Rubinstein Bargaining

Alternating-offer game, in which players are trying to split one unit and the game ends when an offer is accepted. Preferences over outcomes satisfy

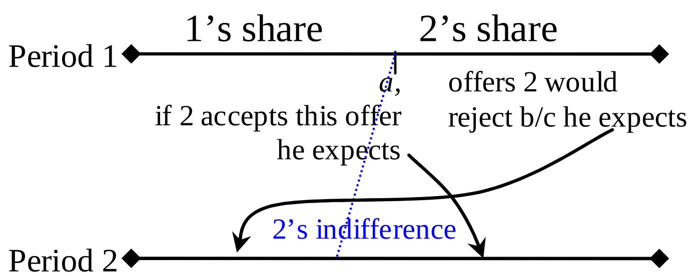
(A-1) More is better
(A-2) Sooner is better
(A-3) Stationarity: preferences over outcomes between times t and $t+1$ independent of t
(A-4) Continuity
(A-5) The compensation $f_i(s)$ to wait for one period (so a player i is indifferent between s now and $s + f_i(s)$ after one period) is non-decreasing in s.

Notation: an offer $s \in [0, 1]$ means a proposed split $(s, 1\text{-}s)$ to players 1 and 2.
A is a set of SPE payoff if player 1 starts bargaining, B if player 2.

Lemma. If $a \in A$ then there is
1. $b \in B$ at least as bad for 2 tomorrow as a today

If in addition $a < 1$ then

2. $b \in B$ at least as good for 2 tomorrow as any offer worse than a today



if 2 rejects period 1 offer and pl.1 has equilibrium payoff *a*, then pl. 2's equilibrium payoff in the subgame after rejection is worse than *a*

Proof.
1. If 2 accepts the current offer, then the current offer must be *a* and 2 must expect something a payoff of *a* or worse in the next period. If 2 rejects the current offer, and 1 gets value *a* from the game, then 2 must a payoff worse than current *a* from the game.
2. Player 2 must be able to reject any offer worse than *a*, otherwise 1 will get more than *a* in equilibrium.

<u>Logical exercise</u>. Suppose $a < 1$, and from 1 and 2 we conclude that there exists $b \in B$ that makes player 2 just indifferent between *a* now and *b* in the next period. Is this argument correct? Why (or why not)?

However, we can argue that A and B are intervals. Then we can conclude that for $a < 1$, we have $a \in A$ if and only if there exists $b \in B$ such that 2 is indifferent between a now and b next period, i.e. $(1-b) = (1-a) + f_2(1-a)$.

Define $d_2(b) = a$ if $(1-b) = (1-a) + f_2(1-a)$ and $d_2(b) = 1$ if $1-b < f_2(0)$. Then d_2 gives a that 2 can replace for b tomorrow and be indifferent. Similarly, define d_1. Then d_1 and d_2 have slopes ≤ 1.

The characterization of SPE payoffs is in terms of the set

$$\Delta = \left\{ (x, y) \in S \times S \mid \begin{matrix} d_1(x) = y \\ d_2(y) = x \end{matrix} \right\}$$

Proposition 1. For any $(x,y) \in \Delta$, there is an SPE in which

- in odd periods player 1 offers x and player 2 accepts
- in even periods, player 2 offers y and player 1 accepts

Propositions 2 and 3. The graph of Δ is a nonempty closed line segment parallel to the diagonal y = x.

How to prove: Parallel, because f_1 and f_2 are non-decreasing. Closed, because preferences are continuous.

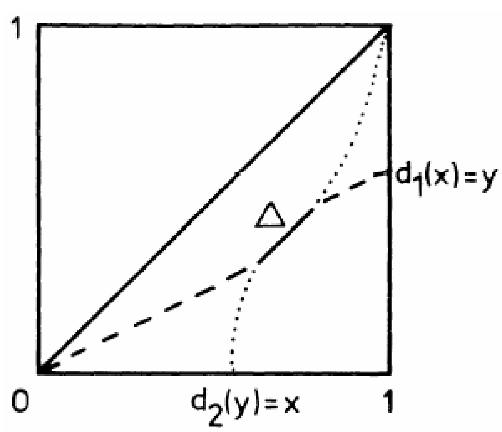


FIGURE 1

Let Δ_1 and Δ_2 be the projections of Δ.

Proposition 4. If $a \in A$ then $a \in \Delta_1$ and if $b \in B$ then $b \in \Delta_2$.

Proof. Suppose $\Delta_1 = [x_1, x_2]$ and $\Delta_2 = [y_1, y_2]$. Let s = sup $\{a \in A\}$. Suppose $x_2 < s$. Then $d_2(d_1(s)) < s$. Let $a \in A$ satisfy $r = d_2(d_1(s)) < a < s$. Because $a \in A$, then there is $b \in B$ equally bad or worse for 2 tomorrow than *a* today, that is $d_2(b) \geq a$. If there is $b \in B$ so bad, then there is $c \in A$ at least as good as for 1 tomorrow as *b* today. So, $d_1(c) \geq b$, which implies that $d_2(d_1(c)) \geq a$. But then c > s, contradiction.

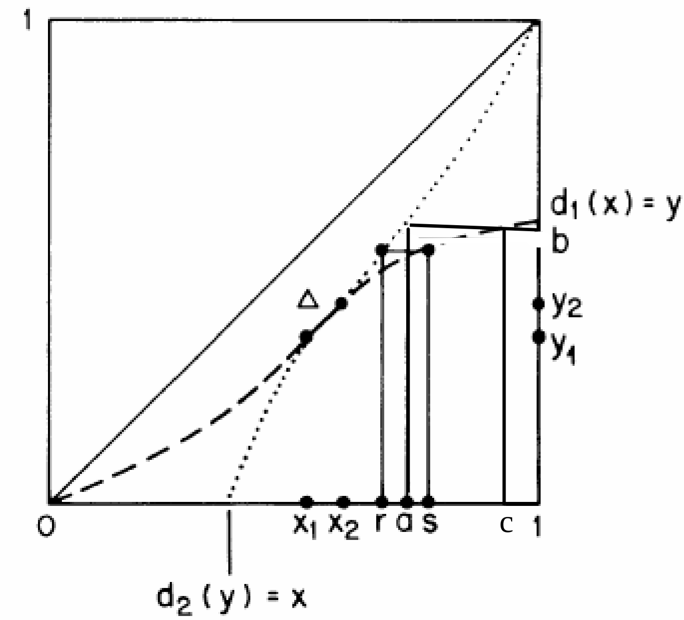


FIGURE 2

The characterization of Proposition 4 allows us to easily solve the game for arbitrary preferences. In particular

- If every player bears a fixed bargaining cost per period (c_1 and c_2) then (i) if $c_1 < c_2$ the only SPE gives all the pie to 1 (ii) if $c_1 > c_2$ the only SPE gives 1 only c_2.
- If each player has a fixed discount factor (δ_1 and δ_2), then player 1 gets $(1-\delta_2)/(1-\delta_1\delta_2)$