightly higher or lower value of W_{t-1}. But then x_{t-1} is also strictly preferred at an open neighborhood, proving the first half of the statement. For the second part, consider first W_t large. Let $\overline{u}$ denote the finite upper bound of period utility. By restricting the consumer to only invest in safe assets and move into a home once and for all, he can obtain lifetime utility approaching $\overline{u}/(1-\beta)$. Denote $\lim_{a\to\infty} u(a,x) = u(\infty,x)$, which is well-defined. Note that $u(a,K) \leq u(\infty,K) < u(\infty,K+1)$ where the last inequality is strict because $u(\infty,K+1) - u(\infty,K) \geq u(1,K+1) - u(1,K)$ by $u_{12} \geq 0$. It follows that if the consumer does not move at time t, his utility will be strictly bounded away from $\overline{u}$ in period t; but then non-adjustment must be sub-optimal.

Now consider W_t small, and first assume that $\lambda_1 > 0$. If $W_t < \lambda_1 x_{t-1}/(1+R_f) + x_{t-1}$, then by not moving today, paying for commitment consumption implies that with positive probability, the consumer will not be able to afford moving at $t+1$. But financing current commitments forever has a cost of $x(1+1/R_f)$ which is higher than the cost of moving. Hence after t, with positive probability, the consumer won't be able to move and won't be able to finance existing commitments. Anticipating this, it will be optimal for him to move today. Now suppose that $\lambda_1 = 0$ but $\lambda_2 > 0$. If $W_t < x_{t-1}$ then the consumer has no choice but to move.

(3) and (4) These statements follow immediately from the fact that given adjustment on a date t, the consumer's problem is fully described by $W_t - \lambda_1 x_{t-1}$ and ω_t.

Proof of Proposition 1. Since the only risky assets for household i are S and S^i, there exists a unique stochastic discount factor in the payoff space associated with the household-specific private market. The following dynamics for adjustable consumption generates a state price density that prices both risky assets as well as the safe asset

$$a_t^i = a_0^i \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t + \frac{\pi_I}{\gamma\sigma_I}z_t^i\right\}$$

and hence must describe the optimal choice of household i. Because $a_0^i = A_0$ for all i, aggregating across i yields, by the strong law of large numbers for a continuum of agents (Sun, 1998)

$$\begin{aligned} A_t &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}\int_i \exp\left\{\frac{\pi_I}{\gamma\sigma_I}z_t^i\right\}di \\ &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2}\left(1+\frac{1}{\gamma}\right) + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}. \end{aligned}$$

Define a new discount rate $\delta = \rho - \left(1 + \frac{1}{\gamma}\right) \pi_I^2 / (2\sigma_I^2)$. Then the dynamics of aggregate adjustable consumption is given by

$$A_t = A_0 \exp\left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + r - \delta \right) t + \frac{\pi}{\gamma\sigma} z_t \right\}.$$

This is exactly the dynamics of adjustable consumption that would obtain for a representative consumer with power utility over A_t and discount rate δ who can invest in the publicly traded risky and safe assets. The existence of a representative consumer obtains even though markets are incomplete because idiosyncratic shocks cancel out in the aggregation, as in Grossman and Shiller (1982). However, the presence of idiosyncratic risk increases both the mean and the variance of household consumption growth. To compensate for the increase in mean consumption growth in the aggregate, the representative consumer must be more patient than the individual households.

Proofs for Section 3

Proof of Proposition 2. Think about the dynamics of y_t^i as being driven by two Brownian motions, z_t and z_t^i. Our goal is to study the evolution of the distribution of y^i conditional on the path of z. Let Q be the measure on the space of sample paths of y that weighs paths of y_t by their share in aggregate consumption. The advantage of the measure Q is that the probability distribution of interest $F(y,t)$ can be written simply as $F(y,t) = \Pr_Q \left[y_t^i < y | A_{[0,t]} \right]$, that is, the marginal under Q of y_t^i, conditional on the path of aggregate shocks. The probability density associated with Q is

$$\frac{dQ}{dP}|_t = \frac{a_t^i}{A_t} = \exp\left[\frac{\pi_I}{\gamma\sigma_I} z_t^i - \frac{\pi_I^2}{2\gamma^2\sigma_I^2} t \right]$$

which is an exponential martingale. By the Cameron-Martin-Girsanov theorem, under Q, the process $d\overline{z}_t^i = dz_t^i - \pi_I / (\gamma\sigma_I)\, t$ is a Brownian motion. We are interested in characterizing the evolution of conditional distribution of y_t^i given a realization of the path of A under Q. Proposition 1 in Caballero derives a stochastic partial differential equation for such conditional densities. To apply this result here, recall that

$$d \log a_t^i = \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} dz^i = \theta dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} d\overline{z}^i$$

where

$$\theta = \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) + \frac{\pi_I^2}{\gamma^2\sigma_I^2}$$

is the drift under Q. Caballero's stochastic differential equation is

$$df(y,t) = \left[\theta \frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y,t)}{\partial y^2}\right] dt + \sigma_A \frac{\partial f(y,t)}{\partial y} dz$$

and substituting in the above expression for θ gives the desired result. The boundary conditions follow directly from Caballero's proposition.

To derive the dynamics of aggregate commitments, note that $X_t = \int_L^U e^y f(y,t) dy \cdot A_t$ and we can use Ito's lemma to write

$$dX_t = A_t \int_L^U e^y \cdot df(y,t) \cdot dy + dA_t \cdot \int_L^U e^y f(y,t) dy + \left\langle \int_L^U e^y \cdot df(y,t) \cdot dy, dA_t \right\rangle.$$

We now evaluate each term on the right hand side. The first term is

$$A_t \int_L^U e^y \cdot \frac{\partial f(y,t)}{\partial y} \left\{ \left(\mu + \frac{\pi_I^2}{2\gamma^2 \sigma_I^2} \right) dt + \frac{\pi}{\gamma \sigma} dz \right\} dy + F_t \int_L^U e^y \cdot \frac{\partial^2 f(y,t)}{\partial y^2} \frac{\sigma_T^2}{2} dt \cdot dy.$$

Integrating by parts, and using the boundary conditions shows that this term equals

$$-X_t \left(\left(\mu + \frac{\pi_I^2}{2\gamma^2 \sigma_I^2} \right) dt + \frac{\pi}{\gamma \sigma} dz \right) + A_t \frac{\sigma_T^2}{2} \cdot \left(f_y(L,t)(e^M - e^L) + f_y(U,t)(e^U - e^M) \right) dt + \frac{\sigma_T^2}{2} X_t dt.$$

The second term is

$$X_t \cdot \frac{dA_t}{A_t} = X_t \left(\left(\mu + \frac{\pi^2}{2\gamma^2 \sigma^2} \right) dt + \frac{\pi}{\gamma \sigma} dz \right)$$

while the third term is simply $-\pi^2 / (\gamma \sigma)^2 X_t dt$. Collecting terms gives the result of the proposition.

Proofs for Section 4

We present a serious of Lemmas and arguments that build up to the proof of Theorem 2 as well as the other results in Section 4. For Theorem 2, we will focus on a sequence Θ_n along which $\sigma_A \to 0$ for much of the proofs; in the end, we will show how to convert this result to a limit where $\sigma_I \to \infty$ with a simple clock change. Along the sequence Θ_n, some endogenous parameters of the model, such as the inaction region $[U, L]$ will also change. While we do not always explicitly indicate this in notation, we always understand those changes to be taking place.

We start by introducing a second measure change that will facilitate some of the arguments below. Recall that $\overline{A}_t = e^{-\mu_A t} A_t$ is an exponential martingale. We define a probability measure R by letting, for any random variable Z_t measurable with respect to $\mathcal{F}_t$, $E^R[Z_t] = E\left[Z_t \overline{A}_t\right]$. The

Girsanov theorem tells us that under R, the process $d\bar{z}_t = dz_t - \sigma_A t$ is a martingale.The key advantage of this measure is that $E_0 \overline{X}_t = E_0^R \left[\overline{X}_t / \overline{A}_t \right]$. This makes it easier to compute the mean and the impulse response of $\overline{X}_t$, because $\overline{X}_t / \overline{A}_t$ is a bounded process (under R as well as under P). We can also write

$$E_0 \overline{X}_t = E_0^R \left[\overline{X}_t / \overline{A}_t \right] = E_0^{QR} \left[x_t / a_t \right]$$

where the superscript QR means that we also apply the measure transformation Q introduced earlier. The idea is that by applying R, we move to using the mean dynamics of $\overline{X}/\overline{A}$; and then, by also applying Q, we can focus on the mean dynamics of a single agent, albeit under a driving process with different drift.

We begin with a technical lemma that establishes the smoothness of conditional expectations of the process y_t. We start by thinking the dynamics of a new process w_t, which is a Brownian motion with some drift μ_w and variance σ_w reborn at some interior point M_w when hitting the boundaries of the interval $[L_w, U_w]$. When the parameters of this process are chosen to match those generated by our model, then w_t will have the same distribution as y_t under QR. We let $h(y, t, \sigma_w, \mu_w, L_w, M_w, U_w) = E\left[e^{w_t} | w_0 = y\right]$. Often we just write $h(y,t)$, in which case we generally assume that the other arguments are at their values as given by the optimal policy of the model, so that $h(y,t) = E^{QR} E\left[e^{y_t} | y_0 = y\right]$.

Let $L_1 < L_2 < M_1 < M_2 < U_1 < U_2$.

Lemma 1 *$h(y, t, \sigma_w, \mu_w, L_w, U_w, M_w)$ is infinitely many times differentiable in $[L_w, U_w] \times (0,\infty) \times (0,\infty) \times [L_1, L_2] \times [M_1, M_2] \times [U_1, U_2]$.*

Proof. We start with the case where w_t is driven by a standard Brownian motion. Let $\zeta_y = \inf\{t \geq 0 : w_t \notin [L, U], w_0 = y\}$. Set $F_w(t) = \Pr\left[\zeta_y \leq t\right]$ and $\overline{h}(y,t) = E\left[e^{w_t} \cdot 1\left\{\zeta_y > t\right\}\right]$ be $h(y,t)$ killed at the boundary. Let $F_y^{(1)}(t) = F_y(t)$ and $F_y^{(n+1)}(t) = \int_0^t F_{y^*}^{(n)}(t-\tau)\, dF_y(\tau) = \int_0^t F_M(t-\tau)\, dF_y^{(n)}(\tau)$ be the the distribution of the $n+1$st exit time. Then

$$h(y,t) = \overline{h}(y,t) + \sum_{n=1}^{\infty} \int_0^t \overline{h}(M, t-\tau)\, dF_y^{(n)}(\tau) = \overline{h}(y,t) + \int_0^t \overline{h}(M, t-\tau)\, dF_y^*(\tau) \tag{12}$$

where

$$F_y^*(t) = \sum_{n=1}^{\infty} F_y^{(n)}(t) = F_y(t) + \int_0^t F_M^*(t-\tau)\, dF_y(\tau) = F_y(t) + \int_0^t F_M(t-\tau)\, dF_y^*(\tau) \tag{13}$$

is the expected number of boundary hits until t.

The transition density of the killed diffusion $p(y,y',t) = \Pr\left[\zeta_y > t,\, y_t = y'\right]$ can be expressed as an infinite sum of normal densities (Revuz and Yor, 1992, p 106), and in particular, is infinitely many times differentiable in $[L,U]\times[L,U]\times(0,\infty)$. This implies that $\overline{h}(y,t) = \int e^{y'}p(y,y',t)\,dy'$ is infinitely many times differentiable in $[L,U]\times(0,\infty)$. The density of the first hitting time ζ_y can also be expressed in closed form as an infinite sum (Darling and Sieger, 1953), and is infinitely many times differentiable in y and t over $[L,U]\times(0,\infty)$. This, combined with (13) implies that $F_y^*(t)$ is C^∞ in $[L,U]\times(0,\infty)$. Combining these observations with (12) shows that $h(y,t)$ is also C^∞ in the $[L,U]\times(0,\infty)$ domain.[13]

We next show that h is also smooth when driven by any Brownian motion with drift and variance, and that it is smooth in the other parameters. Changing the clock of y_t scales both the mean and the variance, and is obviously a smooth transformation of $h(y,t)$ as it just scales the time argument. Shifting and rescaling the vertical axis are smooth operations that shift and rescale the triple $[L,M,U]$. Thus we only need to show smoothness in the drift and in M. The drift can be dealt with using the Girsanov theorem, which implies that the density of the killed diffusion under drift can be obtained as $p^{\mu_w}(y,y',t) = p(y,y',t)\cdot\exp\left[\mu_w(y'-y) - \mu_w^2 t/2\right]$, which is clearly C^∞ in μ_w, and hence so is $\overline{h}(y,t)$. Next, the distribution of the first hitting time is $1 - F_y^{\mu_y}(t) = \int p^{\mu_y}(y,y',t)\,dy'$ is also smooth. The smoothness of h in μ_y now follows from (12). Smoothness in M follows easily from (12).

A key implication of this lemma is that h and it's various derivatives in y and t are all continuous and therefore locally bounded in $(\mu_w,\sigma_w,L_w,M_w,U_w)$. This is useful because when we take σ_A to zero along a sequence, optimal behavior changes, and hence the endogenous parameters $\left(\mu_y,\sigma_y,L,M,U\right)$ vary. By the theorem of the maximum, these parameters will all lie in some bounded open set, and even though $\sigma_A \to 0$, we have σ_y bounded away from zero, since there is positive idiosyncratic risk. The lemma thus implies that along this sequence, $h(y,t)$ and its derivatives in y and t exist and are all bounded. We will exploit this fact later in the proof.

Our next Lemma gives an MA representation of X_t and expresses the weights using the function h.

[13] Grigorescu and Kang (2002) compute the transition density of y explicitly.

Lemma 2 *There exist functions $\xi(u,f)$ and $\xi(u,y)$ so that*

$$\overline{X}_t = \int_0^t \xi(t-s, f(s))\, \sigma \overline{A}_s dz_s + E_0\left[\overline{X}_t\right] \tag{14}$$

where

$$\xi(u, f(s)) = \int_L^U \xi(u,y)\, f(y,s)\, dy \quad \text{and} \quad \xi(u,y) = h(u,y) - h_y(u,y).$$

Proof. We have

$$E_s\left[\overline{X}_t\right] = \overline{A}_s \cdot E_s^R\left[\overline{X}_t/\overline{A}_t\right] = \overline{A}_s \cdot E_s^{QR}\left[x_t/a_t\right] = \overline{A}_s \cdot \int_L^U h(t-s,y)\, f(y,s)\, dy$$

which is a martingale in s. Computing the Ito-differential

$$d_s E_s\left[\overline{X}_t\right] = d\overline{A}_s \cdot E_s^{QR}\left[x_t/a_t\right] + \overline{A}_s \cdot \int_L^U h(t-s,y)\, f_y(y,s)\, \sigma_A dz_s \cdot dy$$

where we used (4) for the evolution of $f(y,s)$ and collected only the dz terms, since the ds terms must cancel by the martingale property. Equivalently,

$$d_s E_s\left[\overline{X}_t\right] = d\overline{A}_s \cdot \left(E_s^{QR}\left[x_t/a_t\right] + \int_L^U h(t-s,y)\, f_y(y,s)\, dy\right) = d\overline{A}_s \cdot \int_L^U \left(h(u,y) - h_y(u,y)\right) f(y,s)\, dy$$

where we integrated by parts. This equation shows the existence of ξ as well as the desired representation.

Proof of Proposition 3. We just need to show that $\xi(u,f)$ as defined in Lemma 2 equals the impulse response of Definition 2. Let $\overline{A}_0^*$ be the point at which we want to differentiate $E_0\left[X_t|f\right]$. The key is to note that we can write the conditional expectation as a function of $\overline{A}_0$ as

$$E_0\left[\overline{X}_t\right] = \overline{A}_0 \cdot E_0^R\left[\overline{X}_t/\overline{A}_t\right] = \overline{A}_0 \cdot \int_L^U h\left(t, y - \left(\log \overline{A}_0 - \log \overline{A}_0^*\right)\right) f(y,s)\, dy.$$

To see the logic, note that when $\overline{A}_0 = \overline{A}_0^*$, the mass of people at any point y is given by $f(y)$, and the conditional expectation given y is summarized by h. When $\overline{A}_0$ changes, the mass of these people is unaffected, and hence $f(y)$ is unchanged; but their y shifts, resulting in a change in the conditional expectation, and hence we must evaluate h at a different point.

Differentiating this in $\overline{A_0}$ gives

$$\frac{\partial E_0\left[\overline{X}_t\right]}{\partial \overline{A}_0} = \int_L^U h(t,y) f(y,s)\, dy - \int_L^U h_y(t,y) f(y,s)\, dy = \int_L^U \left[h(t,y) - h_y(t,y)\right] f(y,s)\, dy$$

which is exactly the definition of ξ given above. This confirms both that the impulse response is well defined, and the MA representation claimed in the proposition.

We next show that the impulse response function converges exponentially fast to its limit value. Intuitively, the impact of a permanent shock that happened in the distant past should be almost fully built into current commitments.

Lemma 3 *There exists $\overline{x}$ such that $\lim_{t\to\infty} E_0\left[\overline{X}_t\right] = \lim_{t\to\infty} \xi(t,y) = \overline{x}$. There exist $K_1, K_2 > 0$ independent of y and σ_A so that $|\xi(t,y) - \overline{x}| < K_1 e^{-K_2 t}$ and $\left|E_0\left[\overline{X}_t\right] - \overline{x}\right| < K_1 e^{-K_2 t}$ for all $(y, \sigma_A) \in [L, U] \times [0, \overline{\sigma}_A]$.*

Proof. Ben-Ari and Pinsky (2009) show that $y_t = \log[x_t/a_t]$ converges exponentially fast to a unique invariant distribution. Ben-Ari and Pinsky (2007) also show that the rate of convergence is uniformly bounded if the drift is from a bounded interval. This implies uniform convergence for all $\sigma_A \in [0, \overline{\sigma}_A]$ through a clock-change argument. Since

$$E_0\left[\overline{X}_t\right] = E_0^R\left[\overline{X}_t/\overline{A}_t\right] = E_0^{QR}\left[x_t/a_t\right],$$

it follows that $E_0\left[\overline{X}_t\right]$ converges exponentially fast to the mean $\overline{x}$ of x/a under the invariant distribution, and that this is uniform in σ_A. Recalling that $h(u,y) = E^{QR}\left[x_u/a_u | x_0/a_0 = e^y\right]$, we also have $h(u,y)$ converge at the same rate to $\overline{x}$ as $u \to \infty$, uniformly in y and σ_A. Letting $F_t^{QR}[y|y_0]$ denote the cross-sectional distribution of y_t given initial value y_0, fixing some $s < u$, we can write

$$\begin{aligned} h_{y_0}(u, y_0) &= \frac{\partial}{\partial y_0} \int_L^U h(u-s, y)\ dF_t^{QR}[y|y_0] = \int_L^U h(u-s,y)\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy \\ &= \int_L^U \left(h(u-s,y) - \overline{x}\right)\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy \end{aligned}$$

where at the last step we used that $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ integrates to zero in y. By the arguments of Lemma 1, $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ is bounded, while $h(u-s,y) - \overline{x}$ converges exponentially fast to zero; hence so does the integral.

The next result will be used in the proof of Theorem 2 to establish that when σ_A is small, the impulse responses of the two models are typically close. We let f^* denote the invariant distribution of y under Q, which is also the long run average cross-sectional distribution of the commitments model.

Lemma 4 $\limsup_{t\to\infty} \left\|\sup_y |F(y,t) - F^*(y)|\right\|_p$ *converges to zero as* $\sigma_A \to 0$.

Proof. We know that EF converges to F^* uniformly in y. Fix $\varepsilon > 0$ and pick s so that for all $t > s$, $|EF_t - F^*| < \varepsilon/8$ for all initial conditions and for all σ small enough. Consider the rectangular set $[-\kappa, \kappa] \times [t-s, t]$, and let G_κ denote the event when the realization of $\log \overline{A}_u - \log \overline{A}_{t-s}$ for $u \in [t-s, t]$ is in this set. Let $F\left(y, t, \overline{A}_{[t-s,t]}, y_s\right)$ denote the distribution of y_t under Q when started at y_s in s, and when the realization of aggregate shocks is given by $\overline{A}_{[t-s,t]}$. We then have that $\left\{\sup_{y_t,y_s}\left|F\left(y,t,\overline{A}_{[t-s,t]}, y_s\right) - F\left(y,t,\overline{A}'_{[t-s,t]}, y_s\right)\right| \;\Big|\; \overline{A}_{[t-s,t]}, \overline{A}'_{[t-s,t]} \in G_\kappa\right\}$ goes to zero as $\kappa \to 0$: two sufficiently close paths of aggregate consumption generate cross-sectional distributions that are themselves close. This is because the share of people for whom the two aggregate paths result in sufficiently different behavior goes to zero. Take κ small enough so that this quantity is less than $\varepsilon/8$. For any fixed κ we can pick σ small enough so that $\Pr\left[\overline{A}_{[t-s,t]} \in G_\kappa\right] > 1 - \varepsilon/8$. This implies that $|E_s F_t - E[F_t|f(s), G_\kappa]| < \varepsilon/4$. Combining these bounds, for $\overline{A}_{[t-s,t]} \in G_\kappa$ we have

$$\left|F\left(y,t,\overline{A}_{[t-s,t]}, f(s)\right) - F^*(y)\right| \leq$$
$$\left|F\left(y,t,\overline{A}_{[t-s,t]}, f(s)\right) - E[F_t|f(s), G_\kappa]\right| + |E[F_t|f(s), G_\kappa] - E_s F_t| + |E_s F_t - F^*(y)| < \frac{\varepsilon}{8} + \frac{\varepsilon}{4} + \frac{\varepsilon}{8} = \frac{\varepsilon}{2}.$$

Using this, we have

$$\left\|\sup_y |F(y,t) - F^*(y)|\right\|_p^p =$$
$$\Pr[G_\kappa] \cdot E\left[\sup_y (F(y,t) - F^*(y))^p | G_\kappa\right] + (1 - \Pr[G_\kappa]) \cdot E\left[\sup_y (F(y,t) - F^*(y))^p \mid \text{not } G_\kappa\right] \leq$$
$$\left[\left(\frac{\varepsilon}{2}\right)^p + 2^p \frac{\varepsilon}{8}\right] < 2^p \varepsilon.$$

Since this is true for all $t > s$, it is also true for the lim sup. But ε was arbitrary, and the bound applies for all σ small enough given ε; hence the desired result follows.

The next technical lemma is used to bound the tails of the MA representations for both habits and commitments.

Lemma 5 *Let $g(u,s)$ be progressively measurable with respect to $\mathcal{F}_s$ satisfying $|g(u,s)| \leq K_1 e^{-K_2 u}$ for all u, s, and let*

$$G_t = \frac{1}{\overline{A}_t} \int_0^t g(t-s,s)\, \overline{A}_s dz_s.$$

For any $1 \leq p < \infty$, for σ_A small enough, there exists $M(p)$ such that $\|G_t\|_p \leq M(p)$.

Proof. We proceed by induction on t. Fix some $k > 0$. We show that (i) the desired bound holds when $t \leq k$, and (ii) if the bound holds for some t, it also holds for $t + k$. We begin by showing (ii), which is the more difficult part.

We can write

$$\|G_t\|_p \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \int_{t-k}^t g(t-s) \frac{\overline{A}_s}{\overline{A}_{u-k}} dz_s \right\|_p + \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_p \cdot \left\| \frac{1}{\overline{A}_{t-k}} \int_0^{t-k} g(t-s)\, \overline{A}_s dz_s \right\|_p$$

where we used independence of the Brownian increments. Denoting $\overline{g}(u,s) = e^{K_2 k} g(u+k,s)$ we can rewrite the final term in brackets as

$$e^{-K_2 k} \cdot \frac{1}{\overline{A}_{t-k}} \int_0^{t-k} \overline{g}(t-k-s,s)\, A_s dz_s$$

where $|\overline{g}(u,s)| \leq K_1 e^{-K_2 u}$ by construction. By our induction assumption, this term has p-norm bounded by $e^{-K_2 k} \cdot M(p)$. To bound the remaining terms, first observe that by lognormality

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_p \leq K_p(\sigma_A, k)$$

for some $K_p(\sigma_A, k)$ that goes to one in σ_A for all k. Next note that

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \int_{t-k}^t g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_p \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \int_{t-k}^t g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p}$$

by the Cauchy-Schwarz inequality. Here

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \leq K_{2p}(\sigma_A, k)$$

where $K_{2p}(\sigma_A, k)$ also goes to one in σ_A for all k. Finally, using standard bounds (e.,g., Karatzas and Shreve, 2008) for moments of the Ito integral, we obtain

$$\left\| \int_{t-k}^{t} g\left(t-s,s\right) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p} \leq K_{2p} \left(\int_{t-k}^{t} K_1^2 \left\| \left(\frac{\overline{A}_s}{\overline{A}_{u-k}} \right)^2 \right\|_p ds \right)^{1/2}$$

which is bounded by $K_{2p} K_1 k \cdot K_{2p}\left(\sigma_A, k\right)$. Combining terms we obtain

$$\left\| G_t \right\|_p \leq K_{2p}^2 \left(\sigma_A, k\right) \cdot K_{2p} K_1 k + K_p\left(\sigma_A, k\right) \cdot e^{-K_2 k} \cdot M\left(p\right).$$

It is easy to see that if

$$M\left(p\right) = \frac{K_{2p}^2 \left(\sigma_A, k\right) \cdot K_{2p} K_1 k}{1 - K_p\left(\sigma_A, k\right) \cdot e^{-K_2 k}}$$

is positive, then the induction step follows. We can make sure that this is the case by first choosing some $k > 0$, and then picking $\overline{\sigma_A}$ small enough so that for all $\sigma_A \leq \overline{\sigma}_A$ we have $K_p\left(\sigma_A, k\right) < e^{K_2 k/2}$. With this choice of $M\left(p\right)$, the induction step follows; and (i) can be verified easily from the argument of the induction step.

We now present some results about habit models. We first show how to convert a habit representation with total consumption weights to one with A-weights, and to convert back.

Lemma 6 *The coefficients in the habit representations*

$$X_t = \int_0^t j(t-s) A_s ds + k(t) X_0$$

and

$$X_t = o(t) X_0 + \int_0^t \zeta(t-s) C_s ds$$

are linked to each other through the Volterra integral equations

$$\zeta(u) = j(u) - \int_0^u \zeta(v) j(u-v) dv \tag{15}$$

$$o(t) = k(t) - \int_0^t \zeta(t-s) k(s) ds \tag{16}$$

with initial conditions $\zeta(0) = j(0)$, $o(0) = k(0)$. In particular, each C-average representation has a unique equivalent A-average representation.

Proof. Consider the process

$$\widetilde{X}_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds.$$

We will show that $\widetilde{X}_t = X_t$ for all $t \geq 0$. First note that

$$\begin{aligned}\widetilde{X}_t &= o(t)X_0 + \int_0^t \zeta(t-s)\left[A_s + X_s\right] ds \\ &= o(t)X_0 + \int_0^t \zeta(t-s)A_s + \zeta(t-s)\left[\int_0^s j(s-u)A_u du \ + k(s)X_0\right] ds \\ &= o(t)X_0 + \int_0^t A_s\left[\zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)du\right] ds + X_0\int_0^t \zeta(t-s)k(s)ds.\end{aligned}$$

Equating coefficients, $X_t = \widetilde{X}_t$ holds if

$$j(t-s) = \zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)\ du$$

or, with $t-s=u$,

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv$$

and

$$o(u) = k(u) - \int_0^u \zeta(u-v)k(v)dv.$$

Substituting in $u=0$ gives $\zeta(0)=j(0)$ and $o(0)=k(0)$. The integral equation for $\zeta(u)$ then yields a unique solution, which can be used to determine $o(.)$. By the above argument, a pair of functions that solve these equations also give $X_t = \widetilde{X}_t$, which is the desired representation.

We next construct the best-fit habit model.

Lemma 7 *Let $\theta(u) = \xi^{*\prime}(u)\cdot e^{\mu_A u}$ and $\theta_0(u) = (\overline{x} - \xi^*(u))\cdot e^{\mu_A u}$, then the habit model*

$$X_t^h = \int_0^t \theta(t-s)A_s ds + \theta_0(t)A_0 \tag{17}$$

generates the impulse repsonse ξ^*.

Proof. Detrending both sides and integrating by parts (using that ξ^* is smooth)

$$\begin{aligned}\overline{X}_t^h &= \int_0^t \xi^{*\prime}(t-s)\,\overline{A}_s ds + [\overline{x}-\xi^*(t)]\,A_0 = \left[-\xi^*(t-u)\,\overline{A}_u\right]_0^t + \int_0^t \xi^*(t-s)\,d\overline{A}_s + [\overline{x}-\xi^*(t)]\,A_0 \\ &= \int_0^t \xi^*(t-s)\,d\overline{A}_s + \overline{x}A_0. \end{aligned} \tag{18}$$

Proof of Theorem 2. We are now ready to prove the main theorem of the section. We first focus on a sequence where $\sigma_A \to 0$. We can write

$$\frac{X_t - X_t^h}{\sigma_A A_t} = \frac{1}{\overline{A}_t}\int_0^t [\xi(t-s,f(s)) - \xi^*(t-s)]\,\overline{A}_s dz_s + \frac{E_0\overline{X}_t - \overline{x}}{\overline{A}_t\sigma_A}.$$

We will bound the p-norm of this expression by breaking it into several pieces. Fix some $\varepsilon > 0$, let $k > 0$, and consider

$$\begin{aligned}\left\|\frac{1}{\overline{A}_t}\int_0^{t-k}[\xi(t-s,f(s)) - \xi^*(t-s)]\,\overline{A}_s dz_s\right\|_p &\le \left\|\frac{\overline{A}_{t-k}}{\overline{A}_t}\right\|_{2p}\cdot\left\|\frac{1}{\overline{A}_{t-k}}\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]\,\overline{A}_s dz_s\right\|_{2p} \\ &\le K_{2p}(k,\sigma_A)\cdot M(2p)\cdot e^{-K_2 k}\end{aligned}$$

where we used Lemma 5. We can chose k large enough so that this entire term is less than $\varepsilon/3$.

Given this k, we next first bound

$$\begin{aligned}\left\|\frac{1}{\overline{A}_t}\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]\,\overline{A}_s dz_s\right\|_p &\le \left\|\frac{\overline{A}_{t-k}}{\overline{A}_t}\right\|_{2p}\cdot\left\|\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]\,\frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s\right\|_{2p} \\ &\le K_{2p}(k,\sigma_A)\cdot K_{2p}(k)\cdot\left[E\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]^{2p}\left|\frac{\overline{A}_s}{\overline{A}_{t-k}}\right|^{2p} ds\right]^{1/2p} \\ &\le K_{2p}(k,\sigma_A)\cdot K_{2p}(k)\cdot\left[E\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]^{4p} ds\right]^{1/4p}\cdot\left[E\int_{t-k}^t\left|\frac{\overline{A}_s}{\overline{A}_{t-k}}\right|^{4p} ds\right]^{1/4p} \\ &\le K_{2p}(k,\sigma_A)\cdot K_{2p}(k)\cdot K_{4p}(k,\sigma_A)\cdot\left[E\int_{t-k}^t[\xi(t-s,f(s)) - \xi^*(t-s)]^{4p} ds\right]^{1/4p}\end{aligned}$$

where we repeatedly used the Cauchy-Schwarz inequality and a martingale moment bound, and where all constants are bounded as σ_A goes to zero. Now we use the idea that for σ_A small, ξ and

ξ^* are close. Note that

$$\xi(t-s,f(s))-\xi^*(t-s)=\int_L^U \xi(t-s,y)\cdot[f(t-s,y)-f^*(y)]\,dy$$
$$=-\int_L^U \frac{\partial}{\partial y}\xi(t-s,y)\cdot[F(t-s,y)-F^*(y)]\,dy.$$

Here, for any fixed k, by Lemma 1, $\partial\xi(t-s,y)/\partial y$ is uniformly bounded in $(y,\sigma_A)\in[L,U]\times[0,\overline{\sigma}_A]$. Denoting this bound by $K(k)$, we have

$$E[\xi(t-s,f(s))-\xi^*(t-s)]^{4p}<K^{4p}(k)\cdot E\sup_y|F(t-s,y)-F^*(y)|^{4p}.$$

Lemma 4 shows that the limsup over t of the last term goes to zero as $\sigma_A\to 0$. Thus given k and $\varepsilon>0$, for all σ_A small enough to make te entire term

$$\left\|\frac{1}{\overline{A}_t}\int_{t-k}^t[\xi(t-s,f(s))-\xi^*(t-s)]\,\overline{A}_s dz_s\right\|_p<\frac{\varepsilon}{3}.$$

Finally, consider

$$\frac{1}{\sigma_A}\cdot\left\|\frac{E_0\overline{X}_t-\overline{x}}{\overline{A}_t}\right\|_p\le\frac{1}{\sigma_A}\cdot\left\|\frac{1}{\overline{A}_t}\right\|_p\cdot K_1e^{-K_2t}\le\frac{1}{\sigma_A}\cdot e^{K_3(p)\cdot\sigma_A^2t}\cdot K_1e^{-K_2t}.$$

If σ_A is small enough, then the limsup of this as $t\to\infty$ is zero. The result now follows for the case when $\sigma_A\to 0$.

We next consider a sequence where $\sigma_I\to\infty$. Here the key is to change the "clock," i.e., the speed with which we go thorugh the Brownian sample paths. This effectively reduces both σ_I and σ_A at the same rate, converting our sequence of models into one where $\sigma_A\to 0$, where the previous result applies. The following Lemma summarizes the key step.

Lemma 8 *Fix $\tau>0$, and let $\left(\widetilde{a}_t^i,\widetilde{x}_t^i\right)$ denote the optimal solution of a model with deep parameters $\tau\cdot\left(\rho,r,\pi,\sigma^2,\pi_I,\sigma_i^2\right)$, fixed costs $\overline{\lambda}=\left(\overline{\lambda}_1,\overline{\lambda}_2\right)$, curvature γ and relative preference κ. Then the process $\left(\widetilde{a}_t^i,\widetilde{x}_t^i\right)$ has the same distribution as $\tau\cdot\left(a_{\tau t}^i,x_{\tau t}^i\right)$: rescaling the time dimension acts the same way as rescaling the parameters of the model.*

Proof. We verify directly that changing the clock is equivalent to rescaling the relevant parameters in the setup of the problem. Maximizing the consumer's problem in the original model is equivalent to maximizing

$$E\int_0^\infty e^{-\rho t\tau}\left(\frac{a_{\tau t}^{1-\gamma}}{1-\gamma}+\mu\frac{x_{\tau t}^{1-\gamma}}{1-\gamma}\right)dt$$

which is proportional to the objective function in the model with new parameters. Similarly, the budget constraint of the original model implies

$$dw_{\tau t}=\left[\left(\tau r+\alpha_{\tau t}\tau\pi+\alpha_{\tau t}^{i}\tau\pi_I\right)w_t-\tau c_t\right]dt+\alpha_{\tau t}w_{\tau t}\sigma\tau^{1/2}dz_{\tau t}+\alpha_{\tau t}^{i}w_{\tau t}\sigma_i\tau^{1/2}dz_{\tau t}^{i}$$

on all non-adjustment dates due to the scaling invariance of Brownian motion. Finally, on adjustment dates, $dw=\overline{\lambda}_1 x_{t-}/r+\overline{\lambda}_2 x_t/r=\overline{\lambda}_1\cdot\tau x_{t-}/\left(\tau r\right)+\overline{\lambda}_2\cdot\tau x_t/\left(\tau r\right)$. Since the optimal policy is unique, the claim follows.

Now consider a sequence of models where $\sigma_I\to\infty$ and let $\tau=\left(\sigma_I\right)^{-2}$. According to the lemma, after changing the clock, dynamics will be identical to a model with parameters

$$\left(\tau\sigma_I^2,\tau\sigma_A^2,\tau r,\tau\mu_A,\gamma,\overline{\lambda}_1,\overline{\lambda}_2,\kappa\right)=\left(1,\tau\sigma_A^2,\tau r,\tau\mu_A,\overline{\lambda}_1,\overline{\lambda}_2,\kappa\right).$$

By construction, along this sequence aggregate risk goes to zero, while other parameters remain bounded as needed. Thus this model is very close to the equivalent habit representation; but then so is the original model where $\sigma_I\to\infty$, as desired.

Proofs for Section 5

Proof of Proposition 4. (1) Excess smoothness. This follows because X is of bounded variation and hence adjusts much more slowly than A. (2) Excess sensitivity. If $s_1\to 0$ and $s_2\to\infty$ then almost all households adjust during $(t+s_1,t+s_2)$ and hence β_2 converges to one.

Proof of Proposition 5. (1) In the habit model this follows from the definition of the habit stock as a time average. With commitments, there exists $p>0$ such that for any t, the probability of the following event is at least p: a mass of at least p in $F\left(t\right)$ is within distance Δ from L, and a mass of at least p is within distance Δ from U. As Δt goes to zero, in the limit all these people in one of these boundary ranges, and none of the people in the other boundary range adjust. Since the adjustment size is bounded away from zero, it follows that for Δt small there exists $K>0$ such that when $\Delta A_t>\Delta A$ we have $\Delta\log X_t>K$ and when $\Delta A_t<-\Delta A$ we have $\Delta\log X_t<-K$. This implies the result.

(2) The existence of such $K > 0$ is also easily seen to imply that the regression coefficient β is bounded away from zero in the commitments model in $\mathcal{B}(\Delta)$. In the habit model, we have $X_t = \int_0^t \theta(t-s) A_s ds$ and hence a change in A_t has zero effect on X_t, implying a regression coefficient converging to zero even in $\mathcal{B}(\Delta)$.

Proof of Proposition 6. (i) We compute the value of the habit agent. Let ψ be defined so that the value of the Marton consumption problem in an economy like that of the representative agent habit consumer, but no habit, is $\psi W^{1-\gamma}/(1-\gamma)$. By the envelope theorem, this Merton agent has consumption policy $c = \psi^{-1/\gamma} W$. We know that the surplus consumption of our habit agent is identical to the consumption of some Merton agent, because the first order conditions are identical. In particular, if the habit consumer chooses a value A_0 for his initial surplus consumption, the dollar cost of his lifetime surplus consumption expenditure is $A_0 \psi^{1/\gamma}$.

We now evaluate the lifetime budget constraint of our habit consumer. Each dollar of consumption spending in a period also creates future expenditure commitments in the form of increased habit. Suppose $1 + B$ dollars is the present value of expenditure commitments for a dollar of consumption spending today, where $B = 0$ with no habits. Then B must satisfy

$$B = \int_{u=0}^{\infty} \theta(u) e^{-ru} du \cdot (1 + B)$$

because each dollar of consumption creates $\theta(u)$ dollars of habit spending u periods ahead, which has total expenditure costs of $\theta(u)(1+B)$ in period u dollars, which we must then discount back at the riskfree rate because these payments are certain. Solving yields

$$B = \frac{1}{1 - \int_{u=0}^{\infty} \theta(u) e^{-ru} du}.$$

At any time t, our habit consumer also has pre-existing habit commitments that are created by his past consumption. The dollar value of these equals

$$Z_t = (1 + B) \cdot \left[\int_{s=0}^{t} C_{t-s} \int_{s}^{\infty} \theta(u) e^{-r(u-s)} du \, ds + \int_{s=t}^{\infty} \theta_0(u) X_0 e^{-ru} du \right]$$

where the term in parenthesis describes the total future consumption expenditures created by habits established before t, discounted back at the riskfree rate because these are certain; and the factor $1 + B$ is included because each dollar of consumption spending has this total expenditure cost.

The consumer's lifetime budget constraint must then satisfy

$$W_t = A_t \cdot \psi^{1/\gamma} (1 + B) + Z$$

and his lifetime utilty from surplus consumption, by the Merton value function, is simply $\psi^{1/\gamma} A_t^{1-\gamma} / (1 - \gamma)$. Combining these equations yields

$$V_t^{habit} (W_t, X_t) = \frac{\psi}{1 - \gamma} \left(\frac{W_t - Z_t}{1 + B} \right)^{1-\gamma}.$$

The welfare of an individual commitment agent for a move-inducing negative wealth shock is proportional to $(w - \lambda_1 x)^{1-\gamma} / (1 - \gamma)$. Now compare the commitment and the habit economies. As wealth falls to zero, if $Z > 0$ then the marginal utility of the habit agent will be driven to infinity even with a finite shock. In contrast, when $\lambda_1 = 0$, the marginal utility of the commitment agent only blows up when all his wealth is taken. It follows that for large finite shocks, the willingness to pay of the habit agent exceeds that in the commitment economy.

(ii) The portfolio share of stocks for the habit agent is inversely related to his coefficient of relative risk aversion over wealth, which approaches infinity as W_t falls to Z_t. In contrast, the portfolio share of stocks in the commitment economy is bounded by a function of the highest possible relative risk aversion in the (S,s) band, which is a finite number.

Proof of Proposition 7. (1) Given the assumptions, by continuity the (S,s) bands along this sequence will converge to the band of the limit economy. When σ_T and μ_a become zero, an agent in the interior of the (S,s) band expects never to adjust. Therefore commitments do not respons at all to an infinitesimal shock, and $T_* (p|x_0) = \infty$. For σ_T and μ_a small, the agent does adjust eventially, but the expected time to adjustment approaches infinity, and hence $T_n (p|x_0) \to \infty$.

(2) The agent in the limit economy never moves, and hence his value function is proportional to $(W - x/r)^{1-\gamma} / (1 - \gamma)$. It follows that $CRRA_* (W_0, x_0) = \gamma W_0 / (W_0 - x_0/r)$. Now consider an agent in economy n. Let p_0 denote the total dollar value at date zero of his total commitment expenditures on his current home. Because of the presence of risk or growth, he moves, implying $p_0 < x_0/r$. One policy available to this consumer at any wealth W is to maintain his spending and moving patterns on current commitments, and adjust spending on adjustables and future commitments in proportion relative to the optimal path at wealth W_0. Given that $\lambda_1 = 0$, this policy yields $V_n (W_0, x_0) (W - p_0)^{1-\gamma} / (W_0 - p_0)^{1-\gamma}$. This is a lower bound for his value function

that is equal to his true value $V_n(W_0, x_0)$ at W_0, and hence has higher curvature at W_0. As a result, $CRRA_n(W_0, x_0) \leq \gamma W_0/(W_0 - p_0)$. Since $p_0 < x_0/r$, the claim follows.

Appendix B: Simulations

Bellman equation and ODE characterization for the commitments model. In the simulations we use an ODE characterization of the optimal policy that builds on a similar characterization for the one-good model by Grossman and Laroque. To develop this ODE, we must study the Bellman equation of the commitment agent. Denote the value function by $V(W, x)$, then the Bellman equation between adjustment dates is

$$\rho V(W, x) = \max_{\alpha, \alpha} \left[\kappa \frac{a^{1-\gamma}}{1-\gamma} + \frac{x^{1-\gamma}}{1-\gamma} + V_1(W, x) EdW + \frac{1}{2} V_{11}(W, x) Var(dW) \right].$$

Following Grossman and Laroque, let $y = W/X - \lambda_1$ and define $h(y) = x^{-1+\gamma} V(W, x) = V(W/x, 1)$. Dividing through by $x^{1-\gamma}$ in the Bellman equation we obtain

$$\rho h(y) = \max_{a, \alpha} \left[\kappa \frac{(a/x)^{1-\gamma}}{1-\gamma} + \frac{1}{1-\gamma} + h'(y) Edy + \frac{1}{2} h''(y) Var(dy) \right]$$

and the budget constraint yields

$$dy = ((y + \lambda_1)(r + \alpha\pi) - 1 - a/x) dt + (y + \lambda_1) \alpha\sigma dz.$$

Maximizing in α, the optimal portfolio satisfies

$$\alpha(y + \lambda_1) = \frac{-h'(y)}{h''(y)} \frac{\pi}{\sigma^2}$$

and adjustable consumption is

$$\frac{a}{x} = \left[\frac{h'(y)}{\kappa} \right]^{-1/\gamma}.$$

Substituting back into the Bellman equation we obtain

$$\rho h(y) = h'(y)^{1-1/\gamma} \kappa^{1/\gamma} \frac{\gamma}{1-\gamma} + \frac{1}{1-\gamma} + h'(y) [(y + \lambda_1) r - 1] - \frac{1}{2} \frac{h'(y)^2}{h''(y)} \frac{\pi^2}{\sigma^2}.$$

This is an ordinary differential equation for $h(y)$. To obtain boundary conditions, note that on an adjustment date the value function equals

$$\begin{aligned}
\frac{V(W,x)}{x^{1-\gamma}} &= \frac{1}{x^{1-\gamma}} \max_{x'} V\left(W-\lambda_1 x-\lambda_2 x', x'\right) \\
&= \left(\frac{W-\lambda_1 x}{x}\right)^{1-\gamma} \cdot \max_{x'} \left(\frac{x'}{W-\lambda_1 x}\right)^{1-\gamma} \cdot V\left(\frac{W-\lambda_1 x}{x'}-\lambda_2, 1\right) \\
&= \left(\frac{W-\lambda_1 x}{x}\right)^{1-\gamma} \cdot \max_{y} \left(y+\lambda_1+\lambda_2\right)^{-1+\gamma} h(y).
\end{aligned}$$

Define

$$M = \max_{y} \left(y+\lambda_1+\lambda_2\right)^{-1+\gamma} h(y)$$

then by the above reasoning, at the edges of the inaction band, denoted y_1 and y_2 we have

$$h(y_i) = M y_i^{1-\gamma}$$

moreover, smooth pasting implies

$$h'(y_i) = M(1-\gamma)\, y_i^{-\gamma}.$$

Finally, the target value of y satisfies

$$y^* = \arg\max \left(y+\lambda_1+\lambda_2\right)^{-1+\gamma} h(y).$$

To numerically solve the ODE subject to these conditions, we follow the approach outlined by Grossman and Laroque. We first pick some M, pick y_1, solve the ODE with initial conditions as given above. If there is no y_2 for which the boundary conditions are satisfied, then we start with a different y_1. If the boundary conditions do hold for some y_2, then we check if M satisfies the equation above; if not, we start with a different M.

Parameters used in simulations. Our general strategy is to choose deep parameters to generate variation in the key consumption risk parameters σ_I and σ_A while holding fixed consumption growth. In all four environments shown in Figures 1-3, the parameters $(\gamma, \kappa, \lambda_1, \lambda_2, \delta) = (2, 1, 1, 0, .0326)$ are held fixed. The other parameters and the implied values of consumption risk, individual and aggregate growth are shown in the table. Figure 4 uses the same parameters as (c) and (d) except that $\kappa = .01$, ensuring that habit is on average about 80% of consumption.

The Sharpe-ratio of the aggregate stock market is lower than in the data, while long-run aggregate consumption risk is higher – this a variant of the equity premium puzzle of Mehra and Prescott (1985).

	π_M/σ_M	π_E/σ_E	r	σ_A	σ_I	μ_a	μ_A
(a) High aggr, low idiosyncr risk	20%	10%	3.24%	10%	5%	1.24%	1.37%
(b) High aggr, high idiosyncr risk	20%	20%	1%	10%	10%	.87%	1.37%
(c) Low aggr, low idiosyncr risk	10%	10%	4.74%	5%	5%	1.24%	1.37%
(d) Low aggr, high idiosyncr risk	10%	20%	2.5%	5%	10%	.87%	1.37%

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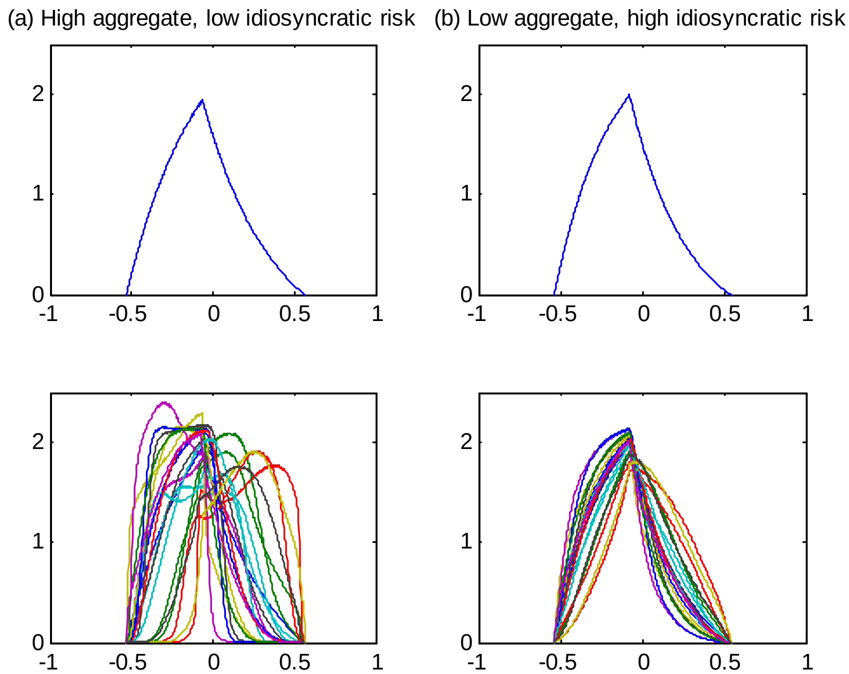
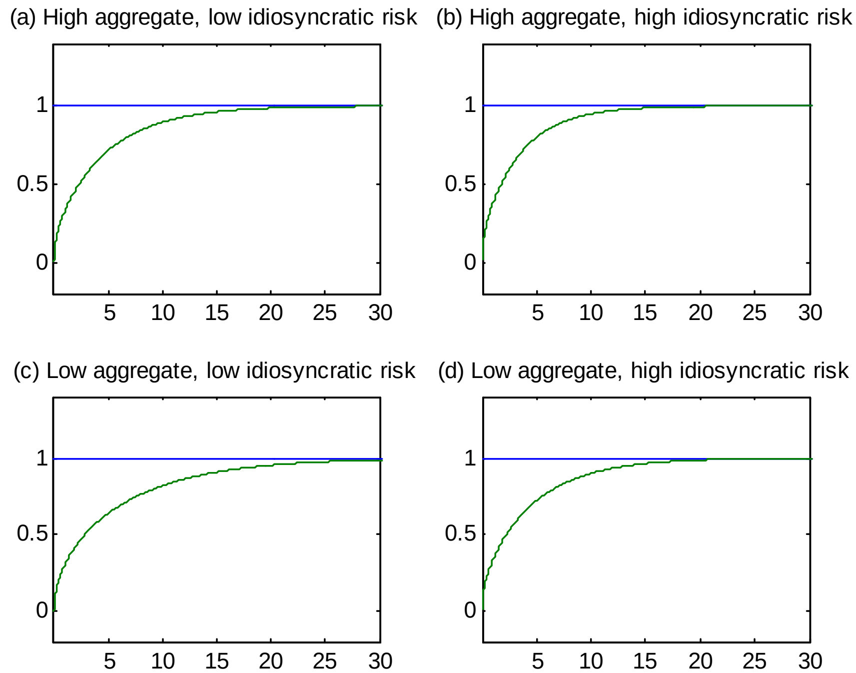
FIGURE 1: Cross-sectional consumption distributions



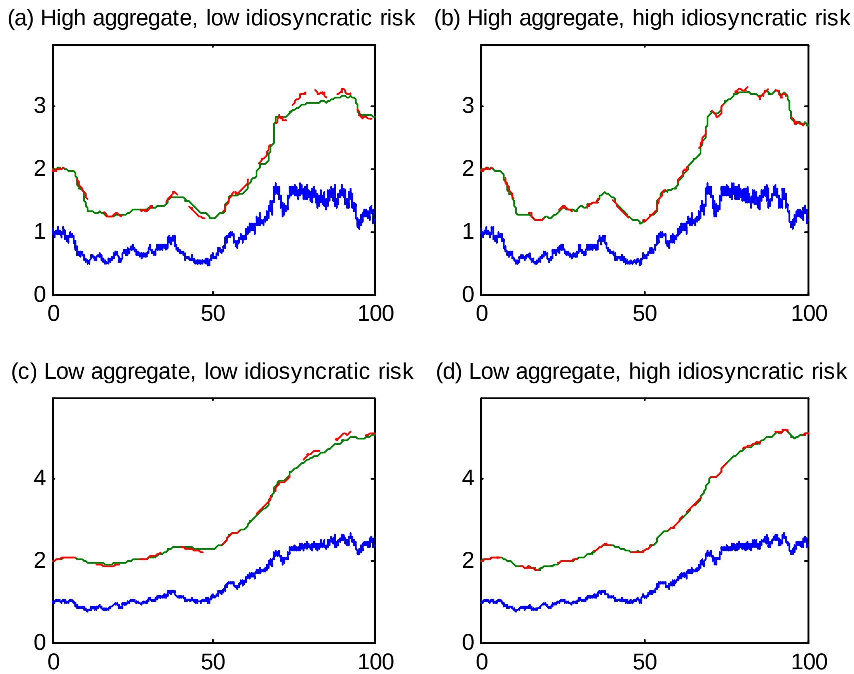
NOTE–Figures show cross-sectional densities of the log commitment to adjustables consumption ratio in two environments. For both environments, the top panel shows the long run steady state f^* while the bottom panel shows twenty realizations over a simulation corresponding to 100 years. Environment (a) has high aggregate risk ($\sigma_A = .1$) and low idiosyncratic risk ($\sigma_I = .05$) while environment (b) has low aggregate risk ($\sigma_A = .05$) and high idiosyncratic risk ($\sigma_I = .1$). The parameters generating these values are described in Appendix B.

FIGURE 2: Normalized impulse response functions



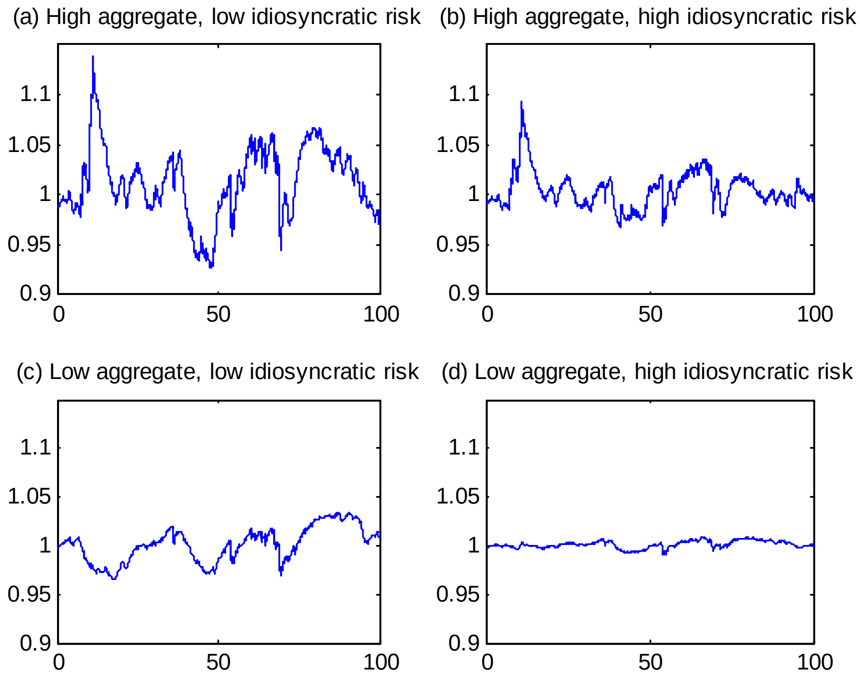
NOTE– Figures plot the normalized cumulative impulse response function of aggregate commitment consumption $\xi^* (t) / \overline{x}$ as a function of time elapsed after a shock in four environments with high (.1) and low (.05) aggregate and idiosyncratic risk. The parameters generating these environments are described in Appendix B.

FIGURE 3A: Aggregate dynamics of commitments and reduced form habit



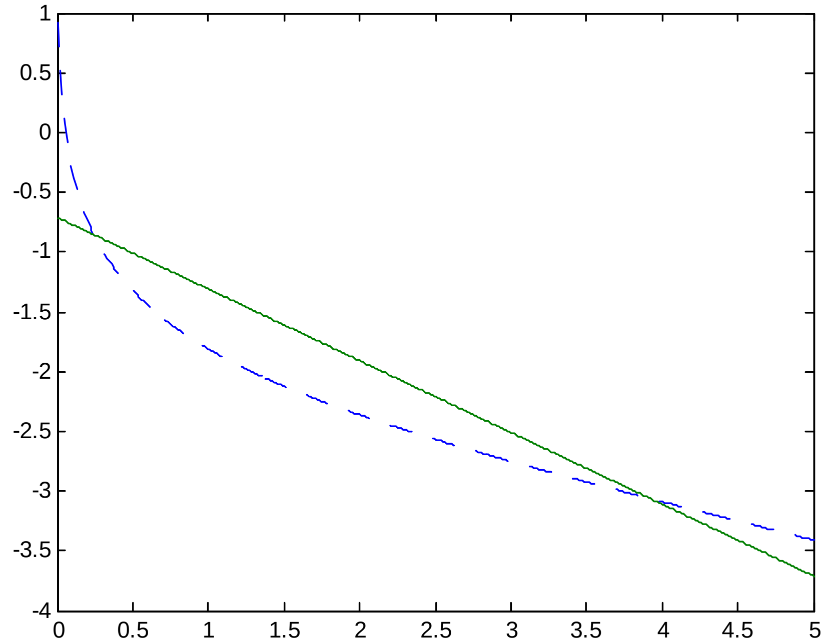
NOTE–Figures show commitments (green, solid) and reduced form habit (red, dashed) paths together with the evolution of permanent income (blue solid line in the bottom) in four environments with high (.1) and low (.05) aggregate and idiosyncratic risk. Figure shows that commitments and habit are closer with low aggregate or high idiosyncratic risk.

FIGURE 3B: Ratio of commitments and reduced-form habit



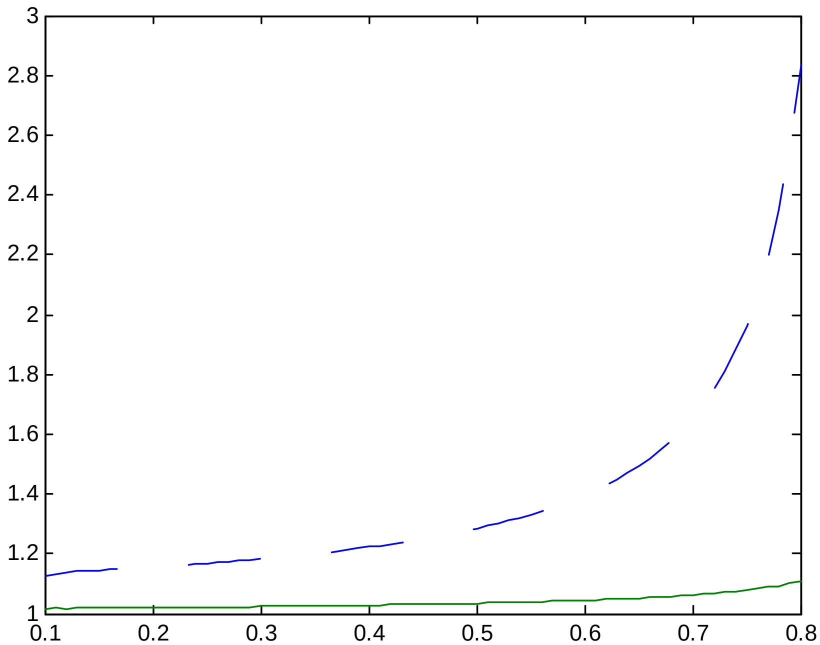
NOTE–Figures show the ratio of aggregate commitments and habit in the same four environments depicted in Figure 3a. Figure shows that commitments and habit are closer with low aggregate or high idiosyncratic risk. The parameters used to generate these figures are described in Appendix B.

FIGURE 4: Log habit weights for exponential and commitment-based model



NOTE–Figure plots log consumption habit weights for the reduced-form exponential habit model (green solid line) and commitments model (blue dashed curve). Expnential habit parameters are from Table 1, column 5 of Constantinides (1990). Commitment model is the low aggregate, high idiosyncratic risk environment of Figure 3, with parameters given in Appendix B.

FIGURE 5: Ratio of risk premium for in habit and commitments model as function of shock size



NOTE– Figure compares proportional risk premium of the commitment and matching habit model for a negative shock realized with probability $p = 1\%$ that reduces wealth by a share b. The blue dashed line plots ratio of proportional risk premium in an environment with low idiosyncratic risk ($\sigma_I = .05$) while the green solid line plots the same ratio with high idiosyncratic risk ($\sigma_I = .1$). When consumption responds sluggishly to shocks, the matching habit model is more persistent and hence exhibits higher risk aversion, particularly for big shocks.

TABLE 1
Speed of adjustment of the commitment-based reference point

Aggregate risk	Idiosyncr risk	Riskfree rate	Individ cons growth	How many yrs till X adjusts p? (p=1 means full adjustment) p=0.25	p=0.5	p=0.75
Adjustment cost= 1* annual consumption						
10%	10%	1%	0.87%	0.44	1.73	4.24
5%	10%	2.50%	0.87%	0.55	2.24	5.6
10%	5%	2.50%	0.87%	0.6	2.34	5.73
5%	5%	4%	0.87%	0.84	3.4	8.64
10%	10%	4%	2.37%	0.4	1.63	4.15
Adjustment cost= 5* annual consumption						
10%	10%	1%	0.87%	1.06	4.15	10.26
5%	10%	2.50%	0.87%	1.15	4.84	12.62
10%	5%	2.50%	0.87%	1.46	5.67	13.79
10%	10%	4%	2.37%	0.73	3.1	8.39

Table computes waiting times till partial adjustment of the commitment based reference point is expected to occur in the aggregate economy. Top panel reports results when adjustment cost of commitments equals annual consumption (λ_1=1); bottom panel when adjustment cost is five times annual consumption value (λ_1=5). Consumption risk is varied by changing the Sharpe-ratio of idiosyncratic and aggregate investments (π/σ=.1 for low and .2 for high risk). Except in last row of each panel, riskfree rate is chosen to hold fixed individual consumption growth across specifications. In all rows, γ=2, κ=1, δ=.0326, λ_2=0. See also Appendix B for details.