Economics 209A
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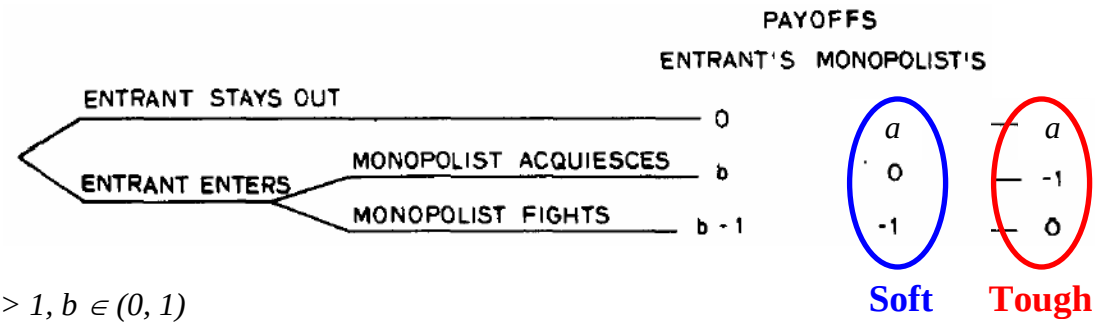
Kreps and Wilson (1982) "Reputation and Imperfect Information."

Chain-store game:



$a > 1, b \in (0, 1)$

Assume that the game is repeated N times with different entrants in each period and the same monopolist. Each entrant cares about his payoffs only in his period. The monopolist cares about the sum of payoffs across all periods. Entrants initially believe the monopolist to be tough with probability $\delta > 0$.

Sequential equilibrium.

Beliefs;

$p_N = p_n = \delta$ if there has been no entry, where n is the number of remaining periods
$p_n = 0$ if there has been an entry which was met by acquiescence. Otherwise…
$p_n = \max(b^{k-1}, \delta)$, where k is the last stage in which entry followed by fighting

Monopolist's strategy:

- Strong monopolist always fights
- The response of weak monopolist to entry depends on n and p_n
  - If n=1, acquiesce
  - If n > 1 and $p_n \geq b^{n-1}$, fight
  - If n > 1 and $p_n < b^{n-1}$, fight with probability such that the posterior upon fighting rises to b^{n-1}

Question: What is this probability?

Entrant's strategy:

- If $p_n > b^n$, entrant n stays out.
- If $p_n < b^n$, entrant n enters
- If $p_n = b^n$, entrant n randomizes, staying out with probability $1/a$.

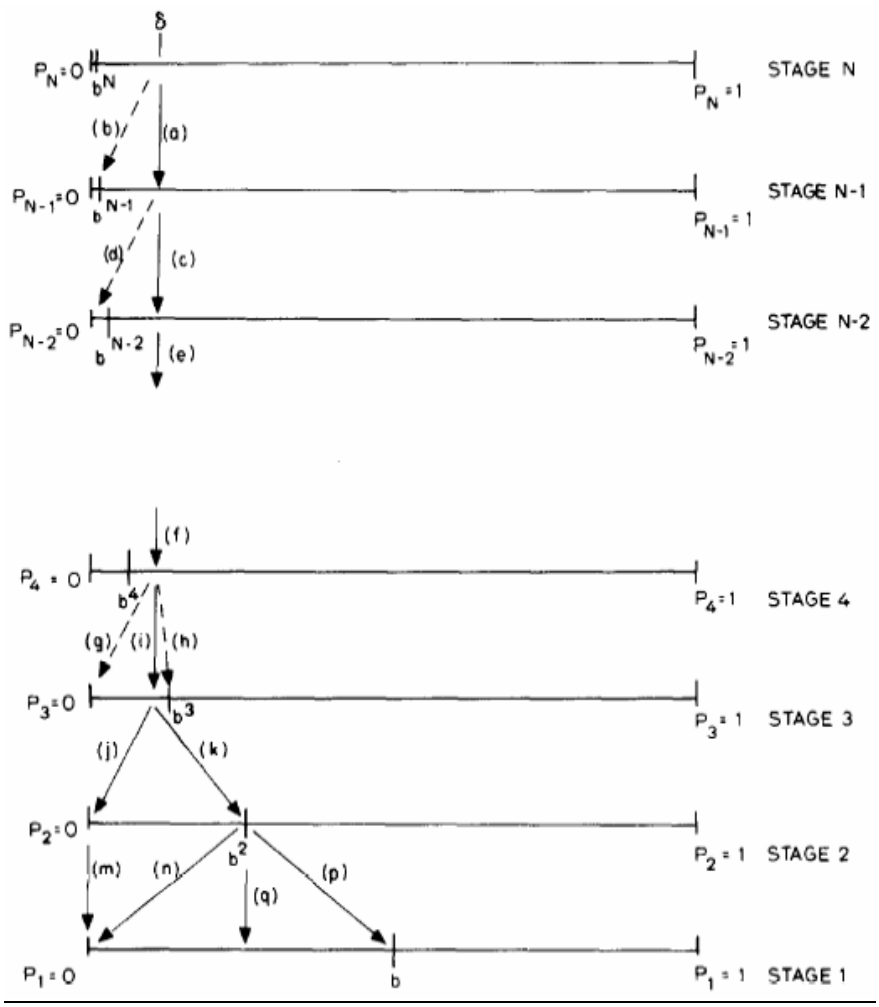
Verification:

- Beliefs are formed by Bayes rule when possible (Bayes rule does not apply if monopolist fights when $p_n = 0$ or acquiesces when $p_n \geq b^{n-1}$)

- Entrant is indifferent between entering or not if and only if $p_n = b^n$, i.e. when the monopolist acquiesces with total probability (1-b)
- Weak monopolist: when he mixes, acquiescing gives a payoff of 0. Fighting gives a payoff of -1 currently, and deters entry for one period with probability 1/*a*. (which gives a payoff of *a*). Therefore, monopolist is indifferent.

<u>Equilibrium play:</u>

Start with $\delta = 1/10$ and b = ½. Then $\delta \in (1/2^4, 1/2^3)$. Entrant enters for the first time in stage 3. Had entrant entered in stage 4, monopolist would mix since $\delta < 1/2^{4-1}$. In stage 3, the monopolist mixes, and conditional on fighting his reputation rises to $1/2^2$, etc.



<u>Is this sequential equilibrium unique?</u>

No. For example, let N = 2, b = ½, δ = 2/3. Then there is also this sequential equilibrium: in stage 2 entrant enters and the monopolist acquiesces. In stage 1, entrant does not enter unless the monopolist fought in the previous stage (then entrant believes the monopolist is weak and enters).

Question: Does this equilibrium satisfy the intuitive criterion?

Instead of intuitive criterion, Kreps and Wilson in introduce the concept of *plausible beliefs*: If histories h_n and h_n' are the same except that some plays of "fight" in h_n are "acquiesce" in h_n', then $p_n(h_n) \geq p_n(h_n')$.

Question: Does the intuitive criterion imply that beliefs are plausible?

Answer: No. Consider the 10-stage game above, and let us modify the equilibrium by specifying that the entrant must enter in the initial stage, and both types of monopolist acquiesce. Thereafter, the game proceeds as in the standard equilibrium, but had the monopolist fought in the initial stage, let the entrants believe that he is weak and enter in all periods. Let us show that these beliefs satisfy the intuitive criterion. Indeed, both the weak and strong monopolist would have fought in the initial stage if it resulted in belief $p = 1$ and deterred entry for all periods. Therefore, the intuitive criterion has no bite.

Proposition 2. *If $\delta \neq b^n$ for $n \leqslant N$, then every sequential equilibrium with plausible beliefs has on-the-equilibrium-path strategies as described previously. Thus every sequential equilibrium with plausible beliefs has the value functions given above.*

Two-sided uncertainty:

Weak entrant: $b \in (0, 1)$, strong entrant: $b > 1$ (strong with probability γ)

Look for an equilibrium where entrant keeps entering, incumbent keeps fighting, until one of them acquiesces and reveals himself to be weak.

Game in continuous time. Time $t \in [T, 0]$. We look for an equilibrium where

(1) The strong monopolist always fights
(2) The strong entrant always enters
(3) If the monopolist acquiesces, it is revealed to be weak
(4) If the entrant fails to enter, it is revealed to be weak

Note that if the entrant is weak for sure, he will always stay out given that the monopolist is strong with a positive probability. If the monopolist is weak for sure, he will always accommodate, given the entrant is strong with a positive probability.

The following figure characterizes equilibrium:

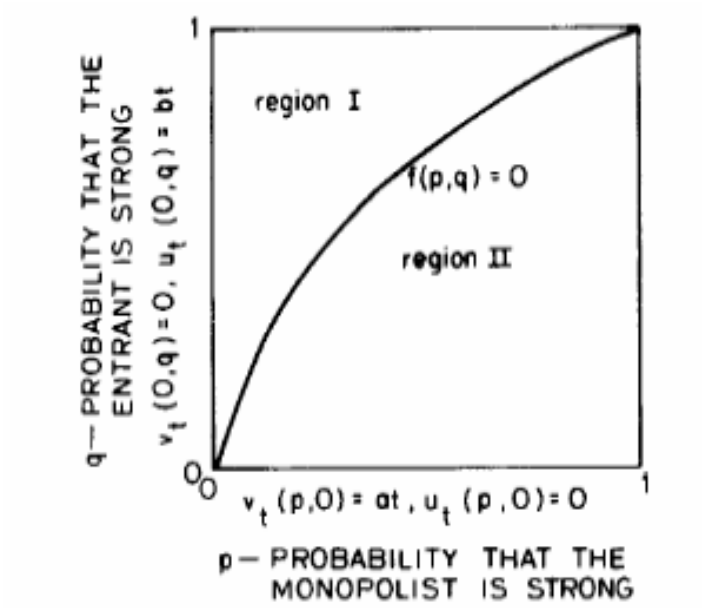


FIG. 5. State space of the continuous-time game.

If the pair (δ, γ) is below the "balanced path" then the entrant is supposed to concede with positive probability right away so γ jumps up to the "balanced path." Thereafter, players concede continuously and at time t=0 the posterior probability that each player is strong becomes 1. Note that games with different T are strategically equivalent.

<u>Exercise:</u> (Kreps and Milgrom meet Morris and Shin) Can you think about an game, where entrants have different information about the type of an incumbent, and an entrant may fail to enter even though it is sure that the incumbent is weak (because he expects the incumbent to fight in order to "manipulate" other entrants, who may not be sure)?

Remark: I think one could develop a good research paper out of this exercise.