

Agency Problems, Screening and Increasing Credit Lines. Preliminary and Incomplete

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Abstract

We consider a setting in which an agent seeks financing for a project with uncertain cash flows. There is both a moral hazard problem: the agent can divert cash for personal consumption, and an adverse selection problem: the investors are initially uncertain about the type of the project. The financing contract must solve the agency problem and be attractive only to agents with a good project. We find that optimal contracts exhibit increasing financial flexibility over time after good performance. Such contracts can be implemented by a credit line agreement, with a credit limit that increases with good performance.

1 Introduction.

This paper develops a setting, in which an optimal financing contract can be implemented by a credit line with a variable credit limit that increases with the agent's good performance. The setting involves an initial adverse selection problem and a moral hazard problem that persists over time. Bolton and Scharfstein (1990) mention both of these informational problems as reasons determining termination and a firm's future access to capital. In our setting the credit line and the current balance determine the amount of losses the investors are willing to tolerate before they have to liquidate the project. Intuitively, the credit line form of the contract solves the moral hazard problem driven by the agent's ability to steal cash. The agent does not steal because by paying down the credit line he gets closer to

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becoming the owner of the project and enjoying its proceeds forever. The fact that the credit limit increases with the agent's performance solves adverse selection: an initially shorter credit limit discourages agents with a bad project from obtaining such financing.

The form of the contract captures the intuitive distinction between adverse selection and moral hazard: the adverse selection is most prominent at the beginning, while moral hazard persists through time. This intuitive distinction also motivates dynamic contracts as an appropriate modeling tool. It would be difficult, if not impossible, to adequately illustrate this dynamic reasoning in a two-period model.

Fortunately, we can make use of the simple dynamic models developed recently by DeMarzo and Fishman (2004) and DeMarzo and Sannikov (2005). In those settings a risk-neutral but cash-constrained agent runs a project that suffers from the moral hazard problem (i.e. the agent can steal cash). Assuming that the agent discounts future cash flows at the market rate and the project may not generate losses, the optimal contract takes the form of a credit line with a market interest rate a *fixed* credit limit equal to the net present value (NPV) of the project's expected future cash flows. The investors liquidate the project if the agent reaches the credit limit, and transfer ownership rights to the agent if he pays down the entire credit line. The contract works because the agent treats a balance on the credit line as cash: his expected future payoff from continuing to run the project is always the project's NPV minus the balance on the credit line, and the interest rate on the credit line equals the interest earned from savings. This contract is shown to be optimal among all history-dependent contracts, but the intuition for optimality is easiest to see by doing a comparative static on the credit limit. Intuitively, a contract with a shorter credit limit would be suboptimal because it would trigger termination too soon, while a contract with a longer credit limit would not be incentive-compatible because it gives the agent too much room to steal.

When we introduce adverse selection, we find that in the optimal contract the credit limit is not constant anymore: it increases with the agent's good performance. Thus, the investors are willing to tolerate more losses if the agent has performed well in the past. This is intuitive because good performance is indicative of good project type. Because we assume that the type of the project is fixed from the start, over time the agent's performance becomes less important to infer his type. Thus, the credit limit eventually converges to its final length, which is the optimal length without adverse selection.

These results resemble many financial arrangements used in practice. Firms and individuals obtain longer credit lines with lower interest rates when they improve their credit ratings. Also, venture-capital financing often takes the form of "staged capital commitments," in which financing at the next stage is conditional upon performance in the previous stage (for example, see Sahlman (1987)).

There is large literature which focuses on financing in the presence of moral hazard or adverse selection. For moral hazard alone, Stiglitz and Weiss (1983) argue that termination is an effective incentive device when the agent has choice over project riskiness. Jensen (1986) points out that paying out cash prevents a manager from investing free cash flows inefficiently. Townsend (1979) introduces a model of costly state verification to study moral

hazard, and Diamond (1984) considers the use of non-monetary punishments. Bolton and Scharfstein (1990) study a two-period model, in which the financing in the second period depends of first-period performance. Hart and Moore (1998) derive an implementation of a two-period contract that involves renegotiation when the agent fails to pay off debt. DeMarzo and Fishman (2004) study a fully dynamic setting, and show that the optimal contract takes the form of a credit line. DeMarzo and Sannikov (2005) study properties like the optimal length of a credit line in a continuous-time setting.

A different but related line of literature studies adverse selection. A number of authors study the role of short-term contracts used to reveal information in a setting with adverse selection (see Flannery (1986) for short-term debt contracts and Hermalin (2002) for a setting where workers signal their ability by short-term contracts). Ross (1977) argues that managers can signal the quality of their firms by taking on debt and exposing themselves to bankruptcy costs. Leland and Pyle (1977) study a setting where entrepreneurs signal by retaining equity of their firms. Brennan and Kraus (1987) study general ways of signaling with securities under asymmetric information.

Diamond (1989) and Holmstrom (1999) have elements of both adverse selection and a moral hazard in their models, but they focus on competitive markets and a series of one-period contracts rather than optimal dynamic contracts. Diamond (1989) finds that financing becomes easier over time with good performance in a setting with simple debt contracts, where the moral hazard problem involves the choice of riskiness of the project. Holmstrom (1999) demonstrates that the adverse selection problem provides workers with incentives to put high effort up front to influence market's learning.

Tchisty (2005) and Battaglini and Coate (2005) consider optimal dynamic contracts in settings that involve both adverse selection and moral hazard. Conceptually more related to our paper, Tchisty (2005) derives an optimal financing contract in a setting where only the agent observes a stream of correlated Markov cash flows. Such a setting involves both moral hazard and elements of adverse selection, because when the agent lies to the investors, he becomes better informed about the future distribution of cash flows. Tchisty (2005) finds that the agent must pay a higher interest on the credit line when he is closer to default: this discourages him large cash flows when the project is in the good state. Thus, borrowing is more costly when the adverse selection problem is more severe. Battaglini and Coate (2005) derive an optimal contract between a monopolist and customers whose preferences follow a Markov process. They find that the initial uncertainty about the type of the customer creates a distortion consumption, and that the distortion becomes less severe over time. This result confirms our intuition that the adverse selection problem becomes less severe with time.

This paper is organized as follows. Section 2 presents a model with two types of projects, good and bad, and formulates the problem of finding the optimal screening contract. Such a contract must finance the good project, which involves moral hazard, subject to the constraint that the agent with a bad project would not undertake financing. Section 3 describes two benchmark optimal contracts with only adverse selection or only moral hazard. Section 4 derives the optimal screening contract that deals with both informational

problems. Section 5 explores to what extent moral hazard and adverse selection affect efficiency of the contract, and draws connection between the two informational problems. Section 6 implements the contract with a credit line, in which the credit limit is increased in two stages. Section 7 studies a generalization, in which the agent with a bad project may “fabricate” cash flows using his own funds. We prove that the optimal contract in this setting exhibits increasing financial flexibility, and eventually converges to the benchmark contract with moral hazard alone, unless termination occurs before that. Section 8 concludes.

2 The Model.

We analyze a setting where a cash-constrained risk-neutral agent seeks outside funding K for a project that involves both moral hazard and adverse selection. To model moral hazard, we assume that the project’s cash flows accrue directly to the agent, who can unobservably divert them to consume or save in a private savings account with market interest rate r . To model adverse selection, we assume that there are two types of projects. Good projects produce cash flows at a Poisson rate λ in amounts X . Bad projects that do not produce any cash flows at all. Both projects produce a private non-pecuniary benefit to the agent at rate μ . The projects are indistinguishable to investors, but the agent knows the type of his project.

We would like to characterize an optimal contract to finance the agent with a good project. We allow fully history-dependent contracts, which specify transfers between the agent and the investor as well as a liquidation time. At liquidation time the principal recovers value $L < \lambda X/r$ from the project’s assets. To specify the problem of an investor, who is risk-neutral and who discounts future cash flows at rate r , we take as given the expected payoff W that the agent with a good project gets from the contract and an upper bound $R < \mu/r$ on the payoff that the agent with a bad project would get. The choice of W is determined by the bargaining powers of the principal and the agent. For any division of bargaining powers, the agent’s value W cannot be less than the value of assets he brings into the project, and the principal’s profit must be at least K . Value R can be interpreted in a number of ways. If agents with bad projects greatly outnumber agents with good projects, so the investors find it optimal not to fund bad projects altogether, then R can be the value of cash and assets that the agent is able to contribute to the project himself. The agents with bad projects do not sign a contract if they get a payoff of R or less from doing so. If the proportion of agents with bad projects is insignificant and the investor is a monopolist, he may choose to offer a menu of two contracts, of which the contract designed for the agent with a bad project bribes him not to undertake the contract of the good type. In this case R is the value that the bad type gets from his contract.

Let us formally describe history-dependent contracts. Due to hidden cash flows the investors must rely on the agent’s reports. By the Revelation Principle, we focus on contracts in which the agent chooses to report truthfully. Denote by \hat{Y}_t the agent’s report of cumulative cash flows up until time t . That is, if the agent reports cash flows at times

$t_1 \leq t_2 \leq t_3 \dots$ then

$$\hat{Y}_t = \begin{cases} 0 & \text{for } t \in [0, t_1), \\ X & \text{for } t \in [t_1, t_2), \\ 2X & \text{for } t \in [t_2, t_3), \\ \dots & \end{cases}$$

As a function of the full history of the agent's reports, the contract must specify a termination time τ and cumulative transfers from the agent to the investors D_t . That is, the agent is supposed to make payments to the principal when D_t increases and receive payments when D_t decreases. Formally, τ is a stopping time and D is a random process on the filtration generated by \hat{Y} .

If the agent accepts the contract and runs the project, he must follow some reporting strategy \hat{Y} and consumption strategy C , where cumulative consumption C_t and reported cash \hat{Y}_t are determined by the true history of cash flows up until time t . For any strategy (\hat{Y}, C) the balance on the agent's savings account evolves as follows:

$$dS_t = rS_t dt + dY_t - dC_t - dD_t,$$

where Y are the true cumulative cash flows. An agent's strategy (\hat{Y}, C) is *feasible* if his savings stay nonnegative until the termination time τ .

A truth-telling contract (D, τ, C) specifies history-dependent transfers, termination and a feasible strategy recommendation $(\hat{Y} = Y, C)$. A contract is *incentive-compatible* if the strategy recommendation maximizes the agent's expected utility given (D, τ) , where we assume that if the agent tries to follow a non-feasible strategy, the project is terminated immediately when the agent fails to make a required payment.

Formally, to optimally finance the good project, we must find an incentive-compatible truth-telling contract (D, τ, C) that maximizes the principal's profit from a good project

$$E \left[\int_0^{\tau(Y)} e^{-rt} dD_t(Y) + e^{-r\tau(Y)} L \right]$$

subject to giving the agent a specific value of

$$W = E \left[\int_0^{\tau(Y)} e^{-rt} dC_t(Y) + \int_0^{\tau(Y)} e^{-rt} \mu dt + e^{-r\tau(Y)} S_\tau(Y) \right]$$

and the agent with a bad project a value less than or equal to R .

Remarks. This framework can be easily adapted to the case where the project also requires a continuous infusion of capital at rate ν until termination, if we define start-up capital to be $K + \nu/r$ and the liquidation value to be $L + \nu/r$.

3 Adverse Selection and Moral Hazard Alone.

In this section we discuss the optimal contract in the presence of only adverse selection or only moral hazard. In both cases the project is terminated with a positive probability in case certain performance targets are not met. The contract with the adverse selection alone is straightforward, and can be implemented using a combination of simple debt and equity. The contract with moral hazard alone is not so straightforward, but it can be derived by the methods of DeMarzo and Fishman (2004) and DeMarzo and Sannikov (2005). This contract is dynamic and can be implemented naturally with a credit line. Termination occurs if the credit limit is reached.

3.1 Adverse Selection Alone.

What happens if the project's cash flows are observable to the investors? Without moral hazard termination is needed only to prevent the agent with a bad project from obtaining financing. Therefore, it is optimal to never terminate the project after the first cash flow is observed. If the cash flows never arrive, the project must be terminated at time T_R such that

$$\frac{1 - e^{-rT_R}}{r} \mu = R,$$

because earlier termination would be inefficient and later termination would not prevent the financing of bad projects. Under such arrangement good projects are terminated with probability $e^{-\lambda T_R}$ and the total surplus shared between the investors and the agent is

$$L e^{-(r+\lambda)T_R} + \frac{\mu \lambda X}{r} (1 - e^{-(r+\lambda)T_R}). \quad (1)$$

Since the cash flows are observable, this cash portion of the surplus can be shared in many different ways and the contract has a number of reasonable implementations. One way to implement the contract is through a bond with face value X and maturity T_R held by the investors, and equity shared by the agent and the investors. If the agent does not get any cash flows before time T_R then the bond cannot be paid and default occurs. If the bond is paid, the remaining cash flows can be paid out as dividends.

3.2 Moral Hazard Alone.

Let us discuss the problem of finding a profit-maximizing contract with a specific value W to the good type, without the extra constraint that the principal must screen out the bad type. This problem is easily solved with the methods of DeMarzo and Fishman (2004) and DeMarzo and Sannikov (2005).¹ These papers first show that without loss of generality, one can look for an optimal contract among those contracts, under which the agent chooses to tell the truth and maintain zero savings.

¹Even though the processes by which cash flows arrive in those papers are different from ours, their methods can be readily extended to our Poisson setting.

With this restriction, it can be shown that the optimal contract is based on a single state variable: the agent's continuation value. Denote by $b(W)$ the principal's profit from an optimal contract with value W to the agent of the good type. Because the agent is risk-neutral in our setting, the agency problem exists only because the agent is cash-constrained. Therefore, the agency problem disappears if $W \geq (\mu + \lambda X)/r$: the optimal contract achieves first best by giving the agent $W - (\mu + \lambda X)/r$ to consume up front and by letting him keep all the remaining cash flows. The principal's profit in this region is

$$b(W) = -(W - (\mu + \lambda X)/r).$$

In the region $W \in [0, (\mu + \lambda X)/r]$ it is optimal to postpone payments to the agent and let the agent's continuation value evolve according to

$$dW_t = (rW_t - \mu - \lambda X) dt + d\hat{Y}_t. \quad (2)$$

The term $rW_t - \mu + \lambda X$ reflects accounting to ensure that W_t correctly reflects the agent's value, and the term $d\hat{Y}_t$ reflects incentives: the agent's value increases by X every time he transfers X to the principal. If W_t hits 0, the project is terminated immediately, and if W_t jumps above $(\mu + \lambda X)/r$, the agent gets a payment of $W_t - (\mu + \lambda X)/r$ and keeps all the remaining cash flows. The principal's profit in the region $[0, (\mu + \lambda X)/r]$ is defined by equation

$$rb(W) = \lambda X + (rW - (\mu + \lambda X))b'(W) + \lambda(b(W + X) - b(W)), \quad (3)$$

with boundary conditions $b(0) = L$ and $b((\mu + \lambda X)/r + w) = -w$. Function b is concave.

If the agent has all the bargaining power, then the agent's value W_0 is determined by $b(W_0) = K$ and $b'(W_0) < 0$. Figure 1 is an example of a computed profit function.

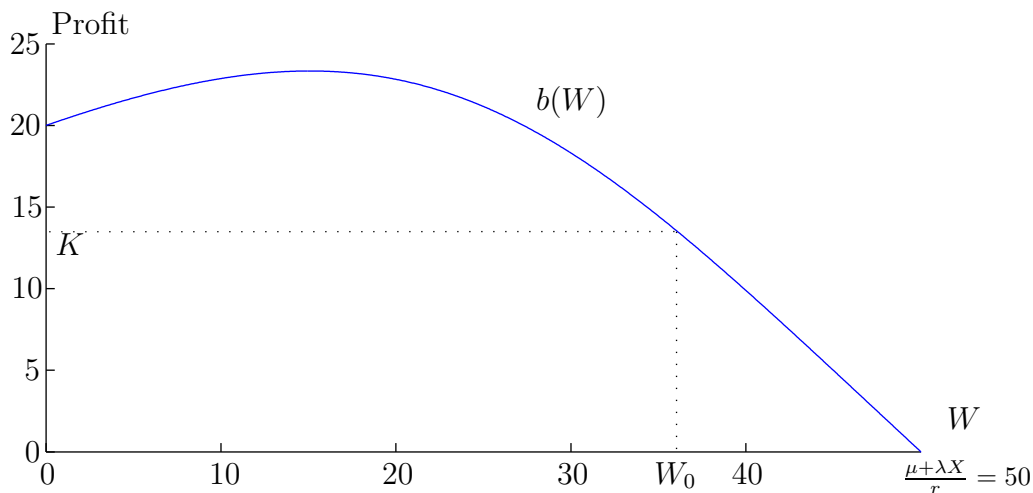


Figure 1: Function $b(W)$ for $r = 0.1$, $X = 10$, $\lambda = 0.3$, $\mu = 2$ and $L = 20$.

The contract has a simple implementation with a credit line that has credit limit $(\mu + \lambda X)/r$ and interest r . Variable $M_t = (\mu + \lambda X)/r - W_t$ can be interpreted as the balance on the credit line, which evolves according to equation

$$dM_t = rM_t - d\hat{Y}_t$$

until the credit line is paid off or the credit limit $(\mu + \lambda X)/r$ is reached. If the credit limit is reached, the project is terminated. If the credit line is paid off, the agent becomes the sole owner of the project and consumes all the remaining cash flows. When that happens, the moral hazard problem goes away.

We will refer to this contract as the one-stage contract. It will be useful as a building block for the optimal screening contract in the next section, which addresses both moral hazard and adverse selection.

4 The Screening Contract.

In this section, we are interested in the profit-maximizing contract which gives the agent with a bad project a value that does not exceed R and the agent with a good project a specific value of $W > R$. Because the one-stage contract is performance-based, it may be successful at screening out bad projects even without extra provisions. In Section 4.1 we derive conditions, under which an agent with a bad project chooses not to accept a one-stage contract and recovers a salvage value of R instead. After that, we solve for the optimal contract for the case when the one-stage contract fails at screening.

4.1 When is the one-stage contract is successful at screening?

To answer his question, let us find the value that the agent with a bad project obtains from a contract which gives value $W \in [0, (\mu + \lambda X)/r]$ to the agent with a good project. First, note that the good type is always indifferent between reporting cash and stealing it for consumption. Therefore, the good type derives his value W from the one-stage contract even if he steals and consumes all cash flows. In case he does so, the amount of time T he has to steal and consume must satisfy

$$(\mu + \lambda X) \frac{1 - e^{-rT}}{r} = W.$$

From time 0 until time T the agent with a bad project would derive value

$$\mu \frac{1 - e^{-rT}}{r} = \frac{\mu W}{\mu + \lambda X}$$

from private benefit. We conclude that the one-stage contract successfully screens out the bad type if

$$\frac{\mu W}{\mu + \lambda X} \leq R.$$

4.2 Optimal screening contracts when the one-stage contract fails.

What happens if the outside option of the bad type is so small that the one-stage contract would still attract him? What is the optimal contract that delivers values (W^B, W) with $W^B < \mu W / (\mu + \lambda X)$? When we solve this problem, we will find the optimal contract whose implementation involves a credit line with an increasing credit limit. The agent starts with a small credit limit, which increases to the credit limit of the one-stage contract after the agent reports cash for the first time.

In search for the optimal contract, it is useful to know that we can focus on contracts in which the agent with a good project does not consume until the first cash flow arrives or until termination.

Lemma 1. *Consider any incentive-compatible contract (D, τ, C, \hat{Y}) . Then there is a payoff-equivalent contract in which the agent with a good project does not consume until the first cash flow arrives or until termination.*

The principal and both types of the agent must derive the same value from a *payoff-equivalent* contract.

Proof. Denote by T' earlier of the termination time or the time of the first cash flow. Consider a modification of the agent's strategy in the contract (D, τ, C, \hat{Y}) : instead of consuming before time T' , he saves what he would have consumed, and consumes the savings at time τ . Then, clearly the agent derives the same value as before. Therefore, the contract with this modification of the agent's strategy remains incentive compatible, and the agent does not consume until termination or until the first cash flow. \square

As in related work without screening, we can show that one can restrict attention to truth-telling contracts without savings.

Lemma 2. *Let (D, τ, C, \hat{Y}) be an incentive-compatible contract that delivers a value pair (W^B, W) . Then $(Y - C, \tau(\hat{Y}), C, Y)$ is a payoff-equivalent incentive-compatible contract, in which the good type chooses to tell the truth and maintains zero savings.*

Proof. DeMarzo and Sannikov (2005) show in a similar setting that it is optimal for the good type to tell the truth and maintain zero savings in the new contract. Moreover, the principal and the agent of the good type will get the same value in both contracts.

Let us show that the bad type will get the same value from the new contract as from the old contract. Because the agent cannot report cash if he has not received any cash flows, $\hat{Y}(0) = 0$. Therefore, $\tau(0) = \tau(\hat{Y}(0))$, so the time when the project is terminated if the agent does not report any cash flows is the same in both contracts. We conclude that the bad type must also get the same value from both contracts. \square

Lemmas 1 and 2 imply that we can focus on contracts that provide incentives to the agent to report cash flows as soon as they arrive and do not make any transfers to the agent until termination or the first cash flow. To attack our problem, we relax the set of incentive constraints that the investors need to satisfy, and find the profit-maximizing contract subject to those weaker constraints. After we find such a contract, we check whether it is fully incentive-compatible.

To relax the incentive constraints, we pick a few important constraints implied by the original problem. First, letting T denote the termination time if the agent does not report any cash, the promise-keeping constraint of the bad type is

$$W^B = \frac{1 - e^{-rT}}{r} \mu + e^{-rT} C_T, \quad (4)$$

where C_T is the consumption the agent receives in case default occurs at time T . Intuitively we expect that $C_T = 0$ and so $T = T_{W^B}$ in the optimal contract, but we have not ruled out the possibility that $C_T > 0$ and $T < T_{W^B}$ yet. When $W^B < \mu W / (\mu + \lambda X)$, the default time T is faster than that of a one-stage contract with value pair $(\mu W / (\mu + \lambda X), W)$.

Second, denote by $V(t) + X$ the value that the good type gets if he reports the first cash flow at time t before default. By the truth-telling constraint, the good type prefers to report cash immediately upon arrival, rather than delay the report and meanwhile consume interest and all other cash flows. Therefore,

$$e^{-rs} ((e^{rs} X - X) + V(t + s)) + \int_0^s e^{-ru} (\mu + \lambda X) du$$

is nonincreasing at $s = 0$. Differentiating with respect to s we find that

$$\mu + \lambda X + rX - rV(t) + V'(t) \leq 0. \quad (5)$$

Let us replace the truth-telling constraint of the good type with a weaker constraint (5). Lastly, we need the promise-keeping constraint for the good type

$$W = \int_0^T e^{-(r+\lambda)t} (\mu + \lambda V(t)) dt + e^{-(r+\lambda)T} C_T. \quad (6)$$

$$\begin{aligned} R &\geq \int_0^T e^{-rt} \mu dt + e^{-rT} C_T \\ &\Rightarrow T \leq T_R \end{aligned}$$

Our relaxed problem involves constraints (4), (5) and (6).

We proceed as follows: we fix T and C_T that satisfy (4) and find the contract that maximizes the investor's profit subject to (5) and (6). After that, we will show that among these contracts, the profit is maximized when $C_T = 0$ and the agent gets as much time as possible without violating (4), i.e. when $T = T_{W^B}$. At the end of this section, we verify the contract that we have derived is fully incentive compatible.

4.3 The Solution to the Relaxed Problem.

For any path of $V(t)$, $t \in [0, T]$ that satisfies (5) and (6), the profit is maximized if the principal switches to the one-stage optimal contract with value $V(t) + X$ in case agent reports cash at time t for the first time. Then at time 0 the principal's expected profit is

$$\max_{T, C_T, V(t)} \int_0^T e^{-(r+\lambda)t} \lambda (b(V(t)) + X) dt + e^{-(r+\lambda)T} (L - C_T) \quad (7)$$

$$\max_{V(t)} \int_0^{T_R} e^{-(r+\lambda)t} \lambda (b(V(t)) + X) dt + e^{-(r+\lambda)T_R} L$$

$$W = \int_0^{T_R} e^{-(r+\lambda)t} (\mu + \lambda V(t)) dt.$$

$$C_T = 0, \quad T = T_R$$

$$\max_{T, C_T, V(t), \text{contract at } t} \int_0^T e^{-(r+\lambda)t} \lambda (b_t + X) dt + e^{-(r+\lambda)T} (L - C_T)$$

Lemma 8 in the Appendix implies that, subject to (5) and (6), expression (7) is maximized by choosing $V(t)$ that solves (5) with equality.² Intuitively, because b is a concave function, it is optimal to choose the path of V with as little variation in V as possible.

By solving (5) with equality and from (6), we find that $V(t)$ is defined by

$$V(t) - \frac{\mu + \lambda X}{r} = e^{rt} \left(V(0) - \frac{\mu + \lambda X}{r} \right) \quad (8)$$

where $V(0)$ is determined by (6). Let us refer to this contract as the *deadline contract*. The deadline contract determined by $V(0)$, T and C_T differs from the one-stage contract with value $V(0)$ only in earlier termination and a terminal payment. Indeed, (2) implies that in a one-stage contract with value $V(0)$ the agent's continuation value would be $V(t) + X$ if he reports cash at time $t \leq T$ for the first time, the same as in the deadline contract. Because in the simple contract $V(t)$ would reach 0 at the termination time, termination occurs earlier in the deadline contract than in the simple contract. The following lemma gives the principal's profit and the agent's value in the deadline contract.

Lemma 3. *The value of the good type in the deadline contract satisfies*

$$W = V(0) - e^{-(r+\lambda)T} (V(T) - C_T) \quad (9)$$

²Indeed, if the path that solves (5) with equality satisfies $V(0) < \frac{\mu + \lambda X}{r}$, then Lemma 8 applies directly. If not, then $b(V(t) + X)$ is linear for all $t \in [0, T]$ and the path that solves (5) with equality clearly maximizes (7) (as any other path for which $V(t) + X \geq \frac{\mu + \lambda X}{r}$ for all $t \in [0, T]$).

and the principal's profit is given by

$$b(V(0)) - e^{-(r+\lambda)T}(b(V(T)) - L + C_T). \quad (10)$$

Among such contracts with T and C_T satisfying (4) the principal's profit is maximized when $T = T_{WB}$ and $C_T = 0$.

Proof. The profit from the deadline contract is equal to the profit from a simple contract with value $V(0)$ less the value lost due to early termination $e^{-(r+\lambda)T}(b(V(T)) - L)$, less the expected cost of the terminal payment $e^{-(r+\lambda)T}C_T$, where e^{-rT} is the discount factor at time T and $e^{-\lambda T}$ is the probability of reaching time T without a single cash flow arrival. Similarly, it follows that the payoff of the good type is given by (9).

To show that among such contracts the principal's profit is maximized when $C_T = 0$, note that rather than pay C_T at time T , it is better to postpone termination until time $T' > T$ such that

$$e^{-(r+\lambda)T}(V(T) - C_T) = e^{-(r+\lambda)T'}V(T')$$

with $V(t)$, $t \in (T, T']$ determined by (8). When $C_T = 0$, condition (4) implies that $T = T_{WB}$. \square

We conclude that $C_T = 0$ in a deadline contract that achieves a pair (W^B, W) in an optimal way. If $W^B = \mu W / (\mu + \lambda X)$ then we get the simple contract, in which $V(0) - X = W$ and the good type of the agent is just indifferent between reporting cash or stealing. When $W^B < \mu W / (\mu + \lambda X)$ then the good type of the agent strictly prefers to report. This is intuitive. To keep the value of the bad agent down in order to "screen him out," we need to precipitate default in case cash flows fail to arrive. At the same time, we need to deliver a high value to the good type. Therefore, we need to reward him extra in case the cash flows are reported.

4.4 Verification.

Let us verify that a deadline contract is fully incentive compatible. It is trivial to see that the bad type would get value W^B from this contract, because he has nothing to report and (4) implies that he gets value W^B . Let us verify that the good type has incentives to tell the truth and not to save. Condition (5) implies if the good type is able to report cash, he gets bigger utility by $V(0) - W$ in time 0 terms if he reports cash once than if he never reports cash. After the first report, in the remaining one-stage contract the good type is indifferent between all alternative strategies, as long as he eventually consumes all his savings. Therefore, the truth-telling and zero savings strategy is optimal for the good type, because he takes advantage of the opportunity to report the first cash flow whenever he can. We conclude that the deadline contract is fully incentive compatible for both types.

We have finished deriving the optimal screening contract. In the next section we discuss our results and the interaction of adverse selection and moral hazard in the optimal contract. In Section 6 we derive an elegant implementation of the optimal contract with a credit line that has an increasing credit limit.

5 Moral Hazard, Adverse Selection and Profit.

In this section we discuss the principal's profit in the optimal solution and the role of moral hazard and adverse selection. First, we need a lemma to determine initial conditions.

Lemma 4. *If $W > (\mu + \lambda X)W^B/\mu$, then the principal's profit is increasing in W^B .*

Proof. To be completed. □

Recall that the principal's problem is to find a profit-maximizing contract with a specific value W to the good type, subject to giving the agent with a bad project a value that is less than or equal to R . Lemma 4 implies that when $W > (\mu + \lambda X)R/\mu$, the principal will offer a contract, from which the agent with a bad project would get a value of exactly R . When $W \leq (\mu + \lambda X)R/\mu$ then the adverse selection problem does not affect the optimal contract.

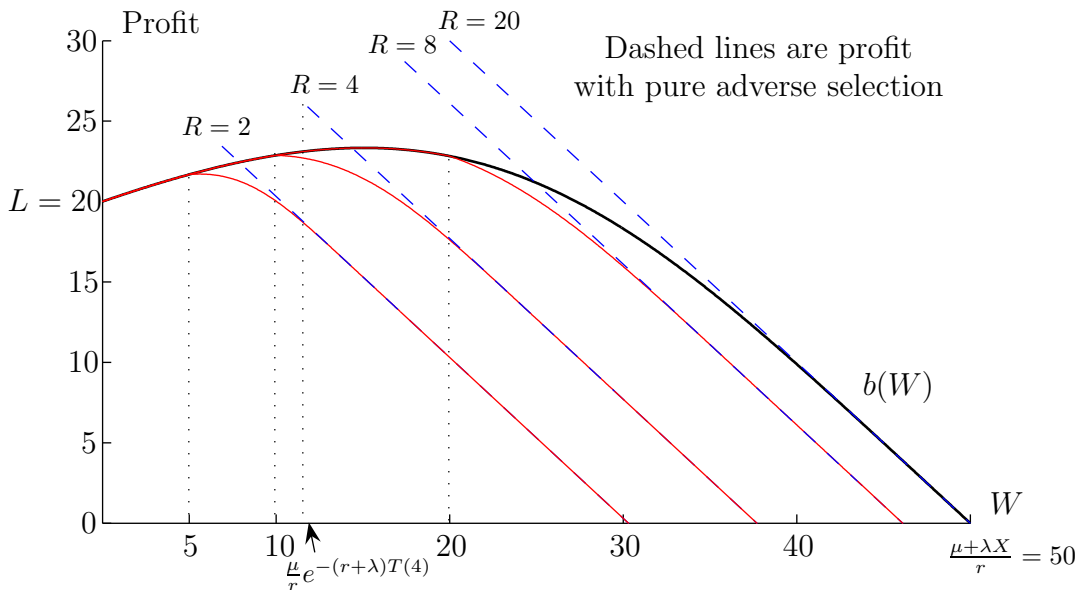


Figure 2: Profit for $R = 2, 4, 8$ and 20 when $r = 0.1$, $X = 10$, $\lambda = 0.3$, $\mu = 2$ and $L = 20$.

Figure 2 shows the principal's profit as a function of W for different values of R . The principal's profit can be computed using (8), (9) and (10) with $T = T_R$ and $C_T = 0$. Note that each profit function coincides with $b(W)$ on the interval $W \in [0, (\mu + \lambda X)R/\mu]$, where the one-stage contract is successful at screening. Also, the principal's profit coincides with profit with adverse selection alone when

$$W \geq \left(\frac{\mu + \lambda X}{r} - e^{-rT_R} X \right) - e^{-(r+\lambda)T_R} \left(\frac{\mu + \lambda X}{r} - X \right).$$

For W in this range the agent consumes all remaining cash flows after the first report, which must be made before the deadline of T_R .³ Therefore, the contract is essentially the same as in the pure adverse selection problem, which determines the termination time T_R . The principal's profit with pure adverse selection is shown by a dotted line in Figure 2.

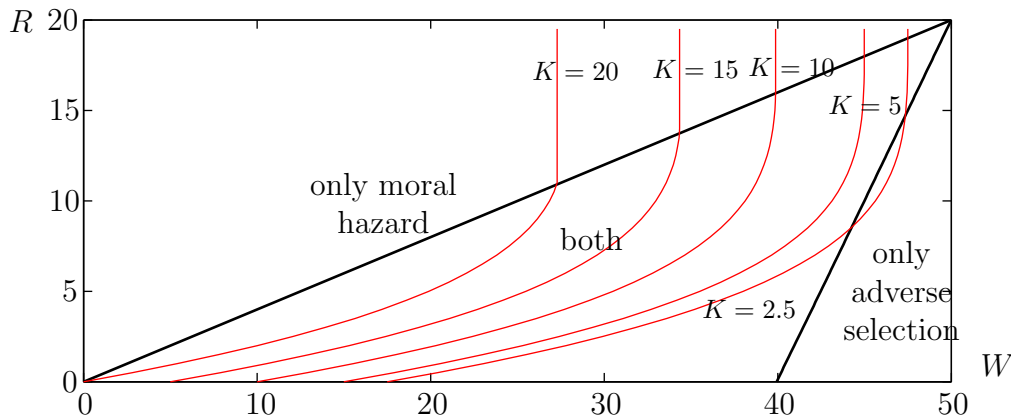


Figure 3: Agent's starting value, for which the principal breaks even, for different values of R and K when $r = 0.1$, $X = 10$, $\lambda = 0.3$, $\mu = 2$ and $L = 20$.

Figure 3 shows regions in the space of W and R where the adverse selection or the moral hazard problem disappears. Also, it shows how the agent's starting value W depends on the required start-up capital K and R when the investors act competitively.

6 Implementation.

In this section we derive an implementation of the optimal screening contract with a credit line that has an increasing credit limit, and summarize the main results. To do the implementation, we map contract state variables into a balance on the credit line. Before the agent reports the first cash flow, it is convenient to look at variable $V(t)$, which evolves as follows

$$\frac{\mu + \lambda X}{r} - V(t) = e^{rt} \left(\frac{\mu + \lambda X}{r} - V(0) \right).$$

If the agent reports cash at time t for the first time, we switch to a one-stage contract with the agent's initial continuation value $W_t = V(t) + X$. Thereafter, the agent's continuation value evolves as

$$\frac{d}{ds} \left(\frac{\mu + \lambda X}{r} - W_s \right) = r \left(\frac{\mu + \lambda X}{r} - W_s \right)$$

between reports, and W_s increases by X whenever the agent reports cash.

³The principal may keep a fraction of the first reported cash flow.

Therefore, the variable

$$M_s = \begin{cases} \frac{\mu + \lambda X}{r} - V(s) & \text{before 1st report} \\ \frac{\mu + \lambda X}{r} - W_s & \text{after} \end{cases}$$

behaves exactly like a balance on the credit line: it grows at an interest rate r between reports, and decreases by X whenever the agent reports cash and gives it to the principal. If $M_s < X$ and the agent reports X at time s , then only M_s is deposited onto the credit line and the agent consumes $X - M_s$ as well as all the remaining cash flows. The credit limit is given by $\frac{\mu + \lambda X}{r} - V(\tau)$ before the first report and $\frac{\mu + \lambda X}{r}$ thereafter.

We summarize our main result in the following theorem:

Theorem 1. *The following implementation gives the optimal contract with value W to the agent with a good project, subject to the constraint that the agent with a bad project would get value W^B from the same contract. The agent gets a credit line with interest r , an initial draw of M and an initial credit limit \bar{M} . After the agent makes the first payment on the credit line, the credit limit increases to $(\mu + \lambda X)/r$. The initial balance and credit limit are determined by*

$$W = \left(\frac{\mu + \lambda X}{r} - M \right) - \left(\frac{M}{\bar{M}} \right)^{\frac{r+\lambda}{r}} \left(\frac{\mu + \lambda X}{r} - \bar{M} \right) \quad \text{and} \quad W^B = \frac{1 - M/\bar{M}}{r} \mu.$$

Under this contract, it is optimal for the agent with a good project to deposit all cash flows he receives until the credit line is paid off. After the credit line is paid off, the agent consumes all remaining cash flows.

6.1 Properties of the Optimal Contract.

In this section we explore the properties of the optimal contract. Assuming that investors are competitive and that the salvage value of an unfunded project is R , we consider the problem of maximizing the value of the agent with a good project subject to the constraints that the investors break even and the agent with a bad project gets at most R . We are interested in how the initial balance and credit limit depend on the scrap value R , start-up capital K , liquidation value L and other parameters.

Figure 4 shows some counterintuitive results: that both the credit limit and the starting balance can be nonmonotonic in R . The left panel shows nonmonotonicity in the credit limit, which is driven by two factors. First, the credit line is affected by the degree of adverse selection. Greater adverse selection causes the principal to cut the initial credit line. The second factor that determines the initial credit limit is the size of the initial draw: a longer credit limit is required to pay interest on a higher initial balance. The U-shaped form of the optimal credit limit as a function of R in the left panel illustrates those effects.

The right panel of Figure 4 shows that the initial draw on the credit line may actually *decrease* when adverse selection becomes worse. This can happen when the project has

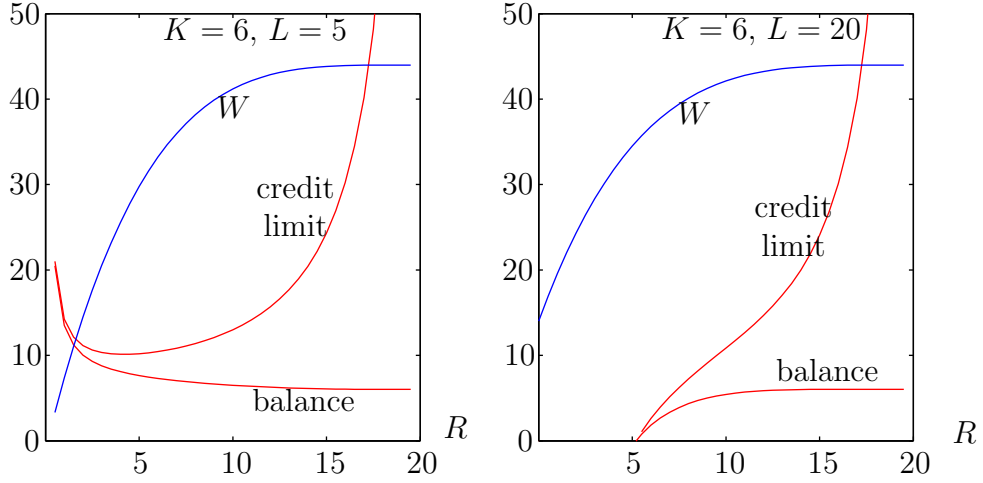


Figure 4: Credit limit, initial balance and the value of the good type as functions of R when $r = 0.1$, $X = 10$, $\lambda = 0.3$ and $\mu = 2$.

a liquidation value higher than the required start-up capital. Then the principal benefits from earlier liquidation due to adverse selection and the agent must borrow less through the credit line to raise start-up capital.

The following lemma illustrates the effects of capital and liquidation value on the initial conditions.

Lemma 5. *With a higher start-up capital K , the optimal contract involves higher initial balance and credit limit. A higher liquidation value involves a lower initial balance, but also a lower credit limit.*

Proof. If we increase K , the agent's starting value

$$W = \int_0^{T_R} e^{-(r+\lambda)t} \lambda(V(t) + X) dt$$

has to fall. Therefore, $V(t)$ has to fall for all t . It follows that the initial balance $\frac{\mu+\lambda X}{r} - V(0)$ and the initial credit limit $\frac{\mu+\lambda X}{r} - V(T_R)$ must increase.

Recall that the principal's profit is given by

$$\int_0^{T_R} e^{-(r+\lambda)t} \lambda(b(V(t) + X) + X) dt,$$

where $V'(t) = -(\mu + \lambda X - rV(t))$. Holding everything else fixed, the principal's profit must be decreasing in the choice of $V(0)$, because otherwise we would be able to make both the agent and the principal better off by increasing $V(0)$. As we increase L , the principal's

profit improves if we keep everything else fixed. In order for the principal's profit to stay constant (equal to R), we must also increase $V(0)$, which causes $V(t)$ to increase for all t . Therefore, as we increase L , the agent gets a lower balance and a lower starting credit limit. \square

In our credit line contract the agent is not allowed to draw balance for consumption: the credit line is drawn only to pay interest on existing balance. This feature is distinct from the credit line contracts of DeMarzo and Fishman (2004) and DeMarzo and Sannikov (2005), in which the agent may be allowed to draw for consumption (but would choose not to do so). Tchisty (2005) studies the adverse selection problem which arises due to correlated cash flows and also discovers that the agent must not be able to draw for consumption. However, both the agent with a good project in our setting and the agent the good state in Tchisty's setting would choose not to draw the credit line for consumption even if they could. The following lemma shows that by drawing a dollar from the credit line, the good type would lose more than a dollar in promised value.

Lemma 6. *We have*

$$\frac{d}{dM} \left(\left(\frac{\mu + \lambda X}{r} - M \right) - \left(\frac{M}{\bar{M}} \right)^{\frac{r+\lambda}{r}} \left(\frac{\mu + \lambda X}{r} - \bar{M} \right) \right) < -1.$$

Proof. To be completed. \square

Of course, allowing the agent to draw from the credit line for consumption would undermine our contract by making it attractive for the bad type.

7 Multi-stage Contracts.

The two-stage contract that we derived relies on the assumption that once the agent reports the first cash flow, the principal becomes convinced that the project is good. However, if the agent with a bad project has his own funds, he may be able to fabricate the first cash flow to mislead the principal. Would he want to do that? If so, how can we modify the contract to take care of this possibility? Will the contract still exhibit short credit limits at first? Will the agent eventually achieve a full credit limit? Although we are unable to characterize the optimal contract in full generality, we answer all of the above questions in this section. In particular, we show that the investors give an increasing degree of financial flexibility to the agent, and the contract eventually converges to the one-stage contract with a fixed credit limit.

7.1 Fabrication of cash flows in a two-stage contract.

Before we formulate a more general model, let us explore the possibility that the bad type may fabricate cash flows from his own funds. The following lemma identifies the range of initial conditions when he would want to do so.

Lemma 7. *The contract of Theorem 1 is vulnerable to the possibility that the bad type may fabricate cash flows if and only if*

$$\bar{M} < \frac{\mu + \lambda X}{r} - \frac{\lambda X^2}{\mu}. \quad (11)$$

Proof. Let us see how a bad type with cash would act in a two-stage contract of Theorem 1. First, let us show that the bad type would fabricate at most one cash flow. When the agent reports cash, it can be seen as an action to buy extra time from the principal to run the project before liquidation occurs. After the first report, the good type is indifferent between “buying time” by reporting cash flows or not. Because the bad type values time less, he would not fabricate any more cash flows after the first one.

Now, let us see if the bad type would fabricate the first cash flow (and when). The good type is indifferent about when he reports the first cash flow, as long as he does so before termination. By reporting later, he buys less time for less money (in time 0 terms). Because the bad type values time less, he would prefer to report the first cash flow at time $T(R)$ rather than earlier. By doing so, he ends up with balance $\bar{M} - X$ on the credit line with a credit limit of $\frac{\mu + \lambda X}{r}$, and so he has time T' given by

$$e^{-rT'} = \frac{\bar{M} - X}{(\mu + \lambda X)/r}$$

to enjoy the non-pecuniary benefits of running the project. This is worth

$$\mu \frac{1 - e^{-rT'}}{r} = \mu \left(\frac{1}{r} - \frac{\bar{M} - X}{\mu + \lambda X} \right)$$

to him. Therefore, the bad type can benefit by fabricating one cash flow if and only if

$$\mu \left(\frac{1}{r} - \frac{\bar{M} - X}{\mu + \lambda X} \right) > X \quad \Leftrightarrow \quad \bar{M} < \frac{\mu + \lambda X}{r} - \frac{\lambda X^2}{\mu}.$$

□

Therefore, the vulnerability of the two-stage contract depends exclusively on the credit limit in the first stage. If the first credit limit is much shorter than the full credit limit, then the bad type buys enough type by fabricating the first cash flow just to prior to default at the first stage. Condition (11) shows that the contract is more likely to be vulnerable when the private benefit μ is bigger and when the cost X to obtain a full credit limit is smaller.

7.2 A generalized model and the possibility of hidden cash.

We modify the model to allow for the possibility that the agent has initial cash funds $Y_0 \in [0, \infty)$. For greater generality, we assume that the cash flows from a good project, which arrive with intensity λ , have a CDF $F : [0, \infty) \rightarrow [0, 1]$. The bad project may also

generate cash flows with intensity $\lambda' \geq 0$ from the CDF $F' : [0, \infty) \rightarrow [0, 1]$. Denote by X and X' the mean of cash flows from the good and bad projects respectively. Assume that

$$L, \frac{\mu + \lambda' X'}{r} < K + R,$$

i.e. it is better to obtain the salvage value of the bad project at time 0 than to fund it.

We seek to characterize the properties of a contract that a good agent with cash Y_0 obtains when investors are competitive. Then the relevant problem is to find a profit-maximizing contract with value W to a good agent with cash Y_0 , subject to giving any agent with a bad project and cash Y'_0 a value of at most $R + Y'_0$. In this formulation we are concerned about the possibility that the agent with a bad project might take the contract, but ignore the possibility that a the good agent with a different amount of cash might take the contract. We do so because a good agent with less cash than Y_0 cannot pretend to have cash Y_0 in a direct revelation contract, and a good agent with more cash than Y_0 would prefer to reveal it to the principal up front and get a better contract, since having more cash diminishes the moral hazard problem.

A full characterization of the optimal contract would be extremely difficult due the the dimensionality of the problem. If one approaches the problem using the methods of Fernandes and Phelan (2000), the recursing structure of the contract would involve two functions as state variables: the continuation values of good and bad types as functions of hidden savings.⁴ However, despite being unable to provide a complete characterization, we are able to derive many intuitive properties of the optimal solution. Proposition 1 the implications of adverse selection on the increasing degree of financial flexibility in an optimal contract. Proposition 2 shows that after the agent reports enough cash to the principal, the adverse selection problem disappears and the contract involves a simple credit line with a fixed credit limit. Finally, we propose a natural contract which satisfies the above properties and involves a sequence of increasing credit lines. We suspect that this contract may in fact be optimal in many settings (but not all).

Remark. In the absense of agents with a bad project, this would be a pure moral hazard problem. As before, the optimal contract would take the form of a credit line with a fixed credit limit of $(\mu + \lambda X)/r$.

7.3 Increasing financial flexibility and eventual convergence to a constant credit line.

As before, we focus on truth-telling contracts and assume that the agent gives all reported cash to the principal until termination or until a time sufficiently removed in the future. Indeed, for an arbitrary contract, it is always possible to postpone payments from the principal and the agent's consumption until termination or an arbitrary date in the future.

⁴The problem may be somewhat simplified by focusing only on the behavior of the bad type with unlimited cash, because he can get at least as much utility from the project as any other bad type.

This modification will not change the principal's profit or the utility of a good type with cash Y_0 because the principal and the agent are risk-neutral and discount consumption at the same rate r . This modification may hurt a cash-constrained agent with a bad project, but may not make any agent with a bad project better off.

With a simpler set of contracts to work with, denote by

$$d_t(\hat{Y}) = \int_0^t e^{-rs} d\hat{Y}_s$$

the time 0 value of the agent's reports to the principal (which the agent must transfer to the principal). Denote by $T(\hat{Y})$ the liquidation time as a function of the agent's reports. The following proposition gives a necessary and sufficient condition under which the agent with a bad project will never undertake financing.

Proposition 1. (Incentive compatibility for the bad type). *A bad type with any initial amount of cash will not take the project if and only if for all reports \hat{Y}*

$$R + d_{T(\hat{Y})}(\hat{Y}) \geq \mu \frac{1 - e^{-rT(\hat{Y})}}{r} + e^{-rT(\hat{Y})} C_{T(\hat{Y})} \quad (12)$$

Proof. The right hand side of (12) is the bad type's non-pecuniary and a final cash benefit when he takes the contract, reports \hat{Y} (assuming he has enough cash) and achieves a termination at time $T(\hat{Y})$. The left hand side is the value the agent gets if he recovers salvage value of the project and consumes cash $d_{T(\hat{Y})}(\hat{Y})$ instead. If (12) fails for some report \hat{Y} , the bad type with enough cash does strictly better by accepting financing than by recovering the salvage value. If (12) holds for all reports, then the bad type is always strictly better off not accepting financing. \square

Proposition 1 implies an upper bound on how long the project can run given the time 0 value d of the amount of cash that the agent gives to the principal. The project must be terminated before time

$$\frac{-\log(1 - r(R + d))}{r} \quad (13)$$

Therefore, initially the financial slackness that the agent gets from the principal can be severely restricted by the adverse selection problem. The following proposition implies that eventually the agent gets a full credit line of length $(\mu + \lambda X)/r$, unless early termination occurs due to unsatisfactory performance.

Proposition 2. (Eventual convergence). *There exists an optimal contract in which, after time 0 value of the agent's reports $d_t(\hat{Y})$ becomes at least $\mu/r - R$, the agent faces a simple credit line with a fixed limit of $(\mu + \lambda X)/r$, assuming that the value of the good type at that point of time is less than or equal to $(\mu + \lambda X)/r$.⁵*

⁵The next version of the paper will deal with this additional assumption.

Proof. Denote by τ the first time when $d_t(\hat{Y})$ becomes bigger or equal than $(\mu + \lambda X)/r$. Denote by W_τ the continuation value of the good type at time τ in case he has zero savings. Let us show that if we replace the continuation contracts at time τ with the one-stage contract with value W_τ (for the good type), this does not alter the incentives of either type of the agent. First, the bad type would not take the original contract and report of cash \hat{Y} to get a one-stage contract with value $W_\tau \leq (\mu + \lambda X)/r$ because his non-pecuniary benefit from such a contract cannot be greater than μ/r . Indeed, if he reaches the one-stage contract at time τ , he maximizes his payoff by simply collecting the non-pecuniary benefit until termination. Second, the replacement at time τ gives the good type exactly the same payoff unless he lies to build up savings for time τ . However, before replacement the value of having extra x dollars of cash at time τ gave the good type a benefit of x or more. Indeed, if not, then if the good type got a cash flow of size x at time τ , then he would not report it. After replacement, the value of having extra x dollars at time τ gives the good type a benefit of exactly x . Therefore, replacement could only diminish the incentives of the good type to lie and build up savings for time τ . \square

Because the optimal contract is very difficult to characterize and it is questionable how much insight can be gained by the complexity of the complete solution, here we propose a very simple and natural contract, which may be sometimes suboptimal. This contract satisfies the conditions of Propositions 1 and 2 and possesses several other appealing properties. It illustrates with simplicity the spirit of restrictions imposed by adverse selection and moral hazard: it screens out the bad type with any amount of cash (Proposition 1) and is incentive-compatible for the good type.

The contract is based on a credit line with a credit limit that increases appropriately every time when the agent reports cash. Between reports, the balance on the credit line grows at rate r . When the agent reports a cash flow of size x , he gives it to the principal and the balance on the credit line falls by x . At the same time, the credit limit increases by the largest amount allowed by the upper bound (13) on time that the agent may be allowed to run the project. The credit limit cannot exceed $(\mu + \lambda X)/r$, however. Once the credit limit reaches $(\mu + \lambda X)/r$, the agent runs a one-stage contract with a fixed credit line. If the balance on the credit line reaches 0, the agent gets to keep all the remaining cash flows. If the agent reaches the credit limit, termination occurs and the agent walks away with nothing.

In this contract, the credit limit is completely determined by the time 0 value of cash that the agent reports. Specifically, the credit limit is

$$\min \left(\frac{\mu + \lambda X}{r}, \frac{(M - d)\mu/r}{\mu/r - R - d} \right),$$

where M is the starting balance on the credit line.

Also, the contract shares the property of the two-stage contract of the previous section that the good type is indifferent about the timing of his cash reports, as long as he does so before termination. Before the full credit limit is reached, the agent strictly prefers to

report cash at some point rather than consume it. After the full credit limit is reached, the agent is indifferent between all reports and consumption choices.

8 Conclusion.

We consider a dynamic financing setting with adverse selection and moral hazard. Both informational problems require contracts, in which termination and future financing are performance-dependent. Depending on initial conditions, we find that sometimes only one problem poses real concern to investors. When the agent with a bad project has a sufficiently high outside option, then only the adverse selection problem is real. When the adverse selection is severe, the incentives that the agent has from trying to convince the investors that he is a good type solve the moral hazard problem.

When both information problems are real concerns, we derive a simple optimal contract in a dynamic setting. Our implementation of the optimal contract involves a credit line, whose credit limit increases over time with the agent's good performance. Because interest is charged on the credit line balance, pushing the agent closer to default when he fails to make sufficient payments, the credit line with a fixed credit limit solves the moral hazard problem. The increasing credit limit solves adverse selection, creating a trial period before giving the agent a full optimal amount of credit. These features of the optimal contract are similar to many arrangements used in practice.

9 Appendix.

Lemma 8. *Let $F : \mathfrak{R} \rightarrow [0, \infty)$ be a strictly positive function and $g : [0, \tau] \rightarrow \mathfrak{R}$ be a density function. Suppose that $V : [0, \tau] \rightarrow \mathfrak{R}$ and $W : [0, \tau] \rightarrow \mathfrak{R}$ satisfy*

$$V'(t) = F(V(t)), \quad W'(t) \geq F(W(t)), \quad \int_0^\tau V(t)g(t) dt = \int_0^\tau W(t)g(t) dt.$$

Then for any concave function b

$$\int_0^\tau b(V(t))g(t) dt \geq \int_0^\tau b(W(t))g(t) dt \tag{14}$$

Proof. Without loss of generality let us carry out the proof for the case when $W'(t) > F(W(t))$. Indeed, if (14) fails for a function W that satisfies $W'(t) \geq F(W(t))$, then it must fail for a slightly perturbed function that satisfies $W'(t) > F(W(t))$.

Denote by t_v and t_w the inverses of V and W . Let

$$G_v(x) = \int_0^{t_v(x)} g(t) dt \quad \text{and} \quad G_w(x) = \int_0^{t_w(x)} g(t) dt$$

and note that

$$\begin{aligned}\int_0^\tau b(V(t))g(t) dt &= \int_{-\infty}^\infty b(x)t'_v(x)g(t_v(x)) dx = \int_{-\infty}^\infty b(x)dG_v(x) \\ \int_0^\tau b(W(t))g(t) dt &= \int_{-\infty}^\infty b(x)t'_w(x)g(t_w(x)) dx = \int_{-\infty}^\infty b(x)dG_w(x)\end{aligned}$$

and

$$\int_0^\tau V(t)g(t) dt = \int_{-\infty}^\infty x dG_v(x) = \int_{-\infty}^\infty x dG_w(x) = \int_0^\tau W(t)g(t) dt$$

Therefore, effectively we need to show that the distribution G_v second order stochastically dominates G_w . Let us show that

$$\int_{-\infty}^x G_w(y)dy \geq \int_{-\infty}^x G_v(y)dy,$$

which would imply second order stochastic dominance.

Let \underline{v} be the left endpoint of the support of G_v . Note that $G_w(\underline{v}) \geq G_v(\underline{v}) = 0$. Moreover, at any point x such that $G_v(x) = G_w(x)$ we have $t_v(x) = t_w(x)$ and

$$G'_v(x) = t'_v(x)g(t_v(x)) = g(t_w(x))/F'(x) > t'_w(x)g(t_w(x)) = G'_w(x).$$

Therefore, there is a critical point $x^* \geq \underline{v}$ such that $G_w(x) > G_v(x)$ for $x < x^*$, $G_w(x^*) = G_v(x^*)$ and $G_w(x) < G_v(x)$ for $x > x^*$, for all x in the support of G_v . Therefore, for all $x \leq x^*$,

$$\int_{-\infty}^x G_w(y)dy \geq \int_{-\infty}^x G_v(y)dy,$$

and for all $x > x^*$,

$$\int_{-\infty}^x G_w(y)dy = \int_{-\infty}^\infty G_w(y)dy - \int_x^\infty G_w(y)dy \geq \int_{-\infty}^\infty G_v(y)dy - \int_x^\infty G_v(y)dy = \int_{-\infty}^x G_v(y)dy.$$

This completes the proof of second order stochastic dominance. \square

Computation.

Here we explain how to compute solutions to equation

$$rb(W) = \lambda X + (rW - \lambda X - \mu)b'(W) + \lambda(b(W + X) - b(W)), \quad (15)$$

with boundary conditions $b(0) = L$ and $b((\mu + \lambda X)/r + w) = -w$. First, on the interval $[(\mu + \lambda X)/r - X, (\mu + \lambda X)/r]$ equation (15) has a family of solutions

$$b(W) = \frac{\mu + \lambda X}{r} - W - C \left(\frac{\mu + \lambda X}{r} - W \right)^{\frac{r+\lambda}{r}}.$$

To solve equation (15), we pick a constant C to obtain the solution on the interval $[(\mu + \lambda X)/r - X, (\mu + \lambda X)/r]$ and continue solving on $[0, (\mu + \lambda X)/r - X]$ using a numerical method. Our guess of C is correct if the resulting solution reaches L at $W = 0$, too large if $b(0) > L$ and too small if $b(0) < L$. Therefore, we can pick the right constant C using a binary search method.

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