Assume that there is a continuum of entrepreneurs with types from the interval $[\underline{\mu}, \overline{\mu}]$.
An entrepreneur with type μ ∈ $[\underline{\mu}, \overline{\mu}]$ generates returns that are distributed N(μ, σ^2).
Assume that the market is risk-neutral, but cannot observe the entrepreneur's type. The entrepreneur is risk-averse with exponential utility, i.e. the certain equivalent of x ~ N(m, s^2) is

$$m - \tfrac{b}{2} s^2$$

Each entrepreneur decides what fraction α of equity to sell, the market infers the type of the entrepreneur from α, and pays him $\alpha \hat{m}(\alpha)$, where $\hat{m}(\alpha)$ is the inferred type. The entrepreneur's utility is given by

$$(1-\alpha)\mu - \tfrac{b}{2}(1-\alpha)^2 \sigma^2 + \alpha \hat{m}(\alpha)$$

Find the separating equilibrium.

F.O.C. gives

$$-\mu - b(\alpha - 1)\sigma^2 + \alpha \hat{m}'(\alpha) + \hat{m}(\alpha) = 0$$

to find the separating equilibrium, we must solve this ODE with BC

$$\hat{m}(1) = \underline{\mu} .$$

$$-b(\alpha - 1)\sigma^2 + \alpha \hat{m}'(\alpha) = 0$$

$$\hat{m}'(\alpha) = \frac{b(\alpha - 1)\sigma^2}{\alpha}$$

$$\hat{m}(\alpha) = -b\sigma^2 \ln \alpha + b\sigma^2 \alpha + K$$

$$\hat{m}(1) = \underline{\mu} = b\sigma^2 + K \Rightarrow K = \underline{\mu} - b\sigma^2$$

$$\hat{m}(\alpha) = b\sigma^2((\alpha - 1) - \ln \alpha) + \underline{\mu}$$