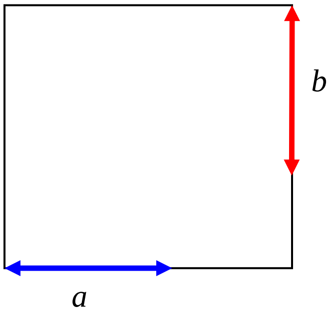
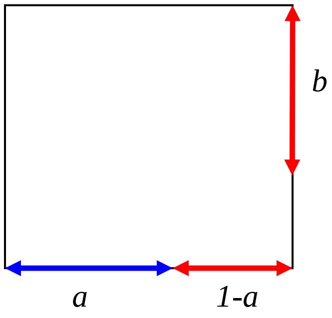
Rubinstein: a geometric interpretation.

Let me draw a unit square.

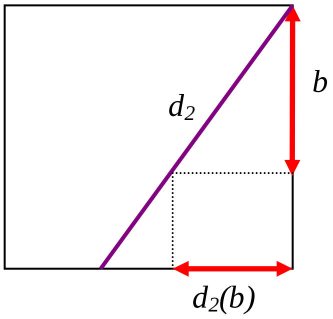




On the horizontal axis, I will represent the SPE payoffs of player 1 when he gets to move first. On the vertical axis, payoffs of player 2 when he gets to move first.

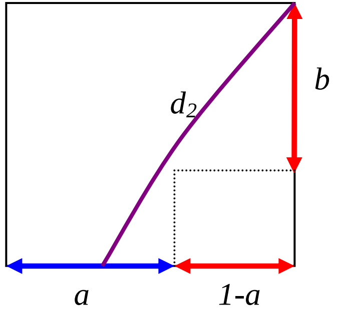
Then, if player 1 makes an offer of a in period 1, player 2 gets $1-a$ if he accepts. If player 2 rejects, he may be expecting an SPE payoff $b \in B$ in the next period. Will player 2 accept or reject?

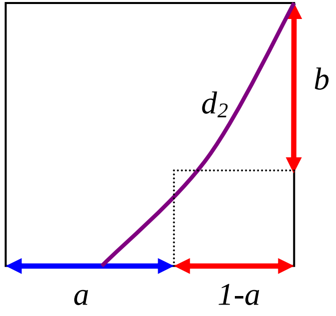
To find out, let us draw a curve d_2, which marks the set of points "indifference" points for player 2:



A point on the curve indicates that player 2 is indifferent between $d_2(b)$ currently and b in the next period. Then by axiom 5, the slope of the indifference curve is 1 or steeper. That is to compensate player 2 for one cent today, he needs one cent or more tomorrow.

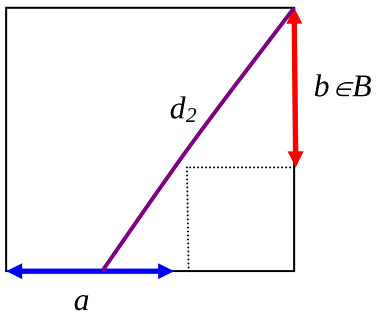
We can graphically determine whether player 2 is supposed to accept the offer or reject.


Reject


Accept

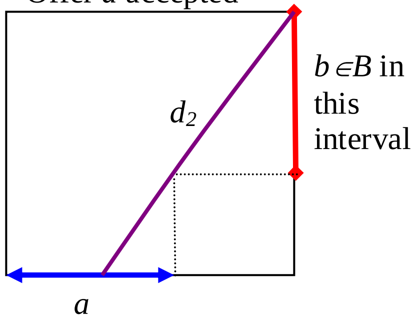
Now, recall that *A* denotes the set of SPE payoffs of player 1 when he acts first, and *B* denotes the set of SPE payoffs of player 2 when he acts first.

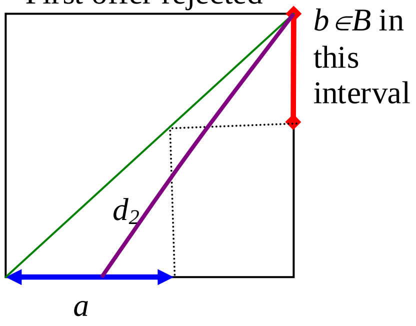
If player 1 gets a payoff $a \in A$, then he must know that player 2 will reject any offer bigger than *a*. That is player 2 must have reasons to reject any such offer, because he is expecting a sufficiently good SPE payoff $b \in B$ if he rejects. This situation is shown in the figure below. This gives us Lemmas 1 (for player 2) and 2 (for player 1) of Rubinstein.



So, $a \in A$ implies that 2 has payoffs sufficiently good to reject anything bigger than *a*.

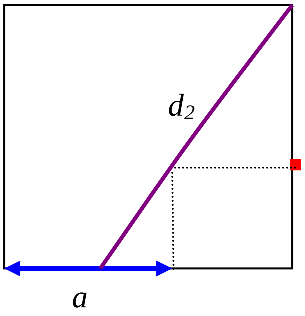
Also, if player 1 gets a payoff of *a* then either he offers *a* and the offer is accepted, or his first offer (which may not be *a*) is rejected. If *a* is accepted then player 2 must have *b* tomorrow that's worse than *a* today (left panel). If the first offer is rejected and player 1 gets *a*, then player 2 gets tomorrow something that's clearly worse than *a* today (since 1 must get at least *a*).

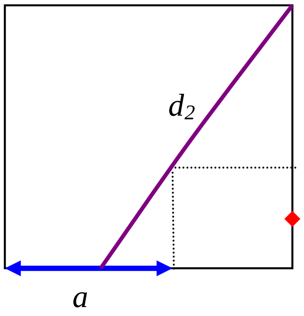

Offer *a* accepted

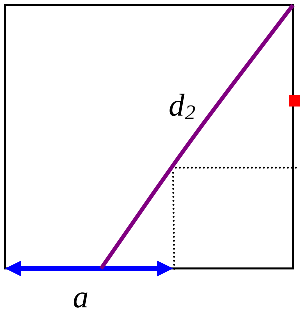

First offer rejected

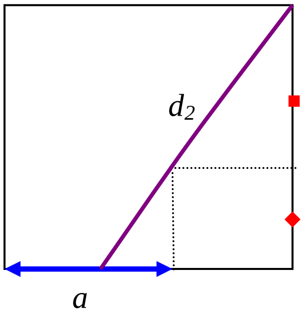
In either case, there is $b \in B$ in the interval on the left panel. This gives Lemmas 3 & 4.

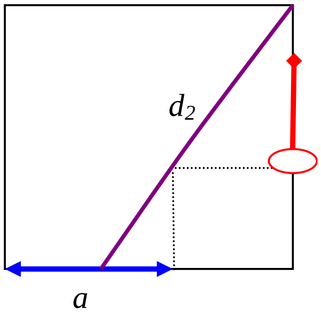
Now, here are a few examples of sets *B*. Given the logic above, and given $a \in A$, which sets are OK?

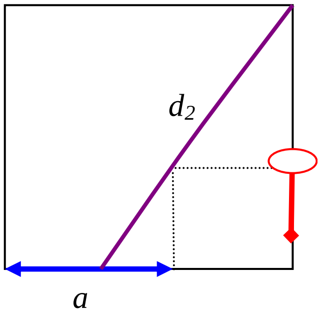




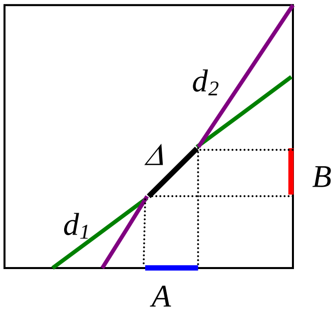








Now, let us draw both curves d_1 and d_2 and consider the set where they intersect (assume they do, for simplicity, although in the example of fixed waiting costs per period $c_1 \neq c_2$ they do not intersect).



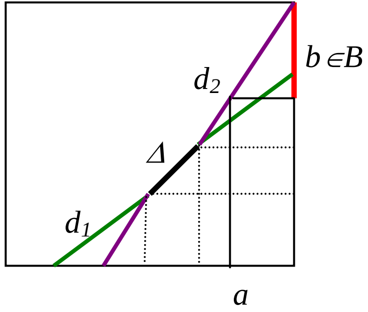
Then the main result of Rubinstein (1982) is that the sets A and B are defined as shown in the picture. To show this, we need to

1. construct an equilibrium for each point in Δ (exercise in class)
2. prove that something outside the projection of Δ on the horizontal or vertical axis cannot give a SPE payoff

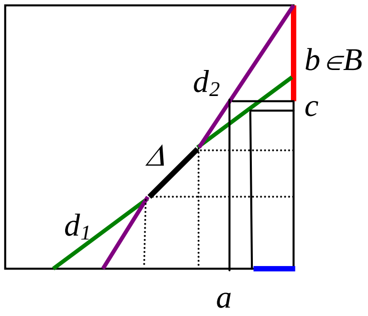
Suppose that there are SPE payoffs of player 1 to the right of the projection of Δ on the horizontal axis. Let s be sup A. Let $a \in A$ be a point sufficiently close to s. Then we want to show (using the lemmas) that (i) player 2 has something sufficiently bad in B and

(ii) player 1 has something sufficiently good in A, which is even better than s, which gives a contradiction.

Player 2 has $b \in B$ in red interval



But then player 2 must know that if he offers c, player 1 will reject it. Therefore, player 1 must have an equilibrium payoff *d* in the blue interval:



This is a contradiction, since we assumed that *a* is very near the sup of A.

Similarly, we can prove that nothing to the left of the projection of Δ can be a SPE payoff of player 1.