<u>Nash Bargaining</u>

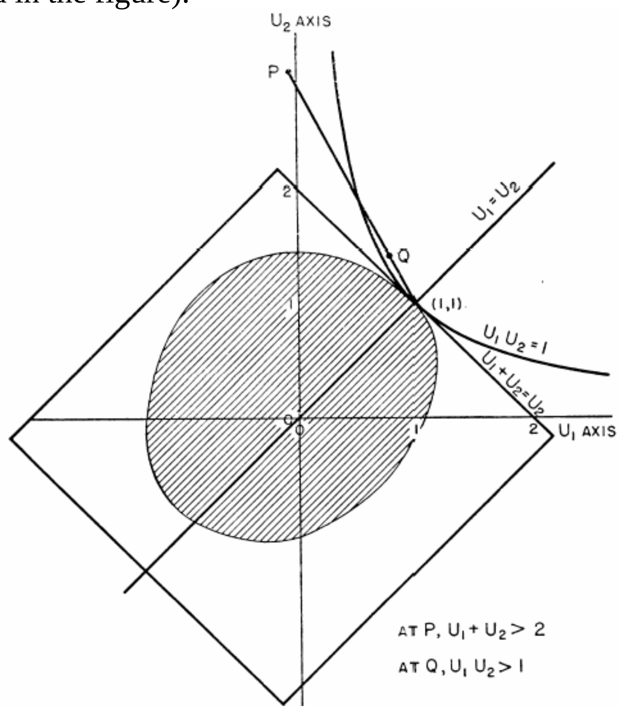
Remark: If u_1 represents the preferences of player 1, then a scaled version αu_1 represents the same preferences.

Let $S \subset R^2$ be a convex and compact set of outcomes in the space of utilities of two players, over which they are bargaining. $(0, 0) \in S$ be the point where the players end up if they fail to agree. We would like to characterize a natural choice $c(S) \in S$ at which the players could arrive through a bargaining process. Assume three natural axioms that we want c(S) to satisfy:

6. If $\alpha \in S$ is strictly Pareto dominated by $\beta \in S$, then $c(S) \neq \alpha$.
7. If c(T), chosen from T, is contained in $S \subset T$, then it would also be chosen from S.
8. If S is symmetric (in some utility representation u_1, u_2), c(S) is of the form *(a,a)*.

Of these axioms, 6 and 8 are relatively straightforward to motivate, but 7 is less so. To motivate 7, if the players know that from T they will choose c(T) and $c(T) \in S \subset T$, then they could also first agree not to consider T\S (because they know that they will not choose those options), and then choose out of a smaller set S. It is intuitive that the choice should be the same.

Let us show that c(S) is the point that maximizes u_1u_2. Without loss of generality, let us scale utilities u_1 and u_2 so that the coordinate of the point that maximizes u_1u_2 is (1,1). Then for any $(u_1,u_2) \in S$, $u_1+u_2 \leq 2$ (otherwise (1,1) does not maximize the product of utilities, as illustrated in the figure).



Because S is bounded, it is a subset of a symmetric square with one of the sides on the line $u_1+u_2 = 2$. The choice from the square must be (1,1) by 6 and 8. Therefore, 7 implies that (1,1) = c(S).