

## Rubinstein Bargaining

Alternating-offer game, in which players are trying to split one unit and the game ends when an offer is accepted. Preferences over outcomes satisfy

(A-1) More is better

(A-2) Sooner is better

(A-3) Stationarity: preferences over outcomes between times  $t$  and  $t+1$  independent of  $t$

(A-4) Continuity

(A-5) The compensation  $f_i(s)$  to wait for one period (so a player  $i$  is indifferent between  $s$  now and  $s + f_i(s)$  after one period) is non-decreasing in  $s$ .

Examples: If each player bears a fixed bargaining cost per period ( $c_1$  and  $c_2$ ), or if each player has a fixed discount factor ( $\delta_1$  and  $\delta_2$ ), then the preferences satisfy (A-1) through (A-5).

Q1: Are there any other preferences that satisfy (A-1) through (A-5)?

Notation: an offer  $s \in [0, 1]$  means a proposed split  $(s, 1-s)$  to players 1 and 2.

$A$  is the set of SPE payoffs of player 1 if he makes the first offer

$B$  is the set of SPE payoffs of player 2 if he makes the first offer

Q2: Is it true that if  $a \in A$  and the first offer is accepted in a corresponding SPE, then there is  $b \in B$  such that 2 prefers  $1 - a$  today than  $b$  tomorrow?

Q3: Is it true that if  $a \in A$  and the first offer is rejected in a corresponding SPE, then there is  $b \in B$  such that 2 prefers  $1 - a$  today than  $b$  tomorrow?

Q4: Is it true that if  $a \in A$ , then there exists  $b \in B$  such that player 2 weakly prefers  $b$  tomorrow rather than  $1 - a$  today?

Q5: Is it true that  $A$  and  $B$  are intervals?

Q6: Prove that if  $a < 1$ , we have  $a \in A$  if and only if there exists  $b \in B$  such that 2 is indifferent between  $1 - a$  now and  $b$  next period, i.e.  $b = (1-a) + f_2(1-a)$ .

Q7: Suppose that a pair  $(a, b)$  satisfies two properties:

- 2 is indifferent between  $1-a$  currently and  $b$  in the next period
- 1 is indifferent between  $1-b$  currently and  $a$  in the next period

Construct a SPE that has value 1 to player 1 (that is, specify all equilibrium offers after all histories, and state which offers are accepted and which are rejected).