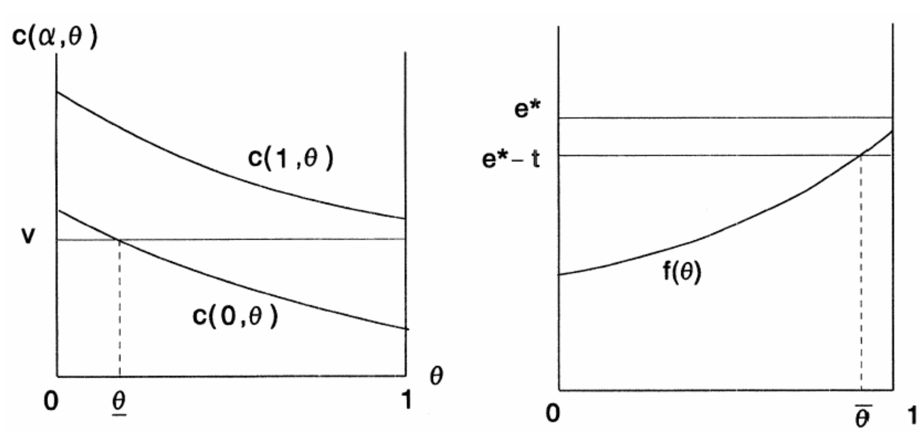
Morris and Shin (1998)

Currency attacks are typically explained by the self-fulfilling nature of beliefs. If speculators believe that an attack will happen, they will all attack, and if they believe it will not happen, they will not attack. However, this explanation is completely useless in explaining the timing of the attack, giving prescription how to curb it, or explaining where the beliefs come from. If one assumes a more sophisticated model of how beliefs are formed, then one can easily get a unique equilibrium in a related model.

The model is concerned with strategic interaction between the government and a group of speculators in foreign exchange markets. The state of fundamentals θ is uniformly distributed on the interval [0, 1], and the natural exchange rate $f(\theta)$ is an increasing function of fundamentals. That is, higher values of θ correspond to "stronger fundamentals." The currency is initially pegged at $e^* \geq f(\theta)$ for all θ. There is a unit mass of speculators. A speculator may attack the currency or not. A speculator gets the payoff of 0 if she does not attack. Attacking means selling short one unit of currency, which entails a transaction cost of t. If the speculator attacks, his payoff is $e^* - f(\theta) - t$ if the currency collapses and $-t$ otherwise. The speculators do not know the true state of fundamentals. Each speculator sees a signal x uniformly distributed on $[\theta-\varepsilon, \theta+\varepsilon]$. The signals are distributed independently across speculators. After observing her private signal, each speculator decides whether to attack or not.

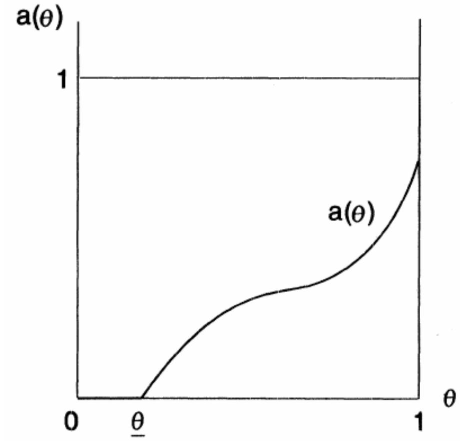
After seeing the mass α of speculators who attack, the government decides whether to defend the exchange rate or not. The government derives value $v > 0$ from defending the exchange rate, but has to pay a cost of $c(\alpha, \theta)$. Function c is continuous, increasing in α and decreasing in θ. Furthermore, functions c and f satisfy the following properties: $c(0, 0) > v$, $c(1,1) > v$, $e^* - f(1) < t$. An example of such functions is shown in the following two figures:



Define by $\underline{\theta}$ and $\overline{\theta}$ as shown in the figures. The government will not defend the currency even if nobody attacks if $\theta < \underline{\theta}$. A speculator would get a negative payoff from attacking if $\theta > \overline{\theta}$ even if the currency is devalued for sure. Assuming by $\underline{\theta} < \overline{\theta}$, the following three regions have an intuitive interpretation:

- Currency *unstable* if $\theta \in [0, \underline{\theta}]$
- Currency *stable* if $\theta \in [\overline{\theta}, 1]$
- Currency *ripe for attack* if $\theta \in (\underline{\theta}, \overline{\theta})$

Denote by a(θ) the critical mass of speculators who would force the government to abandon the peg, which is defined by a(θ) = 0 in the unstable region and c(a(θ), θ) = v everywhere else. A typical form of a is shown in the following figure:



Assume that

- if a speculator is indifferent between attacking or not, she will not attack
- if the government is indifferent between defending or abandoning, it will abandon

Define a speculator's strategy by π(x) = 0 or 1, where 1 denotes the action to attack. When the true state of fundamentals is θ, the fraction of speculators who attack is

$$s(\theta, \pi) = \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x)\, dx.$$

The currency collapses on the set $A(\pi) = \{\theta \mid s(\theta,\pi) \geq a(\theta)\}$.

The expected payoff from attacking the currency is given by

$$u(x, \pi) = \frac{1}{2\varepsilon} \left[\int_{A(\pi)\cap[x-\varepsilon, x+\varepsilon]} (e^* - f(\theta))\, d\theta \right] - t.$$

The rational decision is to attack if and only if u(x, π) > 0.

Unique Equilibrium.

Theorem 1. *There is a unique value of θ^* s. t. the currency collapses if and only if $\theta \leq \theta^*$.*

Lemma 1. *If $\pi(x) \geq \pi'(x)$ for all x, then $u(x, \pi) \geq u(x, \pi')$ for all x.*

Sketch of proof. If $\pi(x) \geq \pi'(x)$, then for any true state of fundamentals θ, the currency is more likely to collapse if everybody plays π than if everybody plays π′. Thus, payoff from attacking is greater under π than under π′ for a speculator with any signal x. QED

Denote by $u(k, I_k)$ the utility from attacking to a speculator with signal k when other analysts attack if and only if they get signal less than k. Then

Lemma 2. *$u(k, I_k)$ is continuous and strictly decreasing in k.*

Sketch of proof. For any k, denote by $[0, \theta_k]$ the interval of fundamentals on which the government abandons the peg. It is easy to see that $d\theta_k/dk \leq 1$. Then

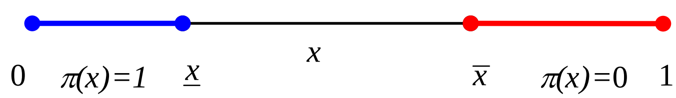
$$u(k, I_k) = \frac{1}{2\varepsilon}\int_{k-\varepsilon}^{\theta_k} (e^* - f(\theta)) - t$$

is decreasing in k, because for larger k the interval $[k - \varepsilon, \theta_k]$ becomes smaller and the payoff from attacking on this interval also becomes smaller.
QED

Lemma 3. *There is a unique x^* such that, in any equilibrium of the game with imperfect information of the fundamentals, a speculator with signal x attacks if and only if $x < x^*$.*

Proof. First, $u(k, I_k) = 0$ has a unique solution because $u(1, I_1) < 0$ (because the payoff from attack is negative even if the currency collapses for sure) and $u(0, I_0) > 0$ (because the government abandons the peg for sure).

Denote $\underline{x} = \inf\{x \mid \pi(x) = 0\}$ and $\overline{x} = \sup\{x \mid \pi(x) = 1\}$:



Then $\underline{x} = \overline{x}$. If not, then one can get a contradiction easily from the fact that speculators with signals $\underline{x}$ and $\overline{x}$ are indifferent between attacking or not:

$$0 = u(\underline{x}, \pi) \leq u(\underline{x}, I_{\underline{x}}) < u(\overline{x}, I_{\overline{x}}) \leq u(\overline{x}, \pi) = 0, \text{ a contradiction.}$$

We conclude that $\underline{x} = \overline{x} = x^*$, which is defined by $u(x^*, I_{x^*}) = 0$. QED

If in equilibrium a speculator attacks if and only if he gets a signal $x < x^*$, then θ^* is easily found graphically from the following figure: